$$\frac{\partial H_{n_1,n_2}(u,t)}{\partial t} = - \lambda H_{n_1,n_2}(u,t) + j\sigma \overline{E}^N_{n_1+n_2} \frac{\partial H_{n_1,n_2}(u,t)}{\partial u}$$

$$- \mu_1 n_1 H_{n_1,n_2}(u,t) - \mu_2 n_2 H_{n_1,n_2}(u,t)$$

$$+ \mu_1 r_1 q n_1 H_{n_1,n_2}(u,t) + \mu_2 r_1 (1-q) n_2 H_{n_1,n_2}(u,t)$$

$$+ \lambda e^{ju} E^N_{n_1+n_2} H_{n_1,n_2}(u,t)$$

$$+ \lambda q H_{n_1-1,n_2}(u,t) - j\sigma q e^{-ju} \frac{\partial H_{n_1-1,n_2}(u,t)}{\partial u}$$

$$+ \lambda (1-q) H_{n_1,n_2-1}(u,t) u - j\sigma(1-q) e^{-ju} \frac{\partial H_{n_1,n_2-1}(u,t)}{\partial u}$$

$$+ \mu_1 r_0 (n_1+1) H_{n_1+1,n_2}(u,t) + \mu_1 r_2 e^{ju} (n_1+1) H_{n_1+1,n_2}(u,t)$$

$$+ \mu_2 r_0 (n_2+1) H_{n_1,n_2+1}(u,t) + \mu_2 r_2 e^{ju} (n_2+1) H_{n_1,n_2+1}(u,t)$$

$$+ \mu_1 r_1 (1-q)(n_1+1) H_{n_1+1,n_2-1}(u,t) + \mu_2 r_1 q (n_2+1) H_{n_1-1,n_2+1}(u,t)$$

$$\tag{1}$$

$$\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\frac{\partial H_{n_1,n_2}(u,t)}{\partial t} = -\lambda\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}H_{n_1,n_2}(u,t) + j\sigma\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\overline{E}_{n_1+n_2}^{N}\frac{\partial H_{n_1,n_2}(u,t)}{\partial u}$$

$$-\mu_1\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_1 H_{n_1,n_2}(u,t) - \mu_2\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_2 H_{n_1,n_2}(u,t)$$

$$+\mu_1 r_1 q\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_1 H_{n_1,n_2}(u,t) + \mu_2 r_1(1-q)\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_2 H_{n_1,n_2}(u,t)$$

$$+\lambda e^{ju}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}E_{n_1+n_2}^{N}H_{n_1,n_2}(u,t)$$

$$+\lambda q\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}H_{n_1-1,n_2}(u,t) - j\sigma q e^{-ju}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\frac{\partial H_{n_1-1,n_2}(u,t)}{\partial u}$$

$$+\lambda(1-q)\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}H_{n_1,n_2-1}(u,t)u - j\sigma(1-q)e^{-ju}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\frac{\partial H_{n_1,n_2-1}(u,t)}{\partial u}$$

$$+\mu_1 r_0\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}(n_1+1)H_{n_1+1,n_2}(u,t) + \mu_1 r_2 e^{ju}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}(n_1+1)H_{n_1+1,n_2}(u,t)$$

$$+\mu_2 r_0\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}(n_2+1)H_{n_1,n_2+1}(u,t) + \mu_2 r_2 e^{ju}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}(n_2+1)H_{n_1,n_2+1}(u,t)$$

$$+\mu_1 r_1(1-q)\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}(n_1+1)H_{n_1+1,n_2-1}(u,t)$$

$$+\mu_2 r_1 q\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}(n_2+1)H_{n_1-1,n_2+1}(u,t)$$

$$(2)$$

$$\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\frac{\partial H_{n_1,n_2}(u,t)}{\partial t} = -\lambda\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}H_{n_1,n_2}(u,t) + j\sigma\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}\frac{\partial H_{n_1,n_2}(u,t)}{\partial u}$$

$$-\mu_1(r_0+r_2)\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_1 H_{n_1,n_2}(u,t) - \mu_2(r_0+r_2)\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_2 H_{n_1,n_2}(u,t)$$

$$+\mu_1 r_1 q\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_1 H_{n_1,n_2}(u,t) + \mu_2 r_1(1-q)\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_2 H_{n_1,n_2}(u,t)$$

$$+\lambda e^{ju}\sum_{n_1=0}^{N}\sum_{n_2=N-n_1}^{N-n_1}H_{n_1,n_2}(u,t)$$

$$+\lambda q\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_2}H_{n_1,n_2}(u,t) - j\sigma q e^{-ju}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}H_{n_1,n_2}(u,t)$$

$$+\lambda(1-q)\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}\frac{\partial H_{n_1,n_2}(u,t)}{\partial u} - j\sigma(1-q)e^{-ju}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}\frac{\partial H_{n_1,n_2}(u,t)}{\partial u}$$

$$+\mu_1 r_0\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_1 H_{n_1,n_2}(u,t) + \mu_1 r_2 e^{ju}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_1 H_{n_1,n_2}(u,t)$$

$$+\mu_2 r_0\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_2 H_{n_1,n_2}(u,t) + \mu_2 r_2 e^{ju}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}n_2 H_{n_1,n_2}(u,t)$$

$$\tag{3}$$

$$\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\frac{\partial H_{n_1,n_2}(u,t)}{\partial t} = +\lambda(e^{ju}-1)\sum_{n_1=0}^{N}\sum_{n_2=N-n_1}^{N-n_1}H_{n_1,n_2}(u,t)$$

$$-j\sigma q(e^{-ju}-1)\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}H_{n_1,n_2}(u,t)$$

$$-j\sigma(1-q)(e^{-ju}-1)\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}\frac{\partial H_{n_1,n_2}(u,t)}{\partial u}$$

$$+\mu_1 r_2(e^{ju}-1)\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}n_1 H_{n_1,n_2}(u,t) + \mu_2 r_2(e^{ju}-1)\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1-n_1}n_2 H_{n_1,n_2}(u,t)$$

$$\tag{4}$$