Chapter 3:
Heterogeneous Agents New Keynesian Models

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I. The Canonical HANK Model

A. *Description of the model*

This section introduces the canonical Heterogeneous Agents New Keynesian (HANK) model (McKay, Nakamura, and Steinsson (2016), Kaplan, Moll, and Violante (2018)). This model replaces the assumption of a representative agent in the basic NK model (Chapter 2) with a standard HA block (Chapter 1) and uses sticky wages instead of sticky prices. In the economy there is no capital but a one-period real asset a_t, there is idiosyncratic income risk e_t, and markets are incomplete. For the HA block of the model we borrow the notation and underlying mathematical structure from Chapter 1. Given the prices, labor supply n_t, states, and initial conditions households decide $\{c_t, a_t\}_{t=0}^{\infty}$ solving the following dynamic program

$$v_t(a_t, e_t) = \max_{c_t, a_{t+1}} u(c_t, n_t) + \beta \int_E v_{t+1}(a_{t+1}, e_{t+1}) dF_{e'|e}$$

$$\text{s.t.} \quad c_t + a_{t+1} = (1 - \tau_t) w_t e_t n_t + (1 + r_t) a_t,$$

$$a_t \geq -\phi.$$

The law of motion of the distribution D_t is given by

$$D_{t+1}(a', e') = \int_X Q_t((a, e), (a', e')) dD_t(a, e). \tag{1}$$

The supply block of the model is isomorphic to the one we studied in Chapter 2. A representative firm produces the final consumption good using a linear technology

$$Y_t = Z_t N_t, \tag{2}$$

where Z_t denotes Total Factor Productivity (TFP) assumed to be exogenous and N_t aggregate labor services. Marginal pricing implies $w_t = mc_t Z_t$. Nominal consumer prices are fully flexible. Since the good market is perfectly competitive, real marginal costs are equal to one. Therefore, $w_t = Z_t$. This implies that real wages in this model are exogenous and household earnings only respond to changes in employment levels. From this relation we see that nominal wage inflation is given by $1 + \pi_t^w = (1 + \pi_t)(1 + z_t)$ where

$z_t := Z_t/Z_{t-1} - 1$ and $\pi_t := P_t/P_{t-1} - 1$. If TFP is constant real wages are constant and wage and price inflation coincide.

In this economy, unions set nominal wages by maximizing the average welfare of the households, and employ all households for an equal number of hours or number of workers n_t. In particular, a competitive recruiting firm aggregates a continuum of differentiated labor services indexed by $j \in [0,1]$ solving the maximization problem

$$\max_{N_{jt}} \quad W_t N_t - \int_0^1 W_{jt} N_{jt} dj,$$
$$N_t = \left(\int_0^1 N_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

where W are the nominal wages and ϵ is the elasticity of substitution across differentiated labor inputs. This implies a demand for the labor services of type j equal to

$$N_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\epsilon} N_t.$$

For each labor input j a union sets the nominal wage to maximize the average welfare of the union members, taking their marginal utility of consumption $u'(c) = c^{-\gamma}$ and the labor disutility $\upsilon(n) := n^{1+\nu}/(1+\nu)$ as given. We also introduce nominal wage rigidities in this model. Specifically, wage adjustment is subject to a quadratic utility cost. Let C_t be aggregate consumption and P_t the consumer price index, the union solves the problem

$$\max_{W_{jt}} \quad \sum_{t=0}^{\infty} Q_t \left(\int_0^1 \frac{W_{jt}}{P_t} N_{jt} - \frac{\upsilon(N_{jt})}{u'(C_t)} - \frac{\theta}{2} \left(\frac{W_{jt}}{W_{jt-1}} - 1 \right)^2 N_t dj \right)$$
$$\text{s.t.} \quad N_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\epsilon} N_t.$$

Let $\mu_w := \epsilon/(\epsilon - 1)$, in a symmetric equilibrium with $W_{jt} = W_t$ and $N_{jt} = N_t$ we obtain a wage Phillips Curve given by

$$(1 + \pi_{wt})\pi_{wt} = \left(\pi_{wt+1}(1 + \pi_{wt+1}) \frac{1}{(1 + r_{t+1})} \frac{N_{t+1}}{N_t} \right) + \frac{\epsilon}{\theta} \left(mrs_t - \mu_w^{-1} w_t \right), \tag{3}$$

where the marginal rate of substitution is

$$mrs_t = \varphi n_t^{\nu} / C_t^{-\gamma}. \tag{4}$$

Note that we assumed that the unions use the market real rate to discount utility. Alternatively, unions could use the household discount factor β. These formulations yield similar results. On the other hand, introducing an incidence function that allows for heterogeneous labor supply across the population could substantially change the results.

I close the model with the policy block. Monetary policy chooses $\{i_t\}$ according to

$$i_t = i + \phi_\pi \pi_t + v_t. \tag{5}$$

In the Taylor rule i is the nominal interest rate in the zero-inflation steady state and v_t is a monetary policy innovation. In the basic model assets are in zero or positive net supply $B \geq 0$. As we did in Chapter 2 we can easily introduce fiscal policy in this setup and assume that the net supply of households' savings is invested in government bonds.

Equilibrium. In the basic HANK model given prices $\{w_t, r_t, \pi_t\}$ households optimally decide $\{c_t, a_t\}$, the law of motion of the distribution is given by (1), unions set nominal wages and labor supply according to (3), (4), firms choose $\{Y_t, N_t\}$ maximizing profits, and monetary policy sets the policy rate $\{i_t\}$ according to (5). Prices are such that the financial market, and the labor markets clear

$$\int_X a_t dD_t(a, e) = B, \tag{6}$$

$$\int_X n_t e_t dD_t(a, e) = N_t. \tag{7}$$

The good market clearing condition $C_t := \int_X c_t dD_t = Y_t$ holds by Walras' law.

B. *Numerical solution and calibration*

Note given the functions $C_t = \mathcal{C}_t(\{w, r, N\}), A_t = \mathcal{A}_t(\{w, r, N\})$ from the HA block of the model we can reduce the system to 7 endogenous variables $\{C, A, N, Y, r, \pi, mrs\}$ and 7 nonlinear structural equations given by

$$(1 + r_t)(1 + \pi_{t+1}) = 1 + i + \phi_\pi \pi_t + v_t,$$

$$Y_t = N_t,$$

$$(1 + \pi_{wt})\pi_{wt} = \left(\pi_{wt+1}(1 + \pi_{wt+1})\frac{1}{(1 + r_{t+1})}\frac{N_{t+1}}{N_t}\right) + \frac{\epsilon}{\theta}\left(mrs_t - \mu_w^{-1} w_t\right),$$

$$C_t = \mathcal{C}_t(\{w, r, N\}), \quad A_t = \mathcal{A}_t(\{w, r, N\}),$$

$$Y_t = C_t,$$

$$mrs_t = \varphi N_t^\nu / C_t^{-\gamma}.$$

We can solve the model given the unknowns $U = (\pi, N)$, the exogenous shocks v, w, and the targtes (6), and (4). Figure 1 shows the directed acyclical graph representation of the equilibrium conditions listed above.

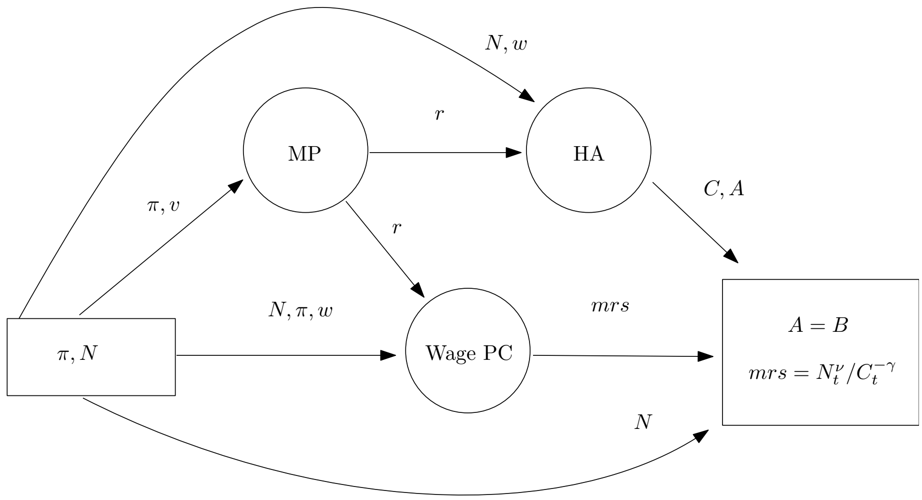

Figure 1: Graph of the basic HANK model.

I calibrate the model to the US economy. Specifically, following the literature and micro estimates I set γ, ν to 1. The borrowing limit is set equal to 1, so that the model features a mass of households close to zero liquid wealth as in the data. In the US this fraction of households is estimated to be around 30%. I set the elasticity of substitution among labor services to 10 implying a wage markup of about 11% and the wage adjustment cost coefficient θ to match a slope of the wage Phillips curve ϵ/θ of 0.1. The remaining parameters ρ, B are jointly calibrated to match a liquid wealth to annual output ratio of 1.5 and an average return on liquid assets of 2%. Table 1 summarizes this parametrization.

Table 1: Parameters in the basic HANK model

Parameter	Description	Value	Source
γ	CRRA/Inverse IES	1	External
ν	Inverse Frisch elasticity	1	External
ϕ	Borrowing limit	1	External
ρ	Individual discount rate	2.5%	Internally calibrated
B	Liquid wealth	5.5	Internally calibrated
θ	Adjustment costs	100	External
ϵ	Elasticity of substitution	10	External
ϕ_π	Taylor coeff.	1.5	External

To keep the model as simple as possible I use a simple two-state Markov process for e_t. In quantitative applications more flexible specifications with more parameters and states are calibrated from micro estimates.

C. Consumption behavior and wealth inequality

How does consumption respond to temporary income changes? This question is central in many applications as it affects the transmission of aggregate shocks and macroeconomic policies to the economy. A key measure of the consumption response to income changes is the Marginal Propensity to consume (MPC), i.e. the fraction of a one-time windfall income change that is spent over h time periods. The MPC out of an income gain of size τ over $h = 1, 2, ...$ time periods is defined as

$$MPC_\tau^h := \frac{C^h(a_t + \tau) - C^h(a_t)}{\tau},$$

where C^h is the cumulative consumption of an individual with initial wealth a_t over h periods. What are the predictions of the Representative Agent (RA) framework for individual consumption behavior? Solving for the consumption policy function we can study the MPC in the RA framework. Define labor income as Y_t^ℓ and $R_t := (1 + r_t)$ as the gross interest rate. The households' budget constraint is given by

$$C_t + A_{t+1} = Y_t^\ell + R_t A_t.$$

Solving this equation forward under the natural borrowing limit $A_t \geq -\bar{A}$ and the transversality condition preventing assets from not being consumed in the limit yields

$$A_t = \sum_{j=0}^{\infty} \prod_{s=0}^{j} \left(\frac{1}{R_{t+s}} \right) (C_{t+j} - Y_{t+j}^\ell).$$

Solving forward the Euler equation for $j \geq 1$ periods $C_t^{-1} = \beta^j C_{t+j}^{-1} \prod_{s=1}^{j} R_{t+s}$ and substituting C_{t+j} from this Euler equation, rearranging terms and using the convergence of geometric series we can rewrite

$$C_t = (1 - \beta) \left(R_t A_t + Y_t^\ell + \sum_{j=1}^{\infty} \prod_{s=1}^{j} \left(\frac{1}{R_{t+s}} \right) Y_{t+j}^\ell \right).$$

In this model consumption depends on lifetime or permanent income. If we consider a one-time income gain τ the MPC is given by $(1-\beta) = r/(1+r)$ where r is the steady state real interest rate. The MPC is close to zero when $\beta \to 1$. Since in typical calibrations we need a discount factor of 0.95 or 0.99 to match a realistic steady-state interest rate or aggregate wealth in the economy, the MPC is around 1% and 5% at best. This is inconsistent with household consumption behavior from micro estimates that often find an average MPC around 20% and even higher for some households.

HA models generate a consumption behavior in line with the micro evidence. A key statistic in these models is the average Marginal Propensity to consume (MPC), i.e. the average fraction of a one-time windfall income gain or transfer that is spent over h time periods, typically one quarter or a year. The average MPC out of an income gain of size τ over $h = 1, 2, ...$ time periods is defined as

$$MPC_\tau^h = \int_X \frac{C^h(a_t + \tau, e_t) - C^h(a_t, e_t)}{\tau} dD(a_t, e_t),$$

where D is the stationary distribution of idiosyncratic states and C^h is the cumulative consumption of an individual with initial states (a_t, e_t) over h periods. Note that, although the two are related, this is in general a different statistic than the slope of the consumption policy function $c'(a)$, i.e. the impact MPC ($h = 1$ MPC). In general, computing the MPC requires to compute the cumulative consumption $C^h(a, e) = E[\sum_h c(a_h, e_h)|(a_t, e_t) = (a, e)]$, with $h > t$ and $C^h(a, e) = E[c(a_t, e_t)|(a_t, e_t) = (a, e)] = c(a_t, e_t)$ when $h = t$. Alternatively, one could recalibrate the model at a different time frequency or aggregate MPCs over time. Finally, we could allow for a dynamic equilibrium and compute the MPCs out of the steady state. Since the model is calibrated at quarterly frequency, for the quarterly MPC ($h = 1$) we can simply use the consumption policy function $C^1(a_t, e_t) = c(a_t, e_t)$ and interpolate to get $C^1(a_t + \tau, e_t)$. Figure 2 plots the quarterly MPCs by wealth (left panel) and the wealth distribution (right panel) in the basic model.

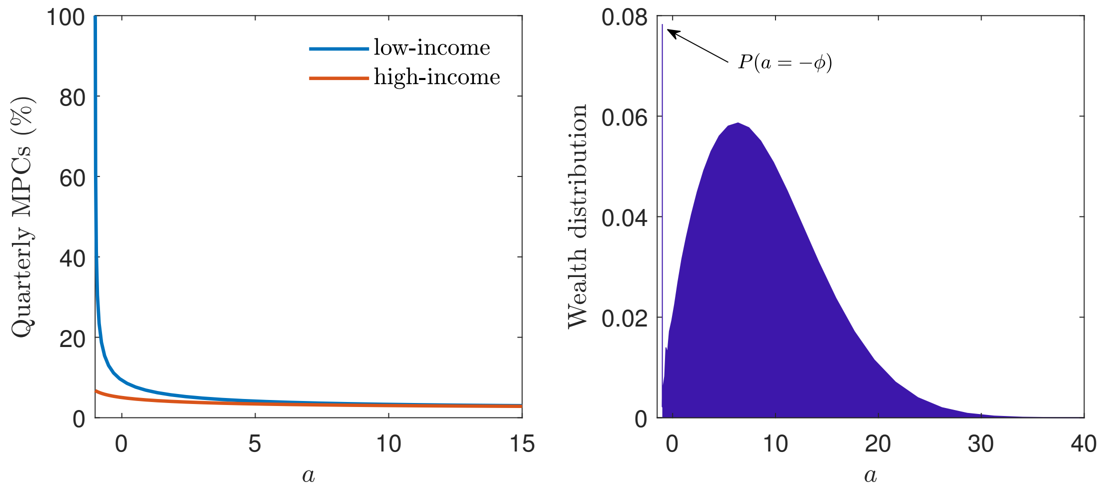

Figure 2: Quarterly MPCs of the basic HANK model.

Empirical estimates typically find that households spend a substantial fraction of windfall income gains (Broda and Parker (2014), Parker, Souleles, Johnson, and McClelland (2013), Fagereng, Holm, and Natvik (2021)). The evidence from micro studies implies an empirical benchmark for the average quarterly MPC out of 500 or 1000 dollars in range between 15% to 25%. This statistic is typically around 2% in models with a representative agent. In our calibrated HANK the average MPC is around 12%, at the lower bound of the empirical estimates, but substantially above the MPC of the representative agent framework. Importantly, the average MPC masks a substantial heterogeneity across households as low-income households can have an MPC above 20%. High MPCs size-up income effects as they imply that households respond more strongly to temporary income fluctuations. This moves the model away from the permanent income hypothesis and towards the micro evidence. Figure 3 shows the annualized MPCs. Note that the MPCs decline with liquid wealth and income, and remain above zero throughout the wealth distribution.

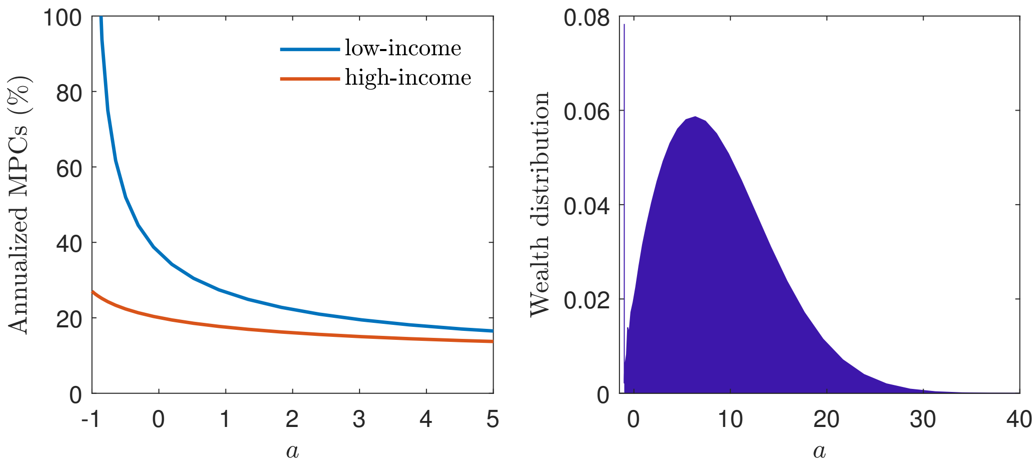

Figure 3: Annualized MPCs of the basic HANK model.

Importantly, the MPCs reflect different consumption behaviors across different regions of the state space. Households at the borrowing limit have an MPC close to 1 since they are constrained and cannot smooth income shocks. Unconstrained households with low income and liquid wealth (close to the borrowing limit) also have high MPCs because they anticipate the possibility of facing a binding borrowing limit in the future as they keep accumulating a sequence of low-income realizations e_t, indeed the income process is very persistent both in the model and in the data. High-income households with low liquid wealth instead are moving in the opposite direction (on average) and accumulate a buffer stock of saving. These households drive the precautionary saving in the economy. These households also have higher MPCs relative to high liquidity households because a one-time income gain relaxes their precautionary saving motive (however this effect is small). On the other hand, they have smaller MPCs than low-income households because they need to save in order to move away from the borrowing limit.

II. Monetary Policy According to HANK

In this section we revisit the effects of monetary policy shocks using the basic HANK framework. As in Chapter 2, I assume that the monetary policy innovation $\{v_t\}$ follows an AR(1) process. We analyze an interest rate cut of 25 basis points, i.e. an exogenous reduction in the short-term nominal interest rate of 0.0025 or 1% annually. For this policy experiment we use the basic version of this model with $B = 0$. The left panel in Figure 4 shows the impulse response functions to the monetary policy shock in our HANK model. As in Chapter 2 the shock has an expansionary effect on the economy.

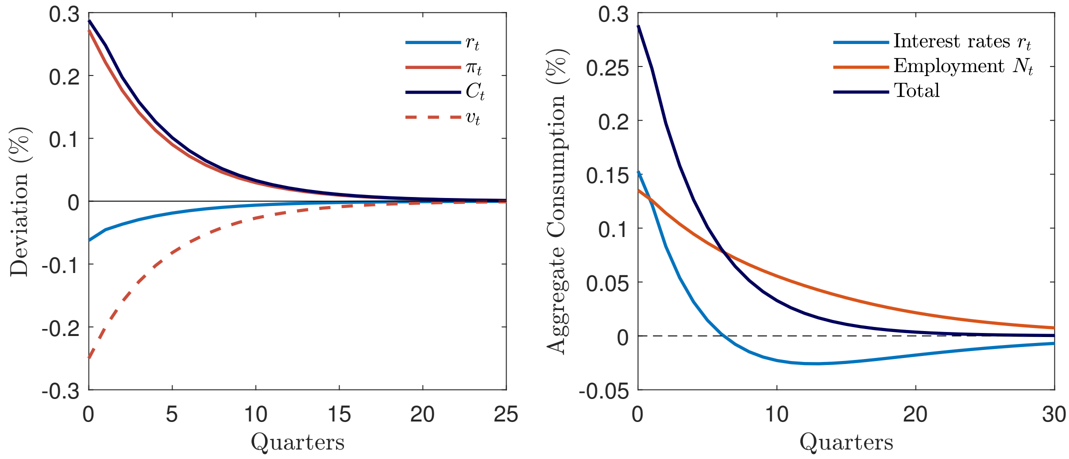

Figure 4: Monetary policy in the basic HANK model.

The right panel in Figure 4 plots a decomposition of the transmission mechanisms of monetary policy to consumption. Specifically, in the HANK framework we have that $C_t(\{r_t, N_t\})$. Totally differentiating this function yields

$$dC_t = \sum_{s=0}^{\infty} \frac{\partial C_t}{\partial r_s} dr_s + \sum_{s=0}^{\infty} \frac{\partial C_t}{\partial N_s} dN_s,$$

this expression provides a decomposition of the transmission channels of monetary policy to aggregate demand into a direct effect related to changes in the real interest rate and an indirect effect due to labor market outcomes and changes in employment. Intuitively, as monetary policy stimulates the economy labor demand increases leading to higher earnings for households (real wages are fixed in the basic model). These income gains feed into consumption for high MPC households increasing aggregate demand. In this calibration, around 1/2 of the initial consumption response is explained by the employment response and over time the importance of these indirect effects increases.

III. Extensions

An important implication of the consumption behavior in HANK is that temporary income effects due to changes in fiscal policy matter for aggregate consumption. As a result in these models the interaction between monetary policy and the response of fiscal policy shapes aggregate demand (Kaplan, Moll, and Violante (2018)). This observation opens important connections between public finance and macroeconomic policy. To study these interactions we need to include fiscal policy in the basic model. Moreover, we can replace the assumption of a competitive good sector with the standard supply block of the basic NK model with market power. Under the assumption of flexible prices, this introduces price markups and firms' profits in equilibrium

$$mc_t = \mu_p^{-1},$$

$$D_t^f = (1 - mc_t)Y_t.$$

Assuming a direct ownership of firms these profits can be distributed to households proportionally to income productivity according to the following rule $d_t = (e_t / \int_X e_t dD_t) D_t^f$. This implies that high-income households receive a larger share of profits, moving the model towards the data. Another important extension of the basic model is to include productive capital and asset pricing in the analysis of monetary policy shocks.

IV. Applications

The HANK literature is quickly developing and there are many exciting research frontiers. A very incomplete list includes: international economics (Ferra, Mitman, and Romei (2020), Auclert, Rognlie, Souchier, and Straub (2021), Bayer, Müller, Kriwoluzky, and Seyrich (2023)), behavioral models (Laibson, Maxted, and Moll (2021), Auclert, Rognlie, and Straub (2020)), and public finance (Angeletos, Lian, and Wolf (2023)).

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