Homework #2

Stat4DS2+DS
https://elearning2.uniroma1.it/course/view.php?id=4951

deadline 23/03/2017 (23:55)

Your Last+First Name Valerio Rossini Your Matricola 1613638

1a) Illustrate the characteristics of the statistical model for dealing with the *Dugong*'s data. Ages (Y_i) and lengths (x_i) of 27 Dugongs have been recorded and the following (non linear) regression model is considered:

$$
\begin{aligned}
Y_i &\sim N(\mu_i, \tau^2) \\
\mu_i = f(x_i) &= \alpha - \beta\gamma^{x_i}
\end{aligned}
$$

Model parameters are $\alpha \in (1, \infty)$, $\beta \in (1, \infty)$, $\gamma \in (0, 1)$, $\tau^2 \in (0, \infty)$. Let us consider the following prior distributions:

$$
\begin{aligned}
\alpha &\sim N(0, \sigma_\alpha^2) \\
\beta &\sim N(0, \sigma_\beta^2) \\
\gamma &\sim Unif(0, 1) \\
\tau^2 &\sim IG(a, b))(InverseGamma)
\end{aligned}
$$

1b) Derive the corresponding likelihood function

1c) Write down the expression of the joint prior distribution of the parameters at stake and illustrate some suitable choice for the hyperparameters.

1d) Compute numerically the maximum likelihood estimate for the vector of parameters of interest $(\alpha, \beta, \gamma, \tau)$ and compare it with the Maximum-a-Posterori estimate

(*write your answers and provide your R code for the numerical solution*)

```
#
#
#
#
#
#
#
#
#
```

2) Consider the Acceptance-Rejection algorithm in the most general form and denote with $\theta = Y^A$ the random variable obtained with the algorithm

2a) Determine the analytic expression of the acceptance probability

2b) Prove that θ has the desired target distribution

2c) Show how in Bayesian inference you could use simulations from the prior (auxiliary density) to get a random draw from the posterior (target distribution) without knowing the proportionality constant

2d) Illustrate analytically possible difficulties of this approach with a simple conjugate model

2e) Verify your conclusions implementing the Acceptance-Rejection approach with your conjugate model (verify empirically that θ has the desired target distribution $\pi(\theta|x_1, .., x_n)$

(*write your answer; provide your R code for the last point*)

```
#
#
#
#
#
#
#
#
#
```

3) Simulate from a standard Normal distribution using pseudo-random deviates from a standard Cauchy and the A-R algorithm. Write the expression the corresponding acceptance probabilityof and evaluate it numerically by MC approximation.

```
#
#
#
#
#
#
#
#
#
```

4) Let us consider a Markov chain $(X_t)_{t\geq 0}$ defined on the state space $\mathcal{S} = \{1, 2, 3\}$ with the following transition

4a) Starting at time $t = 0$ in the state $X_0 = 1$ simulate the Markov chain with distribution assigned as above for $t = 1000$ consecutive times

4b) compute the empirical relative frequency of the two states in your simulation 4c) repeat the simulation for 500 times and record only the final state at time $t = 1000$ for each of the 500 simulated chains. Compute the relative frequency of the 500 final states. What distribution are you approximating in this way? Try to formalize the difference between this point and the previous point.

4d) compute the theoretical stationary distribution π and explain how you have obtained it

4e) is it well approximated by the simulated empirical relative frequencies computed in (b) and (c)?

4f) what happens if we start at $t = 0$ from state $X_0 = 2$ instead of $X_0 = 1$?

```
#
#
#
#
#
#
#
#
#
```

5) Consider again the Bayesian model for Dugong's data:

5a) Derive the functional form (up to proportionality constants) of all *full-conditionals*

5b) Which distribution can you recognize within standard parametric families so that direct simulation from full conditional can be easily implemented ?

5c) Using a suitable Metropolis-within-Gibbs algorithm simulate a Markov chain ($T = 10000$) to approximate the posterior distribution for the above model

5d) Show the 4 univariate trace-plots of the simulations of each parameter

5e) Evaluate graphically the behaviour of the empirical averages $\hat{I}_t$ with growing $t = 1, ..., T$

5f) Provide estimates for each parameter together with the approximation error and explain how you have evaluated such error

5g) Which parameter has the largest posterior uncertainty? How did you measure it?

5h) Which couple of parameters has the largest correlation (in absolute value)?

5i) Use the Markov chain to approximate the posterior predictive distribution of the length of a dugong with age of 20 years.

5j) Provide the prediction of another dugong with age 30

5k) Which prediction is less precise?

```
#
#
#
#
#
#
#
#
#
```

```
## This homework will be graded and it will be part of your final evaluation
##
##
```

```
## Last update by LT: Wed May 17 16:45:54 2017
```