Problem Set #2 - Answer Sheet

1.
a. 0.28

Let J be the event that the engineer programs in Java, and let C be the event that the engineer programs in C++.

Based on the information given in the problem, we obtain:

$P(J) = 0.35$

$P(JC) = 0.28$

$P(C) = 0.4$

$P(JC)$ is the probability that question a is asking for.

b. 0.7
$$P(J \mid C) = \frac{P(JC)}{P(C)} = \frac{0.28}{0.4} = 0.7$$

2.
a. 0.85
Let

H: The visitor is actually a human

R: The visitor is actually a robot

S: The visitor succeeds at a single test

T: The visitor is flagged as a robot (the user fails at a single test)

Based on the information given in the problem, we obtain:

$P(S \mid H) = 0.95$

$P(S \mid R) = 0.15$

Because $P(F \mid R) + P(S \mid R) = 1$, then we can obtain that

$P(F \mid R) = 1 - 0.15 = 0.85$
which is the probability the visitor is flagged when it's actually a robot.

b. 0.05

Similar to a, $P(F|H) = 1 - P(S|H) = 0.05$

c. 0.472222

Based on the information given in the problem, we obtain: $P(R) = 1 - P(H) = 0.05$

We can further calculate:

$$P(RF) = P(F|R)P(R) = 0.85 \times 0.05 = 0.0425$$

$$P(HF) = P(F|H)P(H) = 0.05 * 0.95 = 0.0475$$

$$P(F) = P(RF) + P(HF) = 0.09$$

And finally we can calculate that

$$P(R|F) = \frac{P(RF)}{P(F)} = \frac{0.0425}{0.09} = 0.472222$$

which is the probability the visitor is actually a robot when it get flagged.

3. $\frac{14}{17}$

Let

W: The computer runs operating system W

X: The computer runs operating system I

I: The computer is infected by a virus

Based on the information given in the problem, we obtain:

$$P(W) = 1 - P(X) = 0.7$$

$$P(I|W) = 2P(I|X) = 2p$$

We can further calculate:

$$P(WI) = P(I|W)P(W) = 1.4p$$

$$P(XI) = P(I|X)P(X) = 0.3p$$

$$P(I) = P(WI) + P(XI) = 1.7p$$

Therefore

$$P(W \mid I) = \frac{P(WI)}{P(I)} = \frac{14}{17}$$

4. $\frac{172}{625}$

X: A random variable representing the number of strings hashed to empty buckets of the first 2 strings

We can cumpute:

$$P(X=0) = (\frac{6}{15})^2 \times (\frac{9}{15})^2$$

$$P(X=1) = (\frac{6}{15} \times \frac{9}{15} + \frac{9}{15} \times \frac{7}{15}) \times (\frac{8}{15})^2$$

$$P(X=2) = \frac{9}{15} \times \frac{8}{15} \times (\frac{7}{15})^2$$

$$p = P(X=0) + P(X=1) + P(X=2) = \frac{172}{625}$$

5.

X: A random variable representing the number of servers that are still working after one year

a. $1-(1-p)^3$

$$P(X>=1) = 1 - P(X=0) = 1-(1-p)^3$$

b. $P(X=2) = \binom{3}{2} p^2(1-p)$

The probability for a particular combination of the two malfunctioning servers and one working server is $p^2(1-p)$, becasue there're $\binom{3}{2}$ combinations, therefore the probability should be multiplied by it.

c. $\binom{3}{2} p^2(1-p) + p^3$

$P(X >= 2) = P(X = 2) + P(X = 3)$

$P(X = 3) = p^3$

6. $\frac{25}{129}$

X: A random variable representing the number of the chosen coin

C: An event that the toss results in the sequence H, T, H

Based on the information given in the problem, we obtain:

$P(C \mid X = 1) = (0.1)^2 \times 0.9$

$P(C \mid X = 2) = (0.5)^3$

$P(C \mid X = 3) = (0.9)^2 \times 0.1$

We can further compute:

$P(C, X = 2) = P(C \mid X = 2)P(X = 2)$

$P(C) = P(C \mid X = 1) + P(C \mid X = 2) + P(C \mid X = 3)$

Therefore:

$$P(X = 2 \mid C) = \frac{P(C \mid X = 2)P(X = 2)}{P(C)} = \frac{25}{129}.$$

7. $\frac{14}{41}; \frac{27}{41}$

Let

L_1: An event that the robot is in location L_1

L_2: And event that the robot is in location L_2

W: An event that there is a window

OW: An event that the robot observes the window

Based on the information given in the problem, we obtain:

$P(L_1) = 1 - P(L_2) = 0.7$

$P(OW \mid \overline{W}) = 0.2$

$P(OW \mid W) = 0.9$

$P(L_1 \mid W) = P(W \mid L_1) = 0$

$P(L_2 \mid W) = P(W \mid L_2) = 1$

We can further compute:

$P(W) = P(WL_1) + P(WL_2) = P(W \mid L_1)P(L_1) + P(W \mid L_2)P(L_2) = 0.3$

$$P(W \mid OW) = \frac{P(W, OW)}{P(OW)} = \frac{P(W, OW)}{P(W, OW) + P(\overline{W}, OW)} = \frac{P(OW \mid W)P(W)}{P(OW \mid W)P(W) + P(OW \mid \overline{W})P(\overline{W})} = \frac{27}{41}$$

$P(L_1 \mid OW) = P(L_1 W \mid OW)P(L_1 \overline{W} \mid OW) = P(L_1 \mid W, OW)P(W \mid OW) + P(L_1 \mid \overline{W}, OW)P(\overline{W})$

Therefore

$$P(L_1 \mid OW) = \frac{14}{41}$$

Similarly

$$P(L_2 \mid OW) = \frac{27}{41}$$

8.

a. $\dfrac{10p}{9p + 1}$

Let

S: An event that the email is actually a spam

NS: An event that the email is actually not a spam

G: An event that the program marks the email GOOD

Based on the information given in the problem, we obtain:

$P(NS) = p$

$P(NS \mid G) = q$

$P(G \mid S) = 0.1$

We can further compute:

$$P(NS \mid G) = \frac{P(G \mid NS)P(NS)}{P(G \mid NS)P(NS) + P(G \mid \overline{NS})P(\overline{NS})} = \frac{p}{p + 0.1(1-p)} = \frac{10p}{9p+1}$$

b.

If $q > p$, then $\frac{10p}{9p+1} > p$, leading $p < 1$, which is what p should be in this case. Therefore q is greater.

Because q only represents the probability of a email is non-spam in the universe of G (where the email is marked GOOD by the program). However, p represents the probability of a email is non-spam under no condition. The set of non-spam emails contains the set of non-spam emails marked GOOD by the program. Therefore q is always greater (and equal to) p.

9.

X**:** The number of blue-eyed gene that William posses

a. $\frac{3}{4}$

Because William's sister has blue eyes, then each of William's parents has at least one blue-eyed gene. Because William's parents both have brown eyes, therefore they both have one blue-eyed gene and one brown-eyed gene.

We can further compute that

$$P(X >= 1) = 1 - P(X = 0) = 1 - \frac{1}{4} = \frac{3}{4}$$

b. $\frac{1}{2}$

$$p = P(X = 2) + \frac{1}{2}P(X = 1) + 0P(X = 0) = \frac{1}{2}$$

10.

Based on the information given in the problem, we obtain:

$P(G) = 0.6$

$P(T_1 | G) = 0.7$

$P(T_2 | G) = 0.9$

$P(T_1 | \overline{G}) = P(T_2 | \overline{G}) = 0$

$P(T_1, T_2 | G) = 0.63$

a. Yes

Because $P(T_1, T_2 | G) = P(T_1 | G)P(T_2 | G) = 0.63$

b. Yes

Because $P(T_1, T_2 | \overline{G}) = P(T_1 | \overline{G})P(T_2 | \overline{G}) = 0$

c. 0.42

$P(T_1) = P(T_1 | G)P(G) + P(T_1 | \overline{G})P(\overline{G}) = 0.42$

d. 0.54

$P(T_2) = P(T_2 | G)P(G) + P(T_2 | \overline{G})P(\overline{G}) = 0.54$

e. No

Because $P(T_1T_2) = P(T_1T_2 | G)P(G) + P(T_1T_2 | \overline{G})P(\overline{G}) = 0.378$ and $P(T_1)P(T_2) = 0.2268$, which are not equal to, then T_1 and T_2 are not independent.

11-12.

See the code file