Ricerca Operativa

(30 Aprile 2020)

1. Utilizzando il metodo del simplesso in due fasi, risolvere il seguente problema di PL

$$\begin{aligned} \max\ & 3x_1 + 2x_2 - 5x_3 \\ & x_1 - 2x_2 + x_3 + x_4 = 2 \\ & 3x_1 + x_2 - 2x_3 \leq 6 \\ & x_i \geq 0 \end{aligned}$$

2. Risolvere con il metodo Branch& Bound il seguente problema di PLI

$$\begin{aligned} \max\ & 3x_1 + 2x_2 + x_3 - x_4 + 2x_5 \\ & x1 + x_2 + 2x_3 - 2x_4 + x_5 \leq 2 \\ & x_i \in \{0, 1\} \end{aligned}$$

1. Utilizzando il metodo del simplesso in due fasi, risolvere il seguente problema di PL

$$\begin{aligned} \max\ & 3x_1 + 2x_2 - 5x_3 \\ & x_1 - 2x_2 + x_3 + x_4 = 2 \\ & 3x_1 + x_2 - 2x_3 \leq 6 \\ & x_i \geq 0 \end{aligned}$$

$$\min\ -3x_1 - 2x_2 + 5x_3$$

$$\begin{cases} x_1 - 2x_2 + x_3 + x_4 = 2 \\ 3x_1 + x_2 - 2x_3 + x_5 = 6 \\ x_i \geq 0 \end{cases} \qquad A = \begin{bmatrix} 1 & -2 & 1 & 1 & 0 \\ 3 & 1 & -2 & 0 & 1 \end{bmatrix}$$

Non e' necessaria la fase 1.

$$B = \begin{array}{c} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \begin{matrix} x_4 & x_5 \end{matrix} \end{array} \qquad B^{-1}b = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \qquad B^{-1}N = \begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \end{bmatrix}$$

$$c_B^T = \begin{array}{c} \begin{bmatrix} 0 & 0 \end{bmatrix} \\ \begin{matrix} x_4 & x_5 \end{matrix} \end{array} \qquad c_N^T = \begin{array}{c} \begin{bmatrix} -3 & -2 & 5 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \end{array}$$

$$\gamma^T = \begin{bmatrix} -3 & -2 & 5 \end{bmatrix}$$

$$h = 1 \qquad \bar{u}_h = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad B^{-1}b = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$\bar{\rho} = \min\{2, 2\} = 2 \quad k = 1.$ Esce x_4 entra x_1

$$\rightarrow \left[\begin{array}{c|ccc|c} 1 & 1 & -2 & 1 & 2 \\ 3 & 0 & 1 & -2 & 6 \end{array}\right] = \left[\begin{array}{c|ccc|c} 1 & 1 & -2 & 1 & 2 \\ 0 & -3 & 7 & -5 & 0 \end{array}\right]$$

$$c_B^T = \begin{array}{c} \begin{bmatrix} -3 & 0 \end{bmatrix} \\ \begin{matrix} x_1 & x_5 \end{matrix} \end{array} \qquad c_N^T = \begin{array}{c} \begin{bmatrix} 0 & -2 & 5 \end{bmatrix} \\ \begin{matrix} x_4 & x_2 & x_3 \end{matrix} \end{array}$$

$$\gamma^T = \begin{bmatrix} 0 & -2 & 5 \end{bmatrix} - \begin{bmatrix} -3 & 6 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -8 & 8 \end{bmatrix}$$

$$h = 2 \qquad \pi_h = \begin{bmatrix} -2 \\ 7 \end{bmatrix} \qquad B^{-1}b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\bar{\rho} = \min\{\cdot\,, 0\} = 0 \rightarrow k=2$. Esce x_5 entra x_2.

$$\rightarrow \left[\begin{array}{c|ccc|c} -2 & 1 & 0 & 1 & 2 \\ 7 & -3 & 1 & -5 & 0 \end{array}\right] \sim \left[\begin{array}{c|ccc|c} 0 & 1/7 & 2/7 & -\frac{3}{7} & 2 \\ 1 & -3/7 & 1/7 & -5/7 & 0 \end{array}\right]$$

$$c_B^T = \begin{array}{c} \begin{bmatrix} -3 & -2 \end{bmatrix} \\ \;\; x_1 \;\;\;\; x_2 \end{array} \qquad c_N^T = \begin{array}{c} \begin{bmatrix} 0 & 0 & 5 \end{bmatrix} \\ \;\; x_4 \;\; x_5 \;\; x_3 \end{array}$$

$$\gamma^T = \begin{bmatrix} 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 3/7 & -8/7 & \frac{16}{7} \end{bmatrix} = \begin{bmatrix} -3/7 & 8/7 & 19/7 \end{bmatrix}$$

$$h = 1 \qquad \pi_h = \begin{bmatrix} 1/7 \\ -3/7 \end{bmatrix} \qquad B^{-1}b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\bar{\rho} = \min\{14, \cdot\} = 14 \rightarrow k = 1$ Esce x_1 entra x_4

$$\rightarrow \left[\begin{array}{c|ccc|c} 1/7 & 1 & 2/7 & -3/7 & 2 \\ -3/7 & 0 & 1/7 & -5/7 & 0 \end{array}\right] \sim \left[\begin{array}{c|ccc|c} 1 & 7 & 2 & -3 & 14 \\ 0 & 3 & 1 & -2 & 6 \end{array}\right]$$

$$c_B^T = \begin{array}{c} \begin{bmatrix} 0 & -2 \end{bmatrix} \\ \;\; x_4 \;\;\;\; x_2 \end{array} \qquad c_N^T = \begin{array}{c} \begin{bmatrix} -3 & 0 & 5 \end{bmatrix} \\ \;\; x_1 \;\; x_5 \;\; x_3 \end{array}$$

$$\gamma^T = \begin{bmatrix} -3 & 0 & 5 \end{bmatrix} - \begin{bmatrix} -6 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$$

La SBA attuale è ottima:

$$\bar{x} = \begin{array}{c} \begin{bmatrix} 0 & 14 & 0 & 6 & 0 \end{bmatrix} \\ \;\; x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \end{array}$$

$$\max \; 3x_1 + 2x_2 + x_3 - x_4 + 2x_5$$
$$x1 + x_2 + 2x_3 - 2x_4 + x_5 \leq 2$$
$$x_i \in \{0,1\}$$

Rilassamento lineare: $\max \; 3x_1 + 2x_2 + x_3 - x_4 + 2x_5$

$$\begin{cases} x_1 + x_2 + 2x_3 - 2x_4 + x_5 \leq 2 \\ x_i \in \{0,1\} \end{cases}$$

$c_4 < 0$, $a_4 < 0$

$c_4' = -c_4$ e $a_4' = -a_4$

$\max \; 3x_1 + 2x_2 + x_3 + x_4' + 2x_5$

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_4 + x_5 \leq 2 \\ x_i \in \{0,1\} \end{cases}$$

x_i:	1	2	3	4	5
	3	2	0,5	0,5	2
	y_1	y_2	y_4	y_5	y_3

$\max \; 3y_1 + 2y_2 + 2y_3 + y_4 + y_5$

$$P^\circ \begin{cases} y_1 + y_2 + y_3 + 2y_4 + 2y_5 \leq 2 \\ y_i \in \{0,1\} \end{cases}$$

$\bar{y}^\circ = (1, 1, 0, 0, 0)$

$U^\circ = 5$

$\bar{z} = 5$

La soluzione corrente è ottima:

$$\bar{y}^{\circ} = (\underset{x_1}{1}, \underset{x_2}{1}, \underset{x_5}{0}, \underset{x_3}{0}, \underset{x_4}{0}) \Rightarrow \bar{x}^{\circ} = (1, 1, 0, 0, 0)$$

$$x^* = (1, 1, 0, 0, 0)$$