Energy Shortages and Aggregate Demand: Output Loss and Unequal Burden from HANK

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Abstract

In this paper I build a quantitative Heterogeneous Agents New Keynesian (HANK) model with energy as a critical production factor to study the effects of a reduction in the energy supply. I find that changes in aggregate demand amplify the macroeconomic effects of the energy shock, but these effects remain manageable. In the model a 10% reduction in the energy supply leads to a Gross National Income (GNI) loss in range between 0.8% and 2%. The economic burden is highly nonlinear across the wealth distribution: most households face similar and contained costs, while low-wealth households bear the heaviest burden. I show that monetary and fiscal policy can mitigate the economic costs and the unequal effects of energy shortages.

Keywords: Heterogeneous agents, New Keynesian, energy, fiscal policy, inequality.

JEL Classification: D31, E32, Q43.

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1 Introduction

Estimates based on structural multi-sector models with internationla trade suggest that a reduction in energy imports can lead to a GNI loss in range between 0.3% and 2.3% for Germany (Bachmann et al. (2022)). These estimates vary substantially depending on the calibration of the elasticities of substitution between inputs in the production processes and distribution parameters. In my analysis I take a different approach and leverage a quantitative HANK model in which energy is a key production input to analyze the macroeconomic effects of energy supply shocks on European economies. The model captures business cycle amplification channels that work through labor market adjustments and changes in the level of aggregate demand.

This paper presents three main findings. First, I provide a range of estimates for the GNI loss due to energy shortages. I find that the GNI loss can be large but manageable. Second, I show that aggregate demand channels generate a substantial amplification of energy shocks. Third, I use the model to quantitatively illustrate how an energy shortfall affects household income and consumption across the wealth distribution. The economic burden falls heavily on low-wealth households while other households face similar and relatively more contained costs. I also show that monetary and fiscal policies can substantially mitigate the aggregate income loss and the economic cost faced by households with low income and wealth.

First of all, I find that in the model energy shortages leads to a GNI decline in range between 0.8% and 3.4%. Given the uncertainty surrounding aggregate elasticities of substitution I use the model to analyze different scenarios, including several extreme and conservative cases. In all the simulations the elasticity of substitution between energy and other production inputs is at the lower bound of the empirical estimates. Table 1 reports the main quantitative results from the model for different values of the elasticity of substitution σ. In the more likely outcomes, the energy shortfall leads to a GNI decline between 0.8% and 2%. Throughout these simulations low-income households bear the highest cost.

Table 1: Model simulations

GNI Loss	$\sigma = 0.1$	$\sigma = 0.07$	$\sigma = 0.2$	Fossil gas only
HANK model	1.5%	2%	0.8%	3.4%
CES function	0.6%	0.8%	0.5%	2.3%

Note: The first three columns show the effects of a 10% reduction in the overall energy use. The last column shows the effect of a 30% reduction in the energy inputs from natural gas. In the last column the elasticity of substitution $\sigma = 0.16$.

In the model I focus on the role of energy consumption by firms and assume an exogenous supply of energy. This allows me to keep the model as simple as possible and isolate the role of Marginal Propensities to Consume (MPC) and sticky prices in the amplification of energy shocks. I calibrate several parameters externally following the HANK literature. In the calibration of the remaining parameters I rely on the existing literature based on European micro data. For the parametrization of the elasticity of substitution between inputs and the distribution parameter I follow the empirical literature measuring the elasticity of energy demand. The assumption of an aggregate Constant Elasticity of Substitution (CES) production function allows me to map these estimates directly into the aggregate elasticity of substitution between inputs σ. Overall, the model generates a sizable average MPC and a realistic share of low-liquidity households.

In the main experiment of this paper I simulate a 10% shortfall in the overall energy consumption. This could be the size of the energy supply shock for the German economy (Bachmann et al. (2022)) and for Italy (Bank of Italy, Economic Bulletin, 2, 2022). Given this shock I solve the fully-nonlinear model and compute the impulse response functions of the main equilibrium variables under different parametrizations of the energy elasticity of substitution.[1] The choice of these values is guided by the empirical estimates of energy demand elasticity. In all the simulations I set the elasticity of substitution between energy and other production inputs at the lower bound of the empirical estimates. The resulting income loss for each case is reported in Table 1. Then, for each parametrization I compute a simple counterfactual output loss in absence of the demand amplification features of the quantitative HANK model. In particular, I obtain these counterfactuals by feeding the energy shock in a simple CES production function calibrated as in the quantitative HANK model. This allows me to quantify the amplification effect in all the different scenarios. Table 1 shows that across all parametrizations the GNI response is substantially lower than in the HANK model. In particular, the amplification effect increases with the severity of the recession due to the energy supply shortfall. In the more optimistic case the income loss increases by 0.3 percentage points, in the more pessimistic scenarios the income loss increases by 1.2 percentage points. This implies that heterogeneity in the household Marginal Propensity to Consume (MPC), labor market adjustments, and price rigidities amplify the effects of the energy shock. However, even in the worst-case scenario in the last column of Table 1 the economic cost remains manageable. Finally, I study the cross-sectional predictions of the HANK model to quantify the distributional effects of an energy shortage. I find highly nonlinear effects across the wealth distribution with consumption losses concentrated at the bottom of the distribution. The reason is that these households also experience the largest drop in labor income, and relative to other households they do not have enough wealth to face the income shock and need to reduce consumption expenditures. To the best of my knowledge this is the first paper to document this unequal response of household consumption to an energy shortfall in the HANK framework.

[1] To solve the Hamilton-Jacobi-Bellman (HJB) equation and the Kolmogorov Forward (KF) equations I rely on the algorithm of Achdou, Han, Lasry, Lions, and Moll (2022).

I also consider several extreme and conservative scenarios as well as robustness checks. First, I model the energy input as representing only gas. Therefore, following the calculations in Bachmann et al. (2022) I consider a 30% energy shortage. The result is a GNI loss of 3.4%. In this case the energy shock can cause a very large recession. However, gas is not the only energy input and the assumptions that support this scenario are rather extreme. In the same spirit I consider a counterfactual in which the energy shortage lasts for a long period of time. Moreover, I assume that less than 60% of the energy shortfall can be reversed after 1 year from the shock. This generates a consumption drop of 2.7%, in line with the most pessimistic scenario on the elasticity of substitution considered in the paper ($\sigma = 0.05$). Finally, I also study the sensitivity of the aggregate consumption loss to the size of the MPC. I find that by doubling the average MPC the consumption loss increases by 0.7 percentage points.

In the remaining of the paper I study the macroeconomic effects of monetary and fiscal policies. In particular, I model four distinct policies: interest rate policy, subsidies on energy to industrial consumers, targeted stimulus payments, and an expansion of social insurance programs for low-income households. I find that after an energy supply shock monetary policy faces a trade-off between inflation and economic activity. For example, in the model a more conservative monetary policy stance on inflation by increasing the contraction of output by 0.5 percentage points reduces the increase in inflation by approximately 1 percentage point. However, a more precise quantification of this trade-off crucially depends on the definition of monetary policy rules, and on the effect of the energy shortfall on potential output. Energy subsidies instead are intended to directly address the energy supply problems: subsidies provide incentives to increase energy production that can be used to substitute energy imports and reduce the cost of energy for industrial consumers. Assuming that these policies can reduce the energy supply shortfall by 1% of national energy use, the effects on aggregate production and inflation are quantitatively small. On the other hand, expansions of social insurance programs and stimulus payments targeted to low-income households can substantially mitigate the recession without causing additional inflation. Specifically, I model these policies as fiscal transfers targeted to the most exposed and vulnerable households. However, while stimulus payments increase the consumption expenditures of the recipients, social insurance only reduces the income and consumption losses of these households. It is important to highlight that since both policies target a very small population group these policies do not cause additional inflation. In the baseline calibration with $\sigma = 0.1$ energy subsidies reduce the output loss by 0.18 percentage points. This number is 0.3 with social insurance and 0.6 with stimulus payments. Targeted stimuls payments are the most effective policy against the recession, but also imply an higher cost for the government budget with respect to social insurance mechanisms. In the model the cumulative 1-year cost of stimulus payments is 1% of GNI, while for social insurance the cost is equal to 0.34% of GNI. Therefore, these computational experiments suggest that social insurance targeted to the most vulnerable households can significantly reduce the aggregate demand amplification effects, lower the unequal consequences of the recession, and mitigate the overall economic cost of an energy shortfall.

Finally, it is important to highlight that this study does not consider other important factors that can further amplify or mitigate the macroeconomic effects of energy supply shocks, such as endogenous energy production, direct energy consumption by households, the effects in the global markets, and different combinations of monetary and fiscal policies.

Literature. This paper is related and contributes to two strands of the economic literature. First, it contributes to the literature studying the importance of MPC heterogeneity within HANK models. Second, it is related to the empirical literature on the elasticity of energy demand and to recent quantitative work studying the macroeconomic effects of an energy supply shortfall.

In the first strand of the literature several papers study the amplification or mitigation of aggregate shocks including the effects of monetary and fiscal policies with quantitative HANK models (Alves, Kaplan, Moll, and Violante (2020), Kaplan, Moll, and Violante (2018), Gornemann, Kuester, and Nakajima (2021), Hagedorn, Manovskii, and Mitman (2019), Laibson, Maxted, and Moll (2021), Luetticke (2021), Kaplan and Violante (2021), Auclert, Rognlie, and Straub (2020), Auclert, Rognlie, and Straub (2018), Wolf (2021)). Relative to these papers I introduce energy in the HANK framework to quantify the amplification effect of energy shortages. I also document the unequal consumption and income effects of these type of shocks within the HANK framework.

In the second strand of the literature I closely follow the work by Bachmann et al. (2022). The authors provide a review of the empirical literature on energy demand elasticities (Auffhammer and Rubin (2018), Labandeira, Labeaga, and Lopez-Otero (2017), Steinbuks (2012)). Bachmann et al. (2022) focus mainly on trade and on the macroeconomic implications of supply chains. In this paper I focus more on business cycle effects and aggregate demand amplification channels. I see these different methods as complementary and useful to provide a broader view of the macroeconomics of energy shocks. The estimates for the GNI loss in this paper are somewhat higher than in Bachmann et al. (2022). However, this difference does not change the main conclusions of their study.

Outline. The remaining of the paper is organized as follows. Section 2 presents the model. Section 3 describes the parametrization of the model. Section 4 contains the main quantitative results on the aggregate and distributional effects of energy shocks. I also analyze more conservatice scenarios and the sensitivity of the results to the model calibration. Section 5 shows how monetary and fiscal policies can mitigate the economic cost of an energy shortfall. To this end I study the response of the economy to four distinct policies: interest rate policy, subsidies on energy, stimulus payments, and social insurance mechanisms. Section 6 concludes.

2 The model

This section presents the model that I use for my analysis. I consider an economy in continuous time with incomplete markets and no aggregate risk. Individuals can trade assets a_t, face borrowing constraints, and idiosyncratic labor income risk z_t. Let $M = (X, \mathcal{X})$ be a measurable space where $(a, z) \in X = A \times Z \subseteq \mathbb{R}^2$, $\mathcal{X} = \mathcal{B}(A) \otimes P(Z)$ is the product σ-algebra generated by the Borel σ-algebra $\mathcal{B}(A)$, and the power set $P(Z)$. Moreover, $\psi_t : M \to [0, 1]$ is the probability distribution over idiosyncratic states and f_t the associated density.

2.1 Households

Households solve the following program

$$\max_{c_t, n_t} \ \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{n_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right) dt,$$

$$\text{s.t. } da_t = (w_t n_t z_t + r_t a_t + \tau_t - c_t)dt,$$

$$a_t \geq -\phi,$$

where $\gamma > 0$ is the inverse elasticity of intertemporal substitution, $\nu > 0$ is the inverse Frisch elasticity of labor supply. The lump-sum transfers τ_t include dividends distributed to households proportionally to $z_t / \int_X z_t d\psi_t$. Idiosyncratic labor income risk z_t follows a lognormal process.

$$d \ln z_t = -\nu_e \ln z_t dt + \sigma_e d\hat{w}_{z,t},$$

where σ_e is the standard deviation rate of the log-income process, ν_e the mean reversion parameters, and $d\hat{w}_{z,t} \sim N(0, dt)$ is a standard Brownian motion.

2.2 Firms

A representative firm produces the final good using a continuum of intermediate inputs, indexed by $i \in [0, 1]$. This firm chooses intermediate goods Y_{ist} to maximize nominal profits $P_t Y_t - \int_0^1 P_{it} Y_{it} di$ subject to a CES production function

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}.$$

The final good producer operates in a competitive market, and profit maximization with respect to intermediate good i yields the following demand

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\theta} Y_t.$$

Intermediate good producers demand labor N_{it} and energy E_{it} to minimize production costs $w_t N_{it} + p_{e,t} E_{it}$ where $p_{e,t}$ is the real energy price. These firms use a CES production function

$$Y_{it} = \left(\alpha^{\frac{1}{\sigma}} E_{it}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} N_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where σ is the elasticity of substitution between production factors and α is the distribution parameter that weights each input. These parameters are critical to capture the supply effects of an energy shock even in richer multi-sector models with input-output linkages and international trade like the Baqaee-Farhi framework (Baqaee and Farhi (2021)). These models typically feature a nested CES structure. Therefore, the elasticities of substitution across production inputs are key parameters that shape the supply-chain effects in the model.

The optimal demand of labor and energy obtained from the cost minimization problem are given by the following set of equations

$$N_{it} = (1-\alpha)\left(\frac{w_t}{mc_{it}}\right)^{-\sigma} Y_{it},$$

$$E_{it} = \alpha\left(\frac{p_{e,t}}{mc_{it}}\right)^{-\sigma} Y_{it}.$$

This demand system implies that the real marginal cost mc_t only depends on real wages, energy prices, and structural parameters. Therefore, real marginal costs are the same across all firms, see Appendix A.1 for further details. Let m_{it} denote nominal marginal costs so that $mc_{it} = m_{it}/P_t$. Intermediate producers set prices to maximize profits under price adjustment cost and solve the following problem

$$\max_{\dot{P}_{it}} \int_0^\infty \left[\exp\left(-\int_0^t i_s ds \right) \left((P_{it} - m_{it}) Y_{it} - \frac{\Psi}{2} (\pi_{it})^2 P_t Y_t \right) \right] dt$$

$$\text{s.t. } Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} Y_t.$$

Let $\mu = \theta/(\theta-1)$, the optimization problem in a symmetric equilibrium, i.e. $P_{it} = P_t$, yields a New Keynesian Phillips Curve (NKPC) given by

$$\pi_t \left(r_t - \frac{\dot{Y}_t}{Y_t} \right) = \dot{\pi}_t + \frac{\theta}{\Psi} \left(mc_t - \mu^{-1} \right).$$

The real profits of the firms are given by $D_t = (1 - mc_t) Y_t$. To avoid price adjustment costs becoming a non-trivial fraction of output, I model these costs as being virtual (Hagedorn, Manovskii, and Mitman (2019)). These costs affect optimal choices but do not cause a waste of real resources. The profits are distributed to hoouseholds as dividends, and the proportional distribution rule presented above implies that high-income households receive a larger share of profits moving the model towards the data.

2.3 Equilibrium

The consumption good is chosen to be the numeraire so its price is one in real terms. I assume that energy supply $\{E_t\}$ is exogenous while the energy price is given by the demand of energy by firms. I use the short hand notation m_a to denote the drift of household flow budget constraint and m_z to denote the drift of the stochastic process for $\ln z_t$. I formulate the household problem recursively by means of two partial differential equations the Hamilton-Jacobi-Bellman (HJB) equation and the Kolmogorov Forward (KF) equation. Respectively these equations are

$$\rho v(x_t) = \max_{c_t, n_t} \left\{ u(c_t, n_t) + v_a(x_t)m_a + v_z(x_t)m_z + \frac{1}{2}v_{zz}(x_t)s_z^2 \right\},$$

$$\frac{\partial f_t}{\partial t} = -\frac{\partial}{\partial a}(f_t(x_t)m_a) - \frac{\partial}{\partial z}(f_t(x_t)m_z) + \frac{1}{2}\frac{\partial^2}{\partial z^2}(f_t(x_t)s_z^2).$$

The equilibrium in the model is given by household decisions $\{c_t, a_t, n_t\}$, aggregate variables $\{C_t, Y_t, N_t, E_t\}$, and prices $\{r_t, w_t, p_{e,t}, \pi_t, i_t\}$ such that: (i) the HJB and the KF equations hold, (ii) markets clear

$$B = \int_X a_t d\psi_t,$$

$$N_t = \int_X z_t n_t d\psi_t, \forall s$$

where $C_t := \int_X c_t(x_t)d\psi_t$. (iii) the NKPC and the Taylor rule $i_t = r + \phi_\pi \pi_t + \epsilon_t$ hold. Moreover, I assume that variations in profits relative to the steady state level do not directly enter in household budgets. This implies that household income is less sensitive to the cyclicality of profits and markups, which strongly depends on the particular modelling approach of nominal rigidities. Second, it also prevents that the particular assumptions regarding the distribution of profits across households shape the dynamics of aggregate consumption. The resource constraint in this economy is given by

$$C_t = Y_t - p_{e,t}E_t + Q_t.$$

Since I abstract from energy production and model energy as an exogenous resource the income $p_{e,t}E_t$ should be subtracted from final output to measure value added. The last term Q_t is an endowment component of income that captures exogenous payments of financial assets $r_t B$ net of variations in profits $D_t - D$. Throughout this paper I focus on aggregate consumption to measure the economic losses since it has a welfare interpretation. Moreover, since there is no investment in the model, aggregate consumption is also equivalent to GNI, so the numbers in this paper can also be interpreted as GNI losses.

3 Calibration

In this section I briefly discuss the calibration of the model. The model is calibrated at quarterly time frequency. Given the purpose of this study I calibrate the model with two broad objectives. First, the model should deliver an empirically realistic average MPC and be consistent with the size of household wealth relative to income. Second, the model should match the share of consumption of gas, oil and coal over national income. To achieve this I calibrate most of the parameters externally using standard values in the literature. Then, I calibrate the remaining parameters E, B, ρ to match three statistics measuring household wealth, MPC, and the aggregate energy expenditure share.

I set the preference parameters γ, ν, the borrowing limit ϕ, and the Taylor coefficient ϕ_π to values common in the literature. The value for ϕ implies that the wealth distribution has a point mass of households close to zero as we observe in the data. Following the New Keynesian literature I set the intermediate goods elasticity θ to match a steady state profit share of income $1/\theta$ equal to 10%, and the price adjustment cost parameter Ψ to match a slope of the Phillips curve θ/Ψ of 0.1. Since the main focus of this study is on the aggregate implications of an energy shocks I choose standard values for ν_e, σ_e as well. In particular, the calibration procedure implies an annual autocorrelation for $\ln z_t$ equal to 0.9 and a standard deviation rate of innovations equal to 0.2.

The CES parameters σ, α are taken from Bachmann et al. (2022). I set the distribution parameter $\alpha = 0.04$. This choice is motivated by the fact that in this way my results are directly comparable with those in Bachmann et al. (2022). In particular, in the comparison between a quantitative model and a simple CES production function we use the same CES calibration. In the baseline calibration I set $\sigma = 0.1$. In their study Bachmann et al. (2022) provide a review of the empirical evidence on price elasticities for energy demand. The CES function allows to map these estimates to the elasticity of substitution between inputs. It is important to highlight that in the spirit of being as conservative as possible this value is below the lower bound of empirical estimates. In their meta-analysis of existing elasticity estimates for energy demand Labandeira, Labeaga, and Lopez-Otero (2017) find an average short-run elasticity for natural gas of 0.18 and of 0.22 for energy in general. However, given the uncertainty surrounding the aggregate production elasticity I analyze different scenarios in the more optimistic $\sigma = 0.2$, and in a more pessimistic case $\sigma = 0.07$.

I jointly calibrate E, B, ρ to match the quarterly average liquid wealth to income ratio of 4.2, an average quarterly MPC out of small transfers (500 Euros) between 15%-25%, and the annual energy expenditure share of 4%. The energy share target is taken from Bachmann et al. (2022). The value of the average wealth to income ratio is taken from Carroll, Slacalek, and Tokuoka (2014). Finally, the empirical benchmark for the average MPC is taken from the existing literature and the model based estimates of Carroll, Slacalek, and Tokuoka (2014). I calibrate these statistics on the German economy. However, the values of these targets are also comparable to those of the Italian economy. Table 2 shows the calibrated parameters.

Table 2: Model parameters

Parameter	Description	Value	Source
Households			
γ	CRRA/Inverse IES	1	External
ν	Frisch elasticity of labor supply	1	External
ϕ	Borrowing limit	1	External
B	Net asset supply	6.5	Wealth-income ratio
ρ	Discount rate (p.a.)	8%	Average MPC
Income process			
ν_e	Mean reversion coeff.	0.0263	External
σ_e	S. d. of innovations	0.2	External
Firms and policy			
σ	Elasticity in production	0.1	External
α	Distribution parameter	0.04	External
E	Energy supply	0.0318	Energy share of 4%
Ψ	Price adjustment cost	100	PC slope of 0.1
θ	Intermediate goods elasticity	10	Profit share of 0.1
ϕ_π	Taylor coeff.	1.25	External

Overall, the model fits the targeted statistics quite well. The average wealth-income ratio is equal to 4.6, the energy share is 4%, the average quarterly MPC is 9% and 36% annually, and the fraction of liquidity constrained households is 17%. Jointly matching the dispersion in wealth and average MPC is a well known challenge for heterogeneous agents models. See Carroll, Slacalek, and Tokuoka (2017) for a detailed discussion of this point. The baseline calibration provides a good balance between the wealth target and the MPC target. In particular, it is important to highlight that the model generates sizable MPCs and a realistic share of low-liquidity households. Slacaleky, Tristani, and Violante (2020) using household micro data estimate a share of low-liquidity households of 22% between 2013 and 2015 for both Germany and Italy. However, since the average MPC is at the lower end of the estimates provided by the literature, in the next section I also consider a low-wealth calibration that yields and average quarterly MPC of 20%.

4 Energy supply shock

4.1 Main results

In this section I present the main quantitative exercise. Having calibrated the model, I solve the dynamics of the fully nonlinear economy after an unexpected reduction in the energy supply. Following Bachmann et al. (2022) I assume a 10% reduction in the supply of energy. Figure 1 shows the reduction in energy supply. I calibrate the persistence of the shock so that the energy supply is fully back at the steady state level after 3 years. The half-life of the shock is in the third quarter and more than 70% of the shock is absorbed after 1 year.

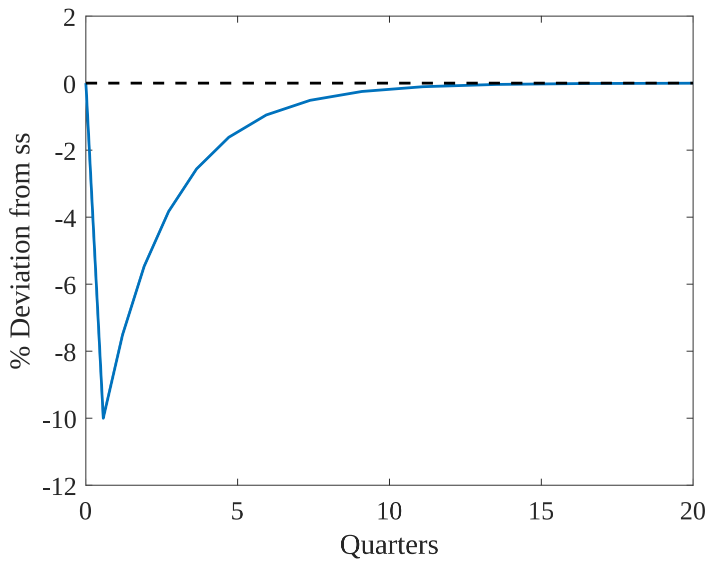

Figure 1: Energy supply shock.

In the baseline calibration I set $\sigma = 0.1$, in a more pessimistic scenario $\sigma = 0.07$, and in the more optimistic case $\sigma = 0.2$. As discussed in Section 3 all these values are at the lower bound of empirical estimates. Figure 2 and Figure 3 show the impulse responses of consumption, labor supply, and prices to the energy shock across calibrations. A reduction in the energy supply leads to lower consumption, higher real energy prices, and an to increase in the inflation rate. In the baseline case household expenditure falls by 1.5% at the peak of the response. This is a consequence of lower wages and higher interest rates. In the baseline case household expenditure falls by 1.5% at the peak of the response. This estimate varies substantially with the production elasticity. In the worst case consumption falls by 2%, while an elasticity of 0.2 leads to a contraction of 0.8%. The energy price increases by a factor of 2 in the baseline calibration and by a factor of 2.5 in the worst-case scenario. The fall in the price of labor is approximately in range between 3% to 8%, the rise in the inflation rate varies from 1% to 4%.

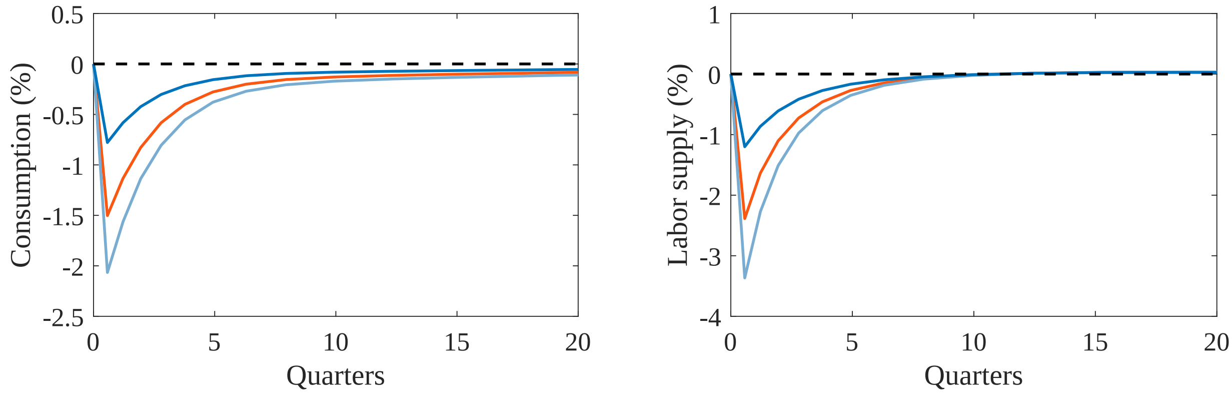

Figure 2: Impulse response of quantities to energy supply shock.

Note: The figures show percentage deviations from steady state. $\sigma = 0.1$ (orange line), $\sigma = 0.07$ (light blue line), $\sigma = 0.2$ (blue line).

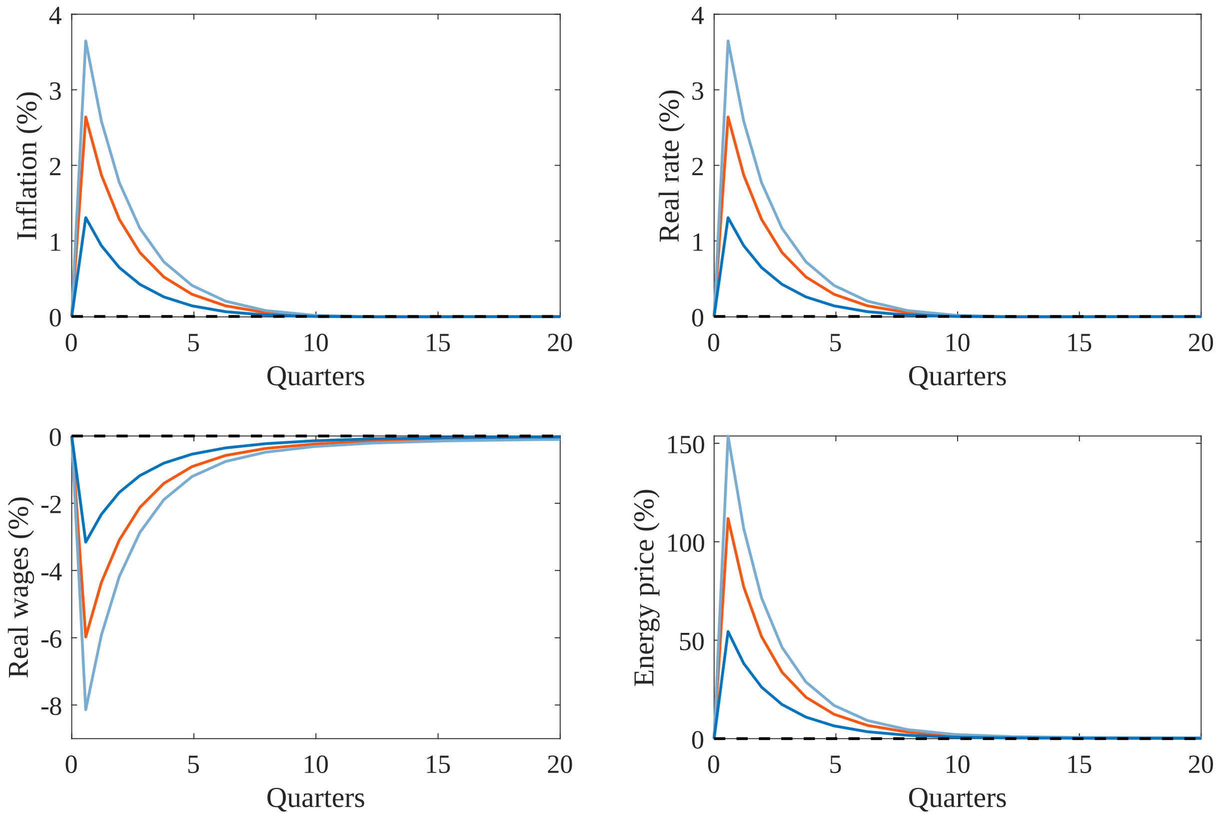

Figure 3: Impulse response of prices to energy supply shock.

Note: The figures show percentage deviations from steady state. $\sigma = 0.1$ (orange line), $\sigma = 0.07$ (light blue line), $\sigma = 0.2$ (blue line).

There are several aspects regarding the response of prices that are important to discuss and clarify. First, since energy prices affect the production costs of the final consumption goods, the energy price increase indirectly contributes to inflation through this channel. Second, from Figure 3 we can see that the real price of energy increases. This reflects an adjustment in relative prices. In particular, the energy shortfall generates an excess demand of energy that is absorbed through higher nominal energy prices. Since for firms is costly to fully transfer the increase in the energy price on the price of the final good that they produce, the relative price of energy increases. This is an important signal to firms to reduce energy consumption. Third, under the parametrization with $\sigma = 0.07$ real wages fall significantly. There are two reasons driving this result. First of all, there is a macroeconomic context in which inflation rises and labor demand declines. Specifically, a reduction in the energy supply leads to high energy prices that increase firms production costs. In turn, firms increase their prices, reduce production, and labor demand. As a result inflation rises, and real wages decline. The central bank increases real interest rates as inflation rises. As a consequence of lower wages and higher interest rates, households reduce consumption and labor supply. These demand effects further amplify the recessions. Moreover, there is a high degree of complementarity between labor and energy in the production function. This increases the impact of an energy shortfall on labor demand. Related to this, also the assumption that labor is the main factor of production besides energy implies that firms adjust only over the labor dimension. Of course, capital is another critical production factor that can provide an additional substitution margin over energy and labor. Therefore, the adjustments do not need to be concentrated on the labor market. Moreover, it is important to highlight that with $\sigma = 0.07$ the model likely overstates the magnitude of labor market effects because in the model wages are fully flexible. The presence of wage rigidities can mitigate these predictions. A more precise quantification of the labor market effects when aggregate elasticities are close to zero goes beyond the scope of this paper, I leave these extensions to future research. Finally, in Figure 3 we also observe an increase in the real interest rate. Two forces contribute to this outcome. Following the Taylor rule the central bank increases the nominal interest rates above the inflation rate. On the other hand, the increase in the real interest rate also reflects an higher demand of saving from households, and given a fixed net supply of assets the market adjustment falls entirely on asset returns.

It is important to emphasize that in the worst-case scenario that I consider in this paper with $\sigma = 0.07$, the elasticity of substitution is half of the median estimate of the short-run energy elasticity from Labandeira, Labeaga, and Lopez-Otero (2017). However, for completeness I further investigate the predictions of the model when the aggregate elasticity of substitution is close to zero. For example, with $\sigma = 0.05$ the GNI loss is 2.7%, energy prices increase by a factor of 3, and inflation by 5%. These responses are somewhat close to those obtained with $\sigma = 0.07$. Overall, the results are robust to the choice of the worst-case scenario. The main conclusion from these simulations is that despite all the uncertainty regarding aggregate elasticities the economic costs of an energy shortage remain bounded and manageable even in worst-case scenarios.

The costs of energy supply shortages are not borne equally across across households. Following the heterogeneous agent literature I analyze the responses of consumption and income by wealth and income. In particular, households are allocated to different groups according to the quintiles of the stationary distribution of wealth a or income risk z, before the energy supply shock materializes. I define total income or disposable income as the sum of earnings $w_t z_t n_t$ and financial incomes $r_t a_t$. As for the aggregate variables I focus on the impact response at the peak of the energy supply shortage. The income and consumption losses are all back to zero after three years. Table 3 shows the cross-sectional effects of the energy supply shock in the baseline case when the elasticity of substitution $\sigma = 0.1$. Specifically, this table shows the average consumption and income losses for each group in percentage deviation from the steady state. In the model, the economic cost of an energy shortfall is highly nonlinear across the wealth distribution. Low-wealth households face the largest cost. The consumption of the bottom 20% of the distribution declines by almost 7%, this is an order of magnitude higher than the 2.2-0.8% decline faced by households in the other wealth groups. Households at the bottom of the wealth distribution also experience a larger fall in their income, with a peak response of around 10%.[2] This effect is driven by earnings and reflects the decline in labor demand and salaries following an energy shortage. Households in the bottom 40% have a comparable income exposure to the energy shock however income changes feed into consumption mostly for the bottom 20%. The reason is that these households do not have a buffer stock of wealth that can help them coping with the income decline.

Table 3: Distributional effects

	Wealth		Income risk	
Percentiles	Consumption	Income	Consumption	Income
0-20	6.8	9.3	4.8	4.2
20-40	2.2	9.9	2.4	4.3
40-60	1.5	7.3	1.6	4.2
60-80	1.3	4	1.4	2.8
80-100	0.8	0	1	0.7

Note: The table reports consumption and income losses for each group in percentage deviation from the steady state. Households are ranked according to the stationary wealth and income distribution.

[2]It is important to keep in mind that the starting levels of these households are very low, so large percentage changes still corresponds to small absolute amounts.

The income of the top 20% of the wealth distribution increases by 3.9 percent. This income gain is driven by higher real interest rates and financial flows. These households respond to higher interest rates by increasing their wealth holdings and by slightly reducing consumption.

The last two columns of Table 3 reports the effects of the energy supply shock across the distribution of the idiosyncratic income component. As in the wealth case there is a stark consumption decline at the bottom 20% of the distribution. The consumption of these households falls by 4.8%, this response is twice as large as that of the next group and three times the response of the group that contains the median household. Specifically, the consumption responses of the other income groups fluctuate between 2.4% and 1%. The income loss is more equally distributed across households. Low-income groups as well as the middle part of the distribution experience an income loss of about 4%. On the other hand, top income groups face an economic burden of 0.7%.

Table 4 shows the predictions of the model on the dynamics of inequality. In particular, this table shows the maximum percentage point increase in the Gini coefficient of consumption, disposable income, and wealth. The energy shortage leads to a large increase in income inequality driven by the earning losses at the bottom of the distribution. The shock also generates a substantial increase in consumption inequality, although to a lesser extent than income inequality. This in turn reflects the fact that at the bottom of the distribution income changes feed into consumption generating a large consumption decline. Also wealth inequality increases, this is due to the fact that while low-wealth groups are harshly affected the income of the top 20% of the wealth distribution increases. Over time this income gain contributes to wealth accumulation at the top of the distribution. The effects on wealth inequality are more persistent than the effects on income and consumption inequality.

Table 4: The effects on inequality

	Consumption	Income	Wealth
Δ Gini coefficient	0.5	0.8	0.3

Note: The table reports the percentage point increase in inequality as measured by the Gini coefficient. The table shows the maximum increase in the Gini.

Overall, the main conclusion from this exercise is that households with low income and wealth face the largest cost while the consumption decline is much less dramatic for all the other households. This stark inequality suggests that to mitigate the negative distributional and aggregate effects of the energy shortfall social insurance and fiscal policies should primarily target low-income households.

4.2 Gas as a separate energy input

Now I also consider an alternative scenario in which I focus only on gas and model the energy input E as that specific energy source. Following the calculations in Bachmann et al. (2022) I consider a 30% reduction in the energy supply. This would be the shortfall of energy inputs from natural gas consumption. The persistence of the shock is the same as before. For this exercise I use an elasticity of substitution $\sigma = 0.16$ at the lower bound of empirical estimates (Steinbuks (2012)). The distribution parameters $\alpha = 0.024$. I calibrate this parameter so that the CES production function generates a GNI reduction of 2.3% as in Bachmann et al. (2022). Figure 4 shows the impulse response functions. Agrgegate consumption falls by 3.4%. This more than doubles the economic loss measured by national expenditure of the baseline case in the previous section. The energy price increases by a factor of about 4, inflation rises by more than 6%. These price responses should be taken with caution since the model does not feature wage rigidities which are an important feature of real economies. Moreover, I should emphasize that since gas is not the only energy input this is not a likely outcome. Rather than providing a realistic prediction on the effects of energy shortages the main purpose of this computational exercise is to quantify the amplification effect in an extreme scenario.

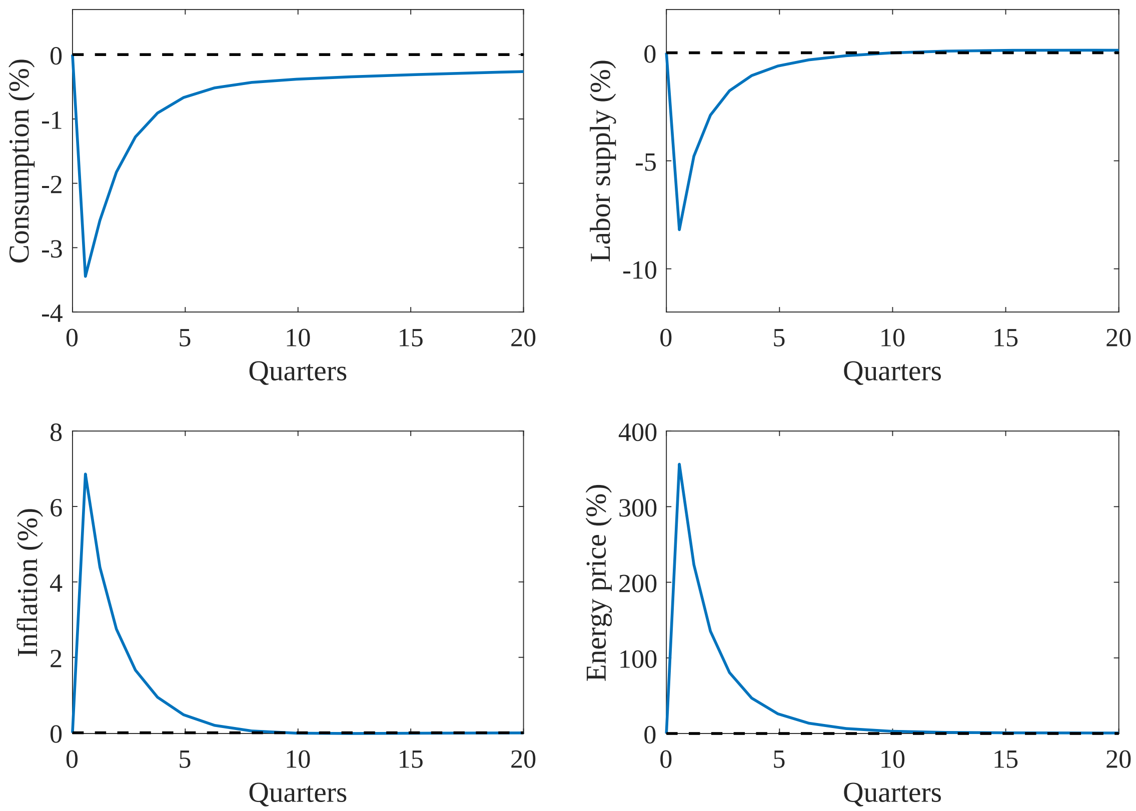

Figure 4: Impulse responses to a -30% energy shock.

Note: The figures show percentage deviations from steady state ($\sigma = 0.16$).

4.3 Long-term energy shortages

In all the simulatons of Section 4.1 the energy shock lasts for 3 years and more than 70% of the energy shortfall is absorbed within 1 year. Here, I consider a more conservative scenario in which the energy supply shortage lasts for 5 years and less than 60% of the energy shortfall is absorbed within 1 year.[3] In this exercise I focus on the baseline calibration with an energy decline of 10% and $\sigma = 0.1$. I choose this value for the elasticity of substitution over the other cases discussed in Section 4.1 to build-in a dose of caution and at the same time acknowledge that macro elasticities tend to be larger in the long run than the short run. Figure 5 contrasts the consumption response under this new scenario with the consumption response to the energy shock in Figure 1. Now consumption falls by 2.7% and returns to its previous level only after 3 years. This implies that the cumulative loss can be substantial. The difference on impact moslty reflects a larger decline in real wages relative to the baseline. The responses of the other variables are similar in both couterfactuals.

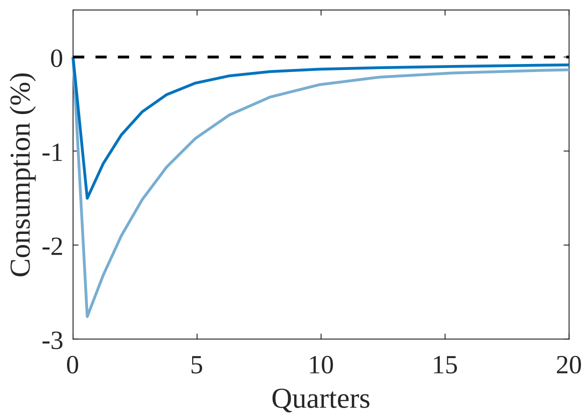

Figure 5: Energy dynamics and aggregate consumption.

Note The figure shows the percentage deviation from steady state of aggregate consumption in the high-persistence case (light blue line) and in the baseline case (blue line).

This exercise shows that the assumptions on the dynamic path of the supply shock are important for the aggregate outcomes. In the baseline calibration the income loss can increase by a factor of 1.8 under a highly persistent energy shock. On the other hand, the responses of inflation and energy prices are close to their baseline counterparts. However, it is important to highlight that despite the uncertainty regarding dynamic aspects of the energy shortfall the economic cost remains within the lower bound of the most pessimistic scenario of Section 4.1 with a fixed elasticity of substitution of 0.05 throughout the entire transition path.

[3] In March of 2022 the European Commission announced a plan to cut out two-thirds of its Russian gas imports by the end of the year, and to make Europe independent from Russian fossil fuels well before 2030.

4.4 A low-wealth calibration

In this section I investigate the sensitivity of the results to the size of the average MPC. The model generates large MPCs for households with low income and wealth, but the MPCs decrease rapidly in wealth leading to an average MPC at the lower bound of empirical estimates, as I discuss in Section 3. The average MPC shapes the demand amplification effect that I study in this paper. Therefore, in the spirit of the "liquid-wealth-only calibration" advocated by Carroll, Slacalek, and Tokuoka (2017), I analyze a low-wealth economy with a substantially higher average MPC. To achieve this I recalibrate the discount rate and the asset size to match a real return on wealth of 1% and an average quarterly MPC of 20%. This generates an economy with a large share of low-liquidity households, around 33% of the population. However, the quarterly MPC in the low-wealth calibration is 17% which is about two times the MPC of the baseline calibration.

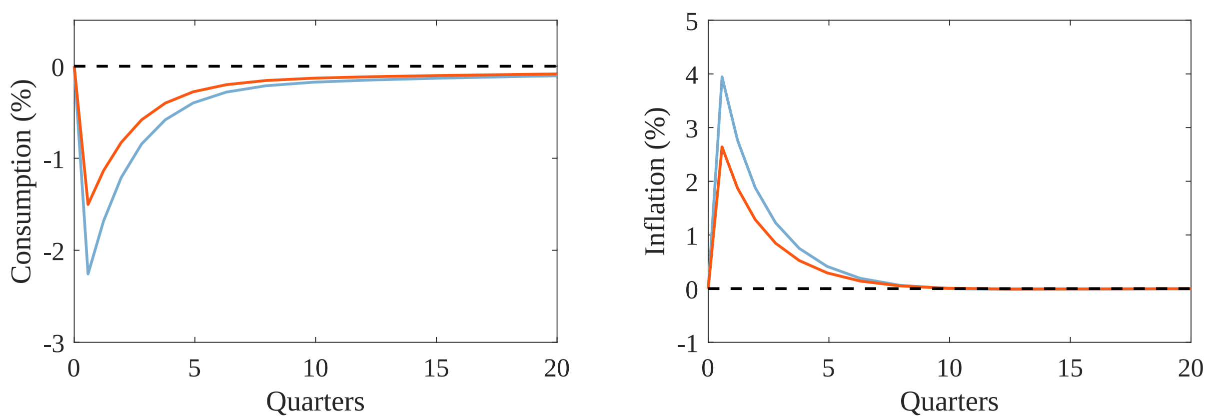

Figure 6: MPC and aggregate consumption.

Note The figure shows the percentage deviation from steady state in the low-wealth case (light blue line) and in the baseline case (orange line). $\sigma = 0.1$.

Figure 6 shows the response of consumption and inflation to the energy shock across calibrations. At its peak the response of inflation increases from 2.6% to 4.4%. On impact, the response of the real interest rate rises from 2.6% to 4.9%, and the energy price increases by 30 percentage points across calibrations. Consumption falls by 2.2%, this implies that doubling the MPC only increases the consumption response by 0.7 percentage points or by 46 percent. This is somewhat a large change. However, it remains a small effect in comparison with the range of estimates generated by the uncertainty regarding the macro elasticity of substitution across inputs. Therefore, I leave a more complete analysis on the determinants of a large average MPC and the implications for the propagation of energy shocks to future work, instead I only emphasize here that the quantitative results in Section 4.1 are likely to be robust.

5 Policy analysis

In this section I analyze how monetary policy and fiscal policy can mitigate the economic cost of energy shortages. To this end I study the response of the economy to four distinct policies: interest rate policy, subsidies on energy, stimulus payments, and social insurance mechanisms. In practice is very likely that a combination of these policies would be needed. However, these policies mitigate the economic loss from an energy shortfall in very different ways. The main objective of these experiments is to isolate and quantitatively assess these different effects.

5.1 Monetary policy

The response of the economy to an energy shock depends on the stance of monetary policy. To analyze the role of monetary policy during an energy supply shortfall, I compare the results of the baseline model with Taylor parameter on inflation $\phi_\pi = 1.25$ and those of an alternative policy in which $\phi_\pi = 1.5$. The first monetary policy rule captures a more accomodative monetary policy stance, while the second a more conservative approach on inflation.

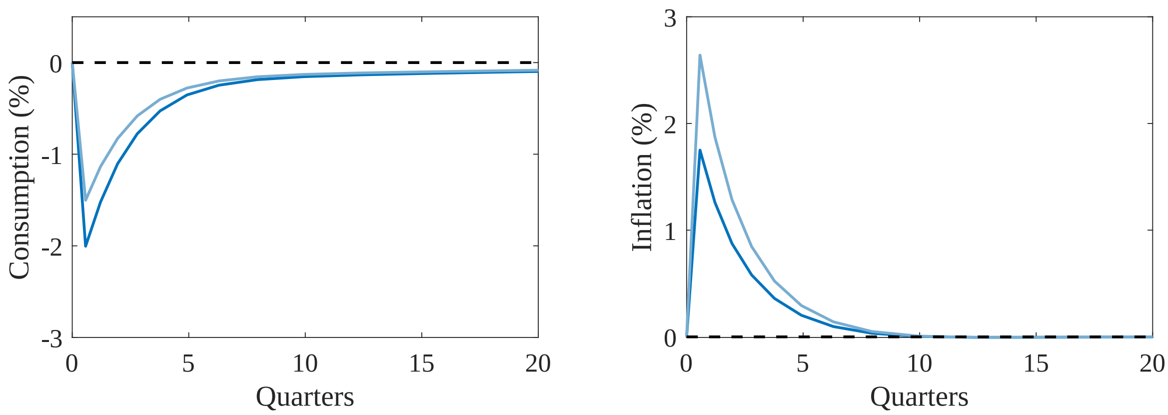

Figure 7: Monetary policy and energy shortages.

Note: The figures show percentage deviations from steady state with $\sigma = 0.1$ across different policy rules. $\phi_\pi = 1.5$ (blue line) and $\phi_\pi = 1.25$ (light blue line).

Figure 7 shows that under the more strict monetary policy rule inflation increases by 1.7% instead of 2.6%, the real interest rate rises by 3.5% instead of 2.6%, energy prices increase by a factor of 1.9, and consumption falls by 2%. Therefore, under the more accommodative monetary policy rule, the severity of the recession caused by the energy supply shock is mitigated, but prices increase more. In conclusion, increasing the contraction of output by 0.5 percentage points reduces the increase in inflation by approximately 1 percentage point. Not surprisingly, there is a trade-off between economic activity and inflation. However, a more precise quantification of this trade-off would require to carefully estimate and model the impact of the energy supply shortfall on potential output.

5.2 Energy subsidies and social insurance

Tax subsidies on energy for industrial consumers are being used by governments across European countries as a tool to contain the increase in the production costs due to high energy prices. Here, I consider energy subsidies intended to increase the energy supply and reduce the energy price paid by industrial consumers. Therefore, this policy can potentially reduce the direct impact of a fossil fuels shortfall on energy inputs. On the other hand, stimulus payments, and the expansion of social insurance programs can be used to contain amplification effects stemming from more indirect channels such as labor market adjustments and aggregate demand fluctuations. In this section I analyze the predictions of the model regarding the macroeconomic outcomes of these policies.

I model energy subsidies as a reduction in the energy shortfall of one percentage point, from 10% to 9%. Stimulus payments and social insurance are given by temporary lump-sum transfers $b_t(x_t)$ to low-income households. The key difference between stimulus payments and an expansion of social insurance lies in the magnitude of the transfers. In the former case, stimulus payments increase total income (including transfers) and consumption of the recipients. In the latter case, the transfers only mitigate the drop in total income and consumption of the recipients, and bring the economic losses of low-income households close to that of all other households. It is important to emphasize that both policies target a small population group.

In particular, I assume that in the economy there is a supply of energy with infinite elasticity. Therefore, the cost of energy subsidies is $S_t := (p_{e,t} - p_{s,t})E_t$ where $p_{s,t}$ is the energy price under a 9% energy shock, $p_{e,t}$ is the energy price without the subsidy under a 10% energy supply shock, and E_t is energy under a 9% shock. These calculations imply that the cumulative cost of the policy over the first year is around 1% of steady state annual GNI. The ammount of the transfer is b_t if $z_t \leq \bar{z}$ and $a_t \leq \bar{a}$, and zero otherwise. The dynamics of the transfers are given by $b_t = e^{-\eta t}b_0$ where b_0 is the value of the transfer on impact ($t = 0$) at its maximum peak. I set $\eta = 0.5$ so that almost all the payments are made during the first year. Then, I set $\bar{a}$ just below steady state average wealth and $\bar{z}$ at the eighth lowest income state that corresponds to the 20th percentile of the steady state distribution of income risk. This is consistent with a policy design that targets low-income househlds that are relatively more exposed and vulnerable to the recession. Specifically, Table 3 shows that with this parametrization the policy exactly includes those households that would face the largest consumption loss after an energy supply shock. Finally, I calibrate the value of b_t at its peak. In the case of stimulus payments I choose the value of b_0 so that the cost of stimulus payments is the same as the cost of energy subsidies. A transfer equal to 3% of mean annual labor income at the steady state yields

$$\int_X b_t(x_t)d\psi_t = S_t.$$

In the case of social insurance I use a value for b_0 equal to 1% of mean annual labor income at the steady state, and the cumulative cost of the policy is only 0.34% percent of annual GNI.

Since the cumulative cost of energy subsidies and stimulus payments over the first year is around 1% of annual GNI, these policies require a significant fiscal adjustment either through taxes and transfers or via government debt, which for simplicity I do not model here. In particular, I focus on the differential aggregate effects of these policies rather than on the question of how these differences can change with the choice of the fiscal instrument used to finance them. However, as a robustness check I also introduce in my computational experiment lump-sum taxes to finance these policies. These taxes are distributed across households proportionally to $z_t / \int_X z_t d\psi_t$ so that high-income households bear most of the fiscal adjustment. I find similar responses of consumption and inflation as in my main analysis without taxes.

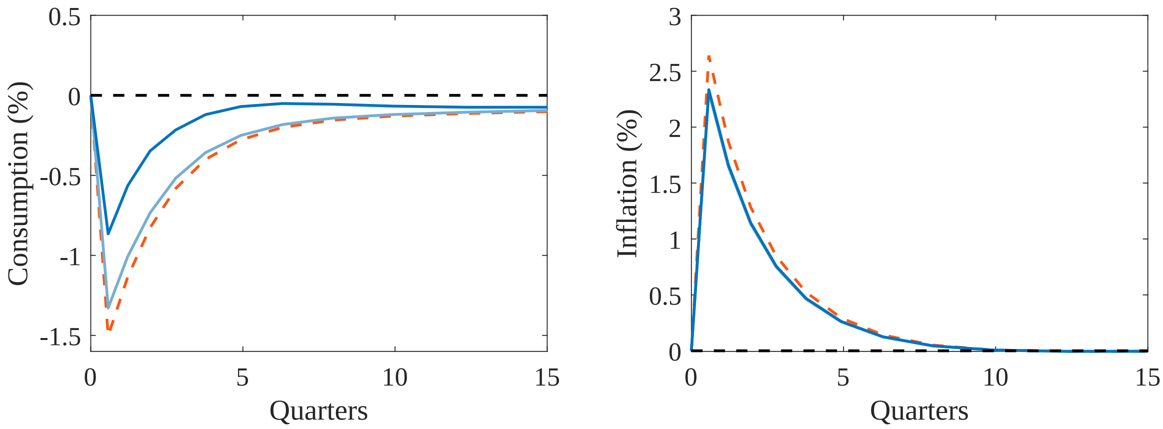

Figure 8: Fiscal policy and energy shortages.

Note: The figures show percentage deviations from steady state with $\sigma = 0.1$ across different fiscal policies. Stimulus payments (blue line), energy subsidies (light blue line), no policy (orange dashed line).

Figure 8 shows the response of consumption and inflation from the model with either stimulus payments (blue line) or energy subsidies (light blue line) relative to the baseline case without any policy intervention (orange line). Energy subsidies incentivize energy producers to increase the supply of energy inputs reducing the size of the energy shortfall and the rise of energy prices for industrial consumers. However, given a 1% reduction in the energy supply shock these effects are very small in the model. Energy prices still rise by a factor of 2 as in the baseline. In turn, we observe an increase in inflation of 2.3% with the subsidies against an increase of 2.6% without this policy. Subsidies also have a small effect on the recession. In particular, subsidies reduce the consumption fall on impact by 0.18 percentage points or by 12 percent. Instead, stimulus payments targeted to the most vulnerable households can reduce the consumption loss by 0.6 percentage points or by 40 percent. Since this policy targets a small population group it does not generate additional inflation. Instead, the reduction in the magnitude of the recession has a marginal but positive effect on inflation: it reduces the contraction in firms markups lowering the incentives of firms to increase prices. Overall, this policy can undo a substantial part of the demand amplification leading to an output loss of 0.9 percent in the HANK model, an income loss much closer to the output contraction of 0.6 percent given by the simple aggregate production function as parametrized in the quantitative model. However, this result crucially depends on the fact that the transfers on top of providing insurance against income losses also increase earnings at the bottom of the distribution stimulating household expenditures.

Therefore, I also recalibrate the value of b_t on impact so that transfers only provide insurance against the economic losses. In particular, with this reparametrization the income and consumption losses of low-income households are of the same magnitude as those of the other households. For example, households in the bottom 20% of the wealth distribution face on average a consumption loss of 1.5% as in the aggregate, while without the policy these households would experience a consumption loss of 7%. Moreover, the income loss of these households is 3% against the 9% that they would face without social insurance. Thus, despite the policy gives relatively little income, it provides substantial insurance against the negative effects of the recession to those who need it the most. At the same time, this policy also mitigates the negative effects for all other households. At the aggregate level the inflation rate is the same as in the baseline. However, aggregate consumption falls only by 1.2%, a reduction of the economic loss relative to the baseline of 0.32 percentage points or 21 percent. Both social insurance and stimulus payments disincentivize labor supply at the bottom of the distribution. The reduction in the aggregate labor supply is respectively about 0.2 and 0.5 percentage points higher than in the baseline. Hence, this effect is smaller relative to the effect on aggregate consumption.

Table 5: The effects of fiscal policies

	No policy	Subsidies	Stimulus	Insurance
Consumption loss	1.5	1.3	0.9	1.2

Note: The table reports percentage deviations from the steady state on impact.

Table 5 summarizes the macroeconomic effects of fiscal policies. The computational experiments suggest that energy subsidies that reduce the energy shortfall by one percentage point of national energy use have small effects on the inflation rate and on economic activity. Instead, a targeted fiscal stimulus can substantially reduce the size of the recession and the consumption loss. Since this policy targets a small population group it does not generate high inflation. However, social insurance is an alternative policy with less extreme implications for the government budget. I find that an expansion of social insurance programs targeted to the most vulnerable households can address the business cycle amplification effects and mitigate the economic cost of an energy shortfall.

6 Conclusion

This paper builds an Heterogeneous Agents New-Keynesian (HANK) model with energy as a critical production factor and studies how much a recession caused by energy shortages can be amplified through aggregate demand channels. A series of numerical simulations highlights a substantial business cycle amplification. However, even in the more conservative scenarios, in which energy inputs have an extremely low aggregate elasticity of substitution, I find manageable income losses. It is important to emphasize that the costs of an energy supply shortfall are not borne equally within the society. Households with low income and wealth face an economic loss which is an order of magnitude higher than the cost faced by all other households.

Going forward there are several questions that can be investigated in future research. First, the analysis should include direct energy consumption by households. This is important to analyze to what extent energy shortages fall on households and firms. Second, the model provides a quantitative framework to evaluate different combinations of monetary and fiscal policies, and study how these economic policies can mitigate the negative effects of energy shortages. Third, differences in the composition of household expenditures can further amplify the heterogeneous effects of energy shocks. So, this is also an important factor to quantitatively model.

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Appendix

A Further details on the model

A.1 Energy demand and marginal cost

In this section I derive the demand for the production factors, and an analytical expression for the marginal costs of the firms. In the model intermediate firms solve the following problem

$$\min_{E_{it},N_{it}} w_t N_{it} + p_{e,t} E_{it},$$

$$\text{s.t.} \quad Y_{it} = \left(\alpha^{\frac{1}{\sigma}} E_{it}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} N_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

The first order conditions $w_t = mc_{it}(1-\alpha)^{\frac{1}{\sigma}} Y_{it}^{\frac{1}{\sigma}} N_{it}^{\frac{-1}{\sigma}}, p_{e,t} = mc_{it}\alpha^{\frac{1}{\sigma}} Y_{it}^{\frac{1}{\sigma}} E_{it}^{\frac{-1}{\sigma}}$ yields

$$N_{it} = (1-\alpha)\left(\frac{w_t}{mc_{it}}\right)^{-\sigma} Y_{it},$$

$$E_{it} = \alpha\left(\frac{p_{e,t}}{mc_{it}}\right)^{-\sigma} Y_{it}.$$

Substituting the demand functions in the cost function and taking the derivative with respect to Y_{it} delivers the real marginal cost of each firm, which is the same across firms and given by

$$mc_t = \left((1-\alpha) w_t^{1-\sigma} + \alpha p_{e,t}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$