NUMERICAL LINEAR ALGEBRA

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Politecnico di Milano

https://antonietti.faculty.polimi.it

TA: Dr. Michele Botti

MODELLISTICA E CALCOLO SCIENTIFICO
MOX
MODELING AND SCIENTIFIC COMPUTING

POLITECNICO MILANO 1863 | DEPARTMENT OF MATHEMATICS

P0: Introduction

Course information

Lectures/TA:

Prof. Paola F. Antonietti (paola.antonietti@polimi.it)

Dr. Michele Botti (michele.botti@polimi.it)

Timetable: The calendar is available on WeBeep [Link]. Please check it regularly as there might be changes.

Office hours: Upon request. Please send an email to paola.antonietti@polimi.it

Recorded lectures: Links to the recorded lectures will be progressively made available on the WeBeep page course [Link].

Slides: Slides will be progressively made available on the WeBeep page

Exam rules

The exam consists of two components: course hands-on challenges (completed during the course) and a written test.

An optional oral exam may be taken upon the student's request.

Read carefully the exam guidelines on WeBeep!!

Exam - Written test

The written exam will be held in the computer room. It covers all theoretical and practical topics discussed in the lectures and lab sessions. Some of the questions and problems will require numerical solutions, including the implementation and programming of numerical algorithms.

The exam focuses on the definition and application of key lemmas, theorems, and important examples.

Light calculations may be required. The use of course materials, including books, notes, or any form of preparation aids, is not permitted.

First written examination session:
7 November 2025, 11:15 (date TBC)

Exam - Course Hands-on Challenges

Two hands-on exercises are conducted during the course.

Course Hands-on Challenges are designed to apply both theoretical and computational knowledge to practical scenarios.

The exercises are completed in small groups (3 students per group) during the course and consist of simple tasks on practical topics. Students are required to collaborate and submit their assignments (via a Microsoft MS form) according to the following schedule:

- **Course Hands-on Challenge I: Assigned on September 22, 2025. Submission deadline: October 6, 2025 (h 23:59)**
- **Course Hands-on Challenge II: Assigned on October 13, 2025. Submission deadline: October 27, 2025 (h 23:59)**
- **Groups of three students must be formed by September 21, 2025 (23:59), compiling a dedicated form.**

Groups - please adhere to the following guidelines

- Group formation:
  - Each group must consist of three students. All group members should collaborate to submit a single answer for the challenges assigned.
- Deadline
  - Groups must be formed by **September 21, 2025, at 23:59**.
- To register your group, please fill out the dedicated Group Registration Form by the deadline.
- Submission process:
  - The form will require you to list the names and student person code of all group members.
  - Ensure all group members agree on the composition before submission, as no changes will be allowed after the deadline.
  - Please organise yourselves and complete the form on time to

If you encounter any issues or have any questions, do not hesitate to reach out for assistance.

Groups - please fill in the form

Deadline: **September 21, 2025, at 23:59**

https://forms.cloud.microsoft/e/HgHVTbRhC7

Modelling and Scientific Computing

Modelling and Scientific Computing

- Scientific computing uses computational methods and algorithms to solve complex scientific problems, integrating mathematics, computer science, and domain-specific knowledge.
- The field fosters collaboration among applied mathematicians, engineers, and domain-specific scientists, facilitating physics-based decision-making that supports sustainable development.

Key Enabling technologies in Scientific Computing

Mathematical Models and Numerical Algorithms

Enable the simulation of complex system interactions, with a rigorous physics-inspired

High-Performance Computing (HPC)

HPC systems enable the processing of complex simulations, allowing researchers to model complex systems, optimize processes and predict impacts with unprecedented accuracy

Machine Learning Algorithms

Analyze large volumes of data to identify patterns and trends, inform physics-based models facilitating predictive analytics and enhance decision-making

Superpowers of supercomputers

1985: CRAY-2

4 processors

1.95 GigaFLOPS (Rpeak)

2025: El Capitan (Lawrence Livermore National Laboratory, United States)

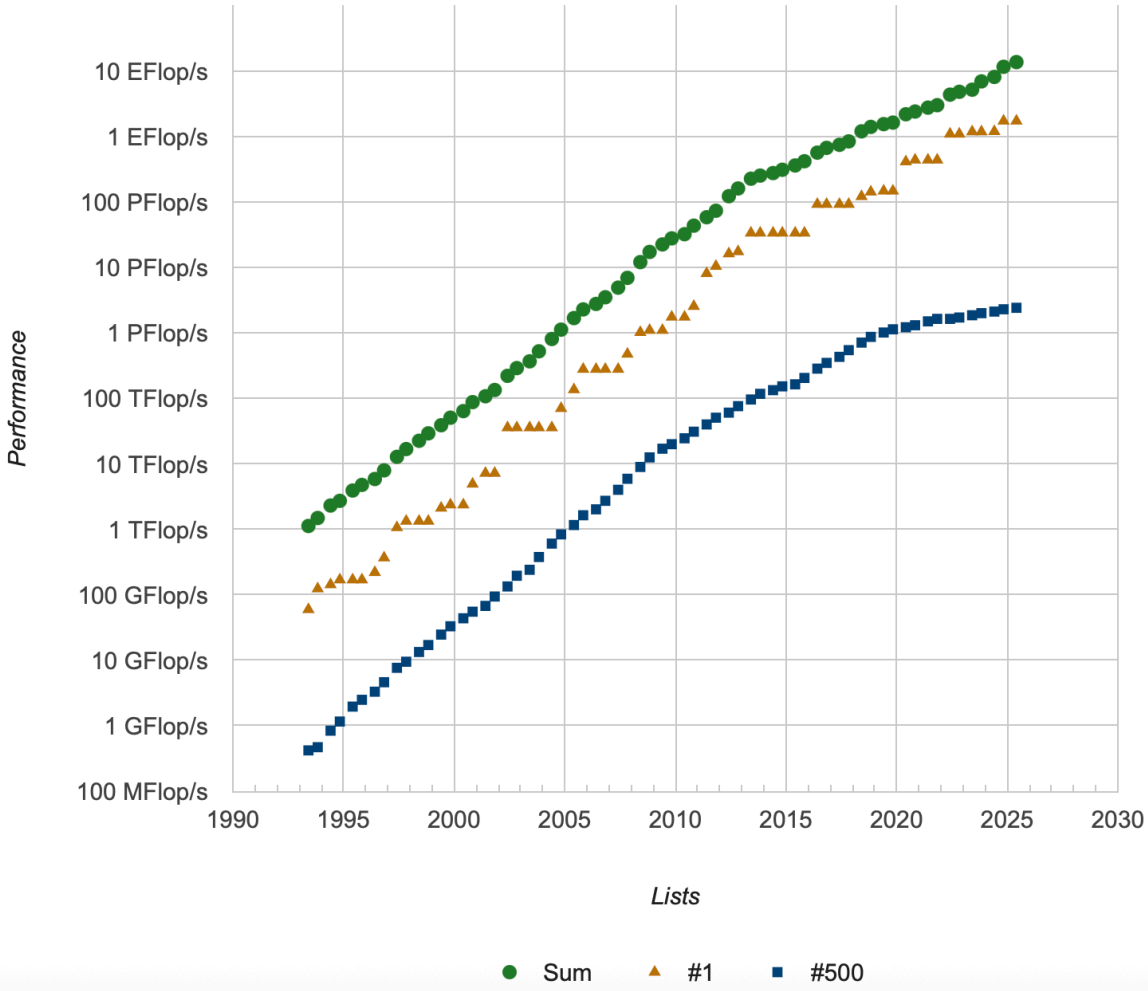
11,039,616 Cores

2.746,38 PFlop/s (RPeak)

- 1 GigaFLOPS $\approx 10^9$ flops
- 1 TeraFLOPS$\approx 10^{12}$ flops
- 1 PetaFLOPS $\approx 10^{15}$ flops
- 1 ExaFLOPS $\approx 10^{18}$ flops

Performance development

https://www.top500.org



yottaFLOPS	10^{24}
zettaFLOPS	10^{21}
exaFLOPS	10^{18}
petaFLOPS	10^{15}
teraFLOPS	10^{12}
gigaFLOPS	10^{9}
megaFLOPS	10^{6}
kiloFLOPS	10^{3}
FLOPS	1

Modeling, Scientific Computing and Sustainability

Predictive Insights

Modeling provides essential predictive insights into complex systems, allowing to make informed decisions based on potential outcomes.

Resource Optimisation

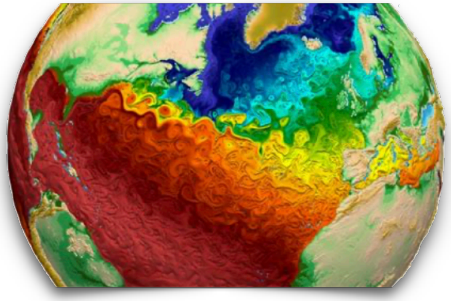
Through simulations, models help optimise the use of resources

Exploring Scenarios

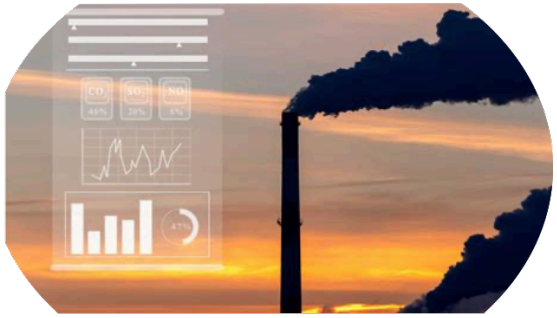
Modeling enables the exploration of multiple scenarios, which is crucial for effective policy-making and strategic planning.

Some successful stories

Personalised medicine



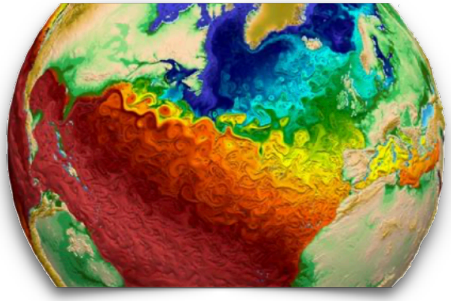
Enviromental modeling



Sustainable production

Some successful stories

Personalised medicine



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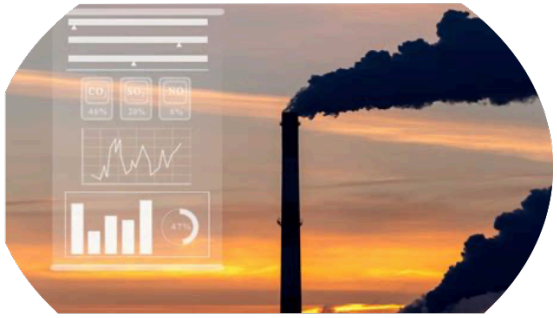
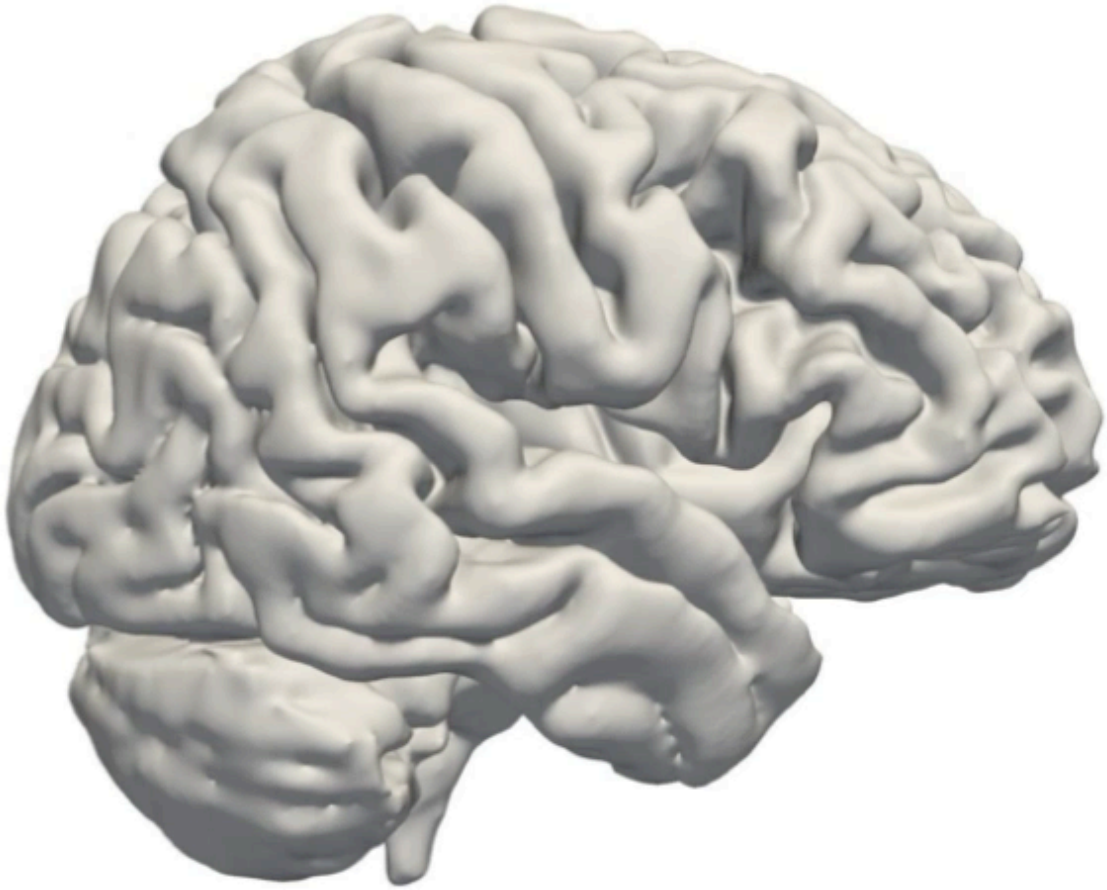
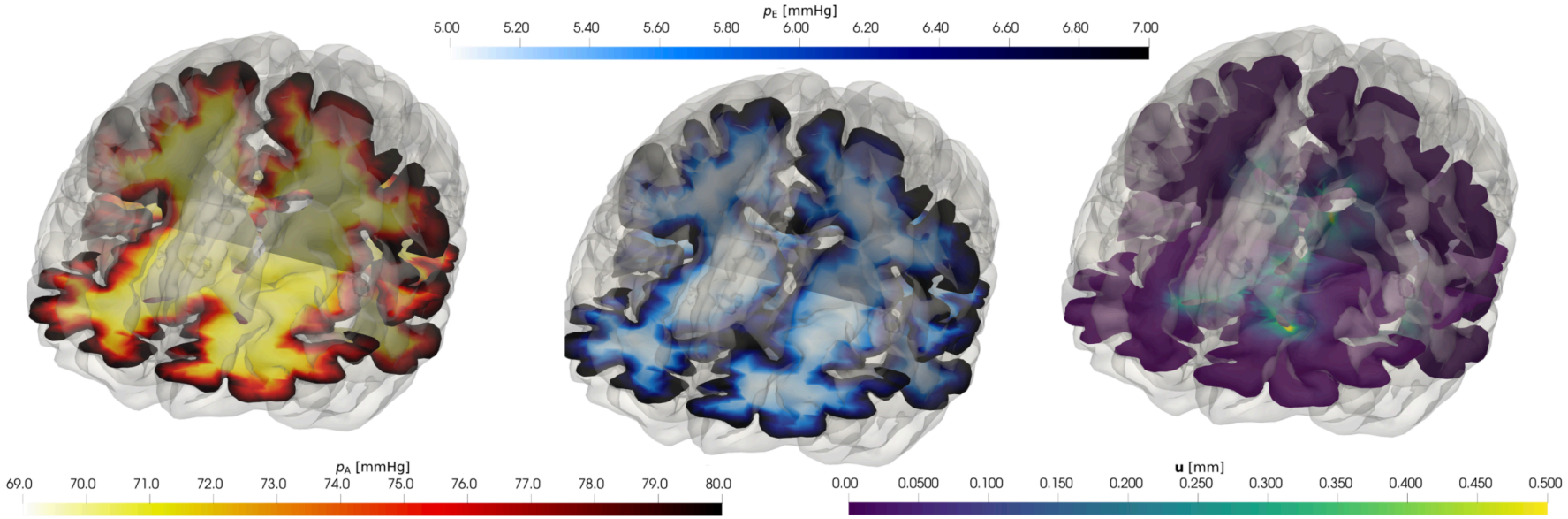


Sustainable production

Computational neurosciences



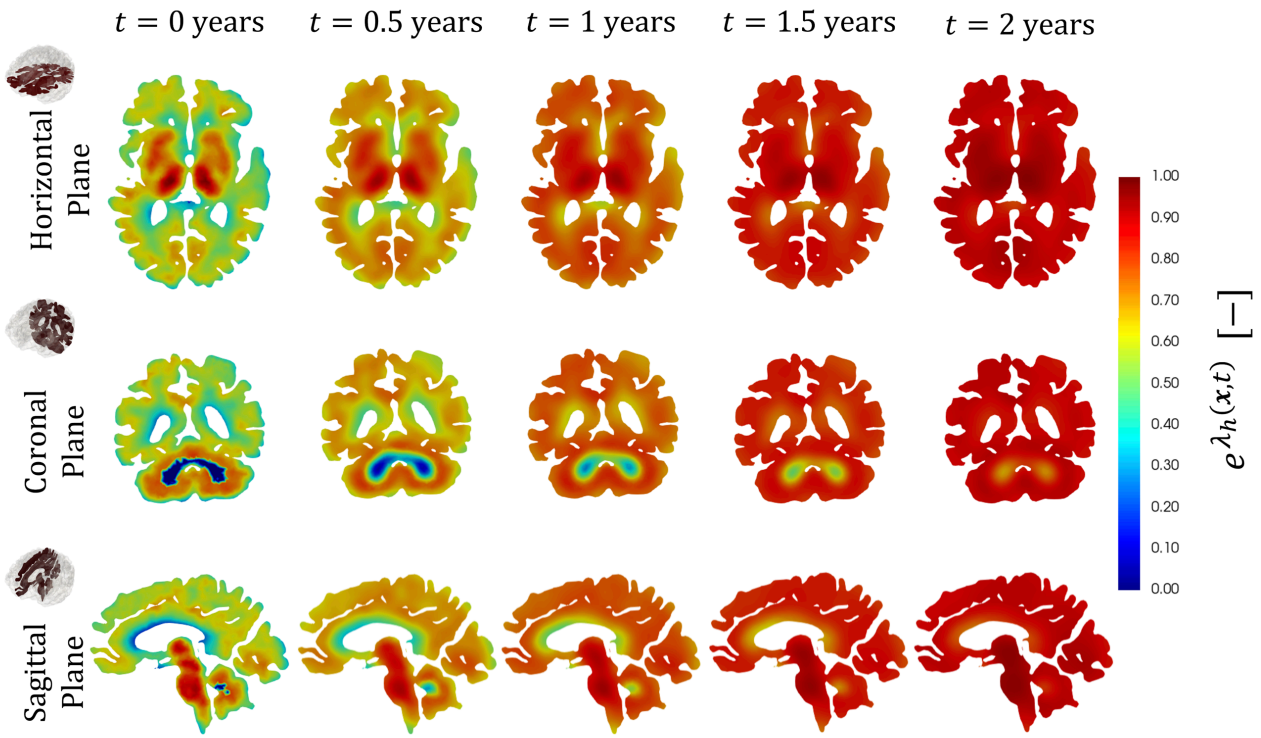
Computational neurosciences



Arterial pressure, CSF pressure, and tissue displacement resulting from a multiple network poroelasticity simulation in physiological conditions [Corti, Antonietti, Dede', Quarteroni. M3AS, accepted (2023)]

https://brainum.mox.polimi.it

Computational neurosciences



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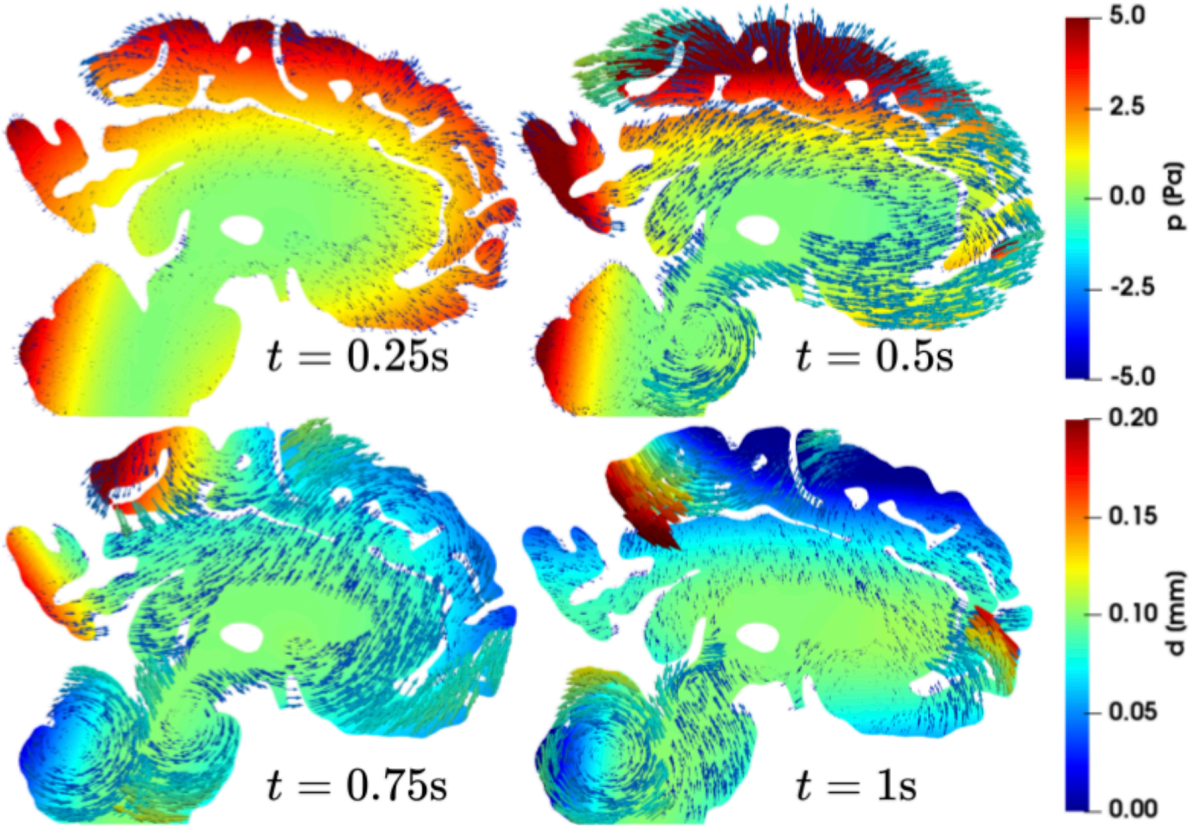
Computational neurosciences

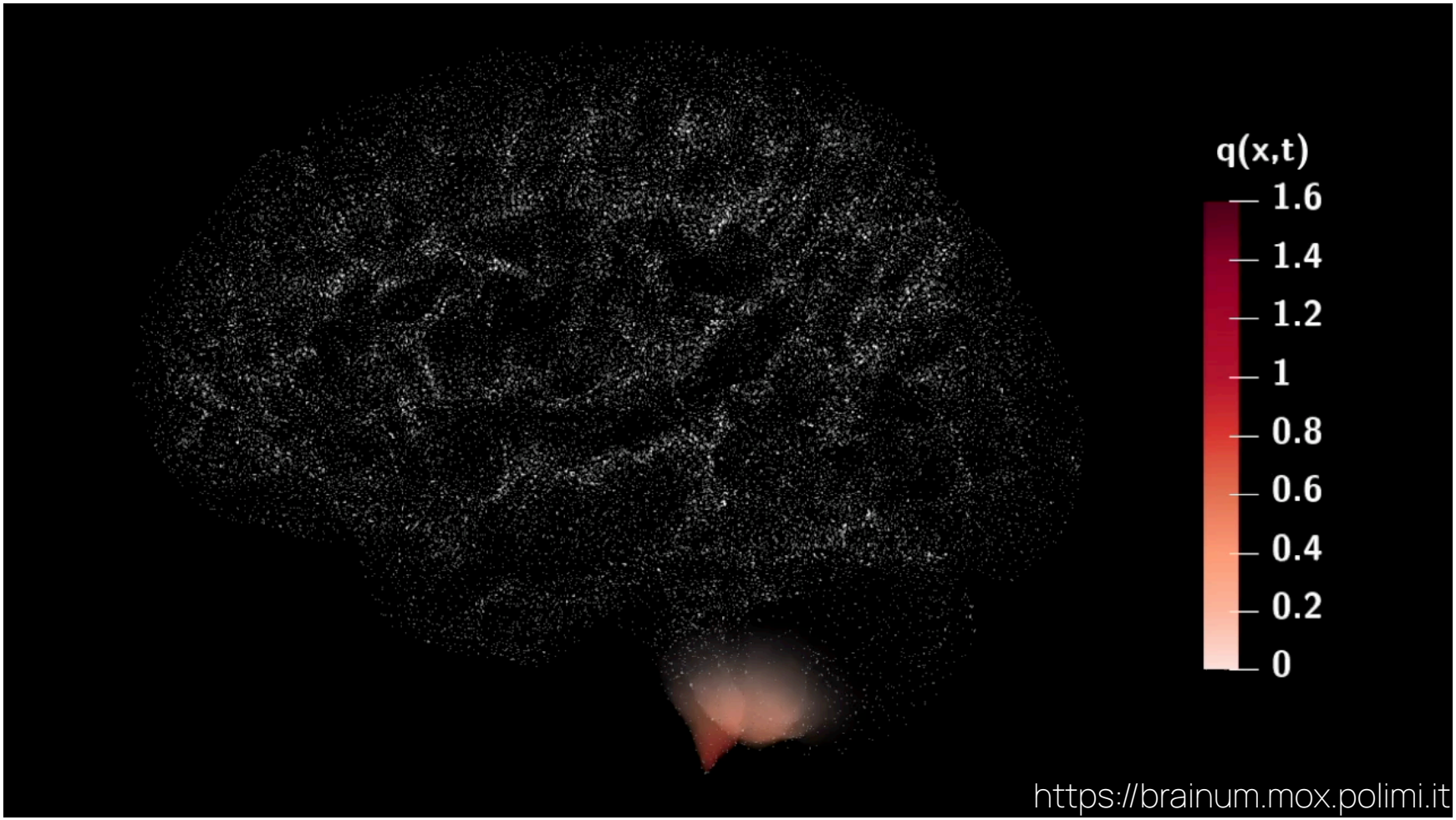


Intracranial pressure and parenchymal displacement obtained simulating the coupling between the CSF in the brain ventricles and the surrounding poroelastic tissue [Fumagalli, Corti, Parolini, Antonietti. arXiv:2310.07651 (2023)]

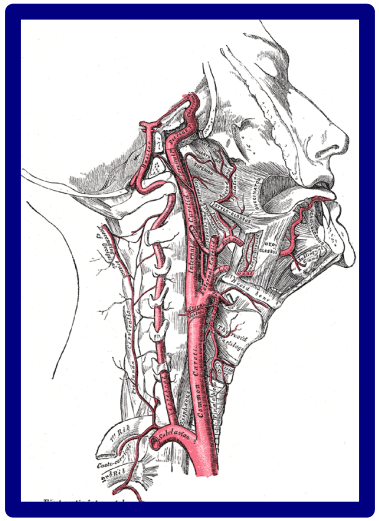
https://brainum.mox.polimi.it

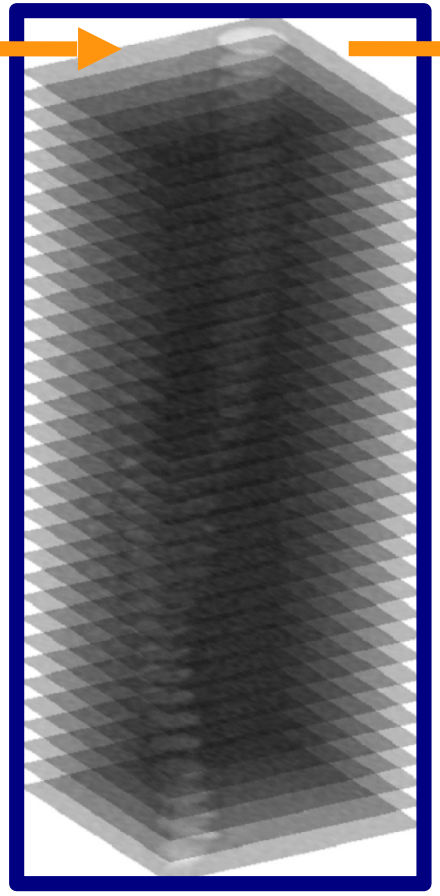
Computational neurosciences

Patterns of α-synuclein concentration at different stages of Parkinson's disease with volume rendering

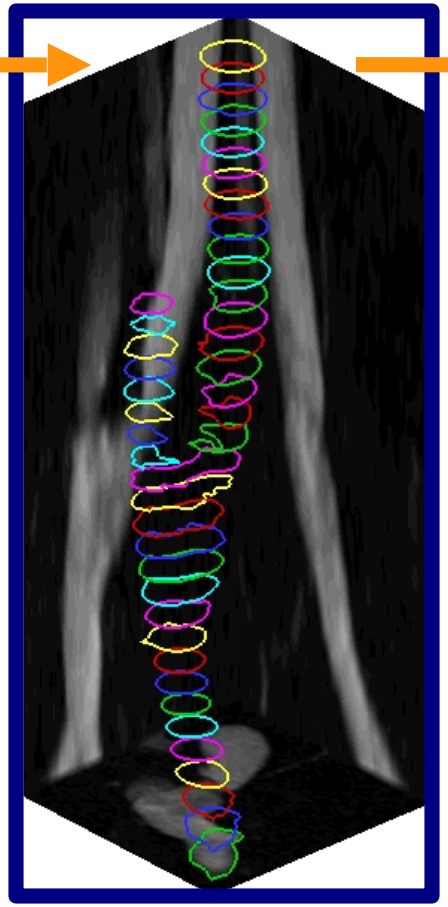


Cardiovascular mathematics

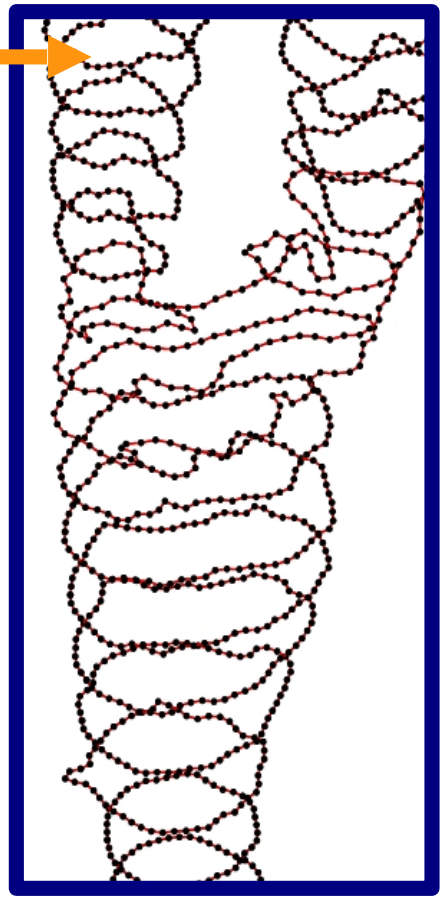




MRI



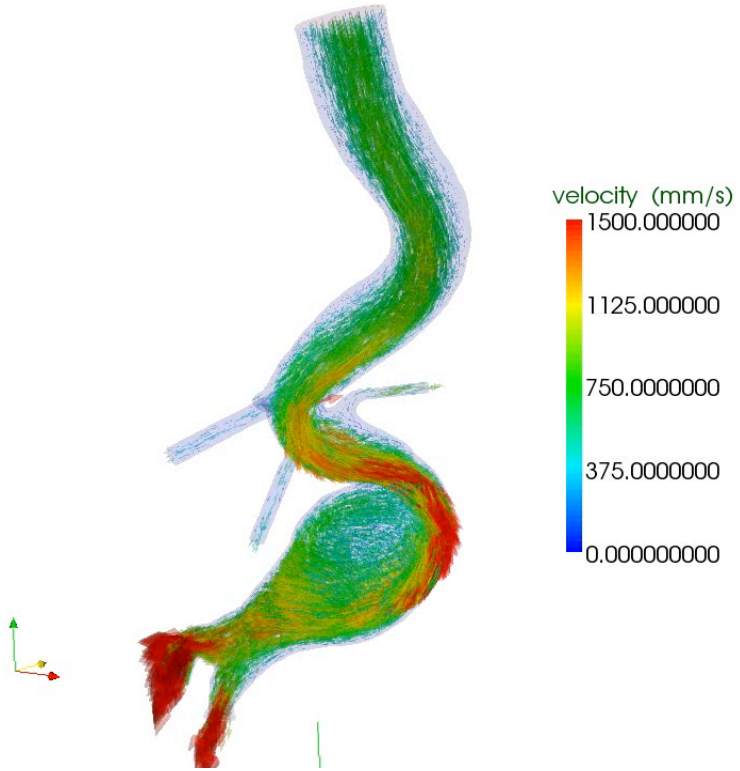
Contours segmentation



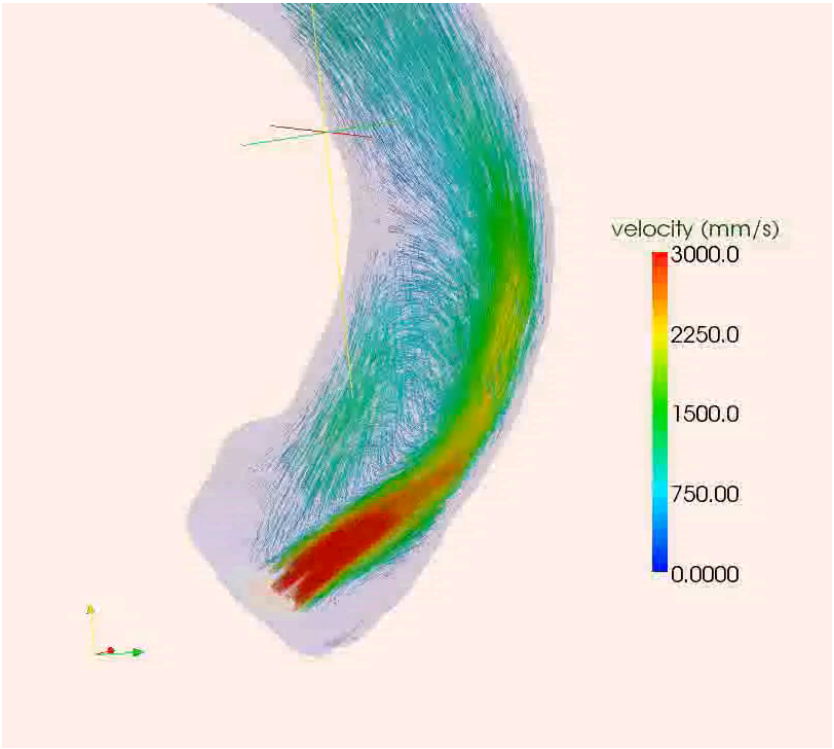
Reconstructed geometry

http://mox.polimi.it

Blood flow simulations



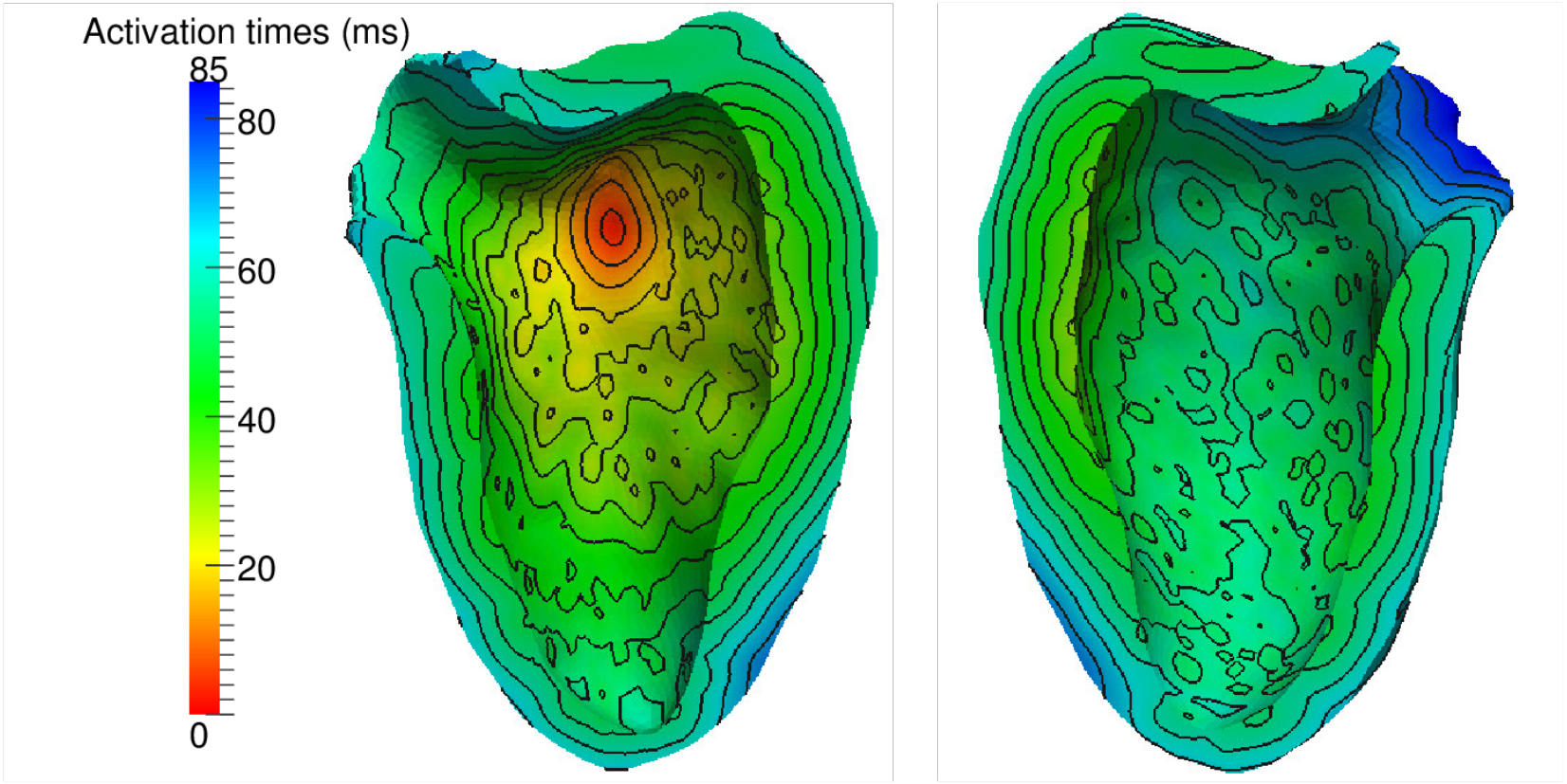
Velocity in an abdominal aneurysm



Velocity streamlines in a human carotid (bicuspid valve)

http://mox.polimi.it

Electrical activity of the heart



Map of activation time

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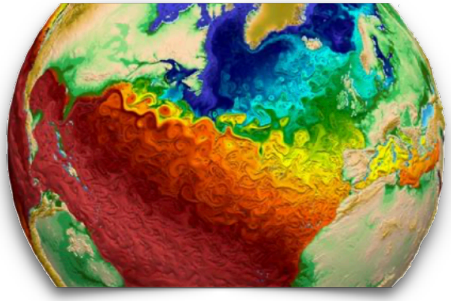
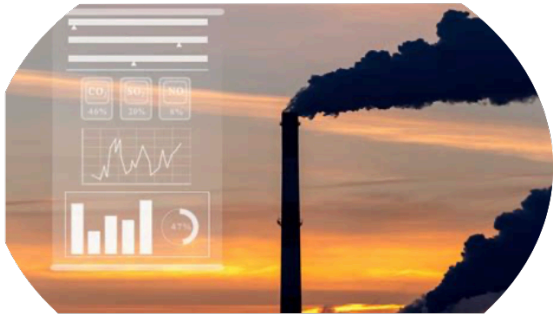
Some successful stories

Personalised medicine



Enviromental modeling



Sustainable production

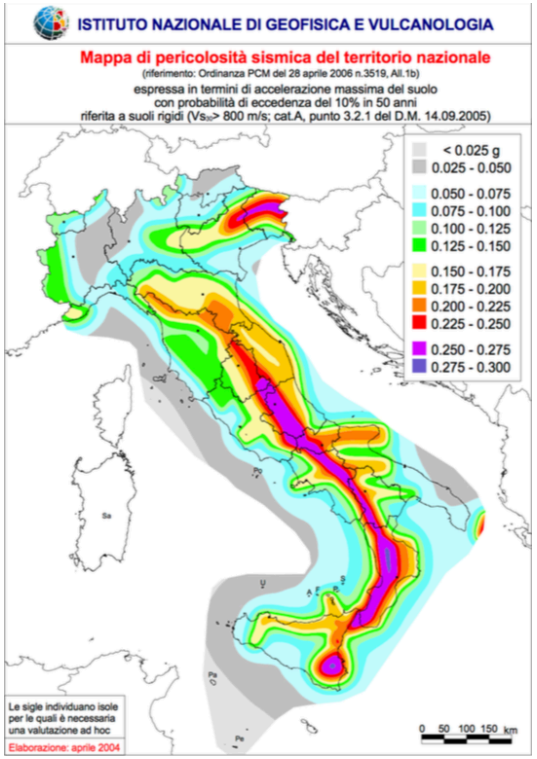
Extreme natural events

Italy has a high exposure to natural risks: earthquakes, floods, landslides, volcanic eruptions, fires. Our territory is fragile.

Hotel Duca degli Abruzzi (L'Aquila)

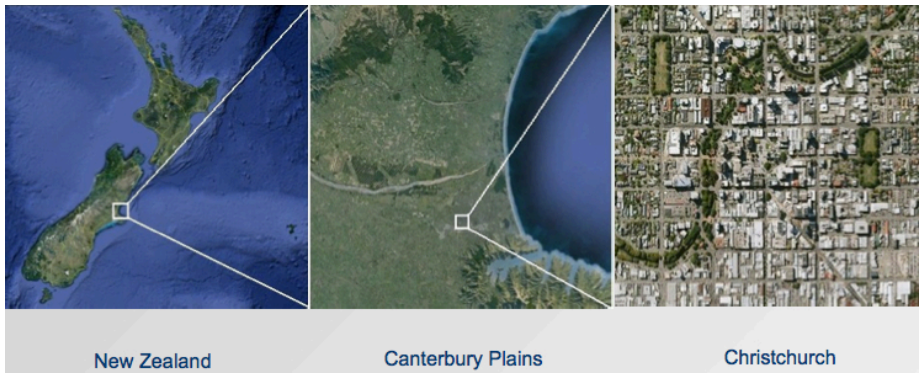
Edificio massiccio su pilastri sottili, altezza irregolare dei pilastri, presenza di corpi strutturali di altezza diversa.

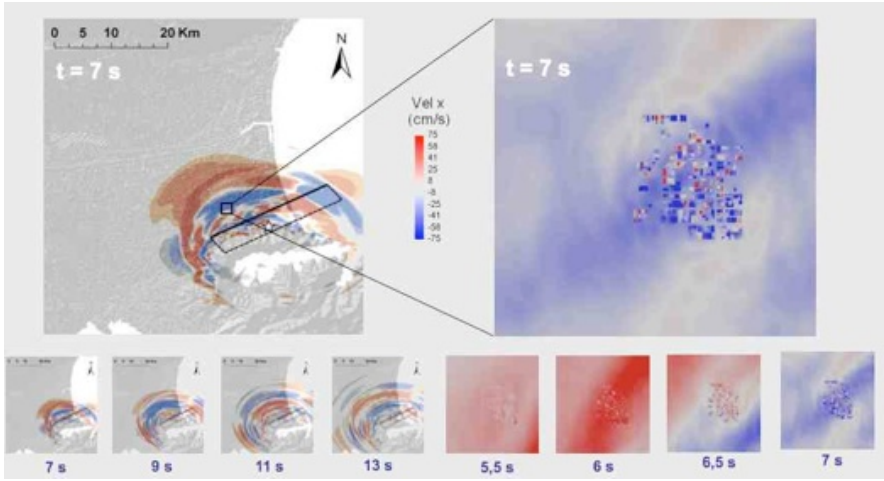
Courtesy of Prof. Paolucci

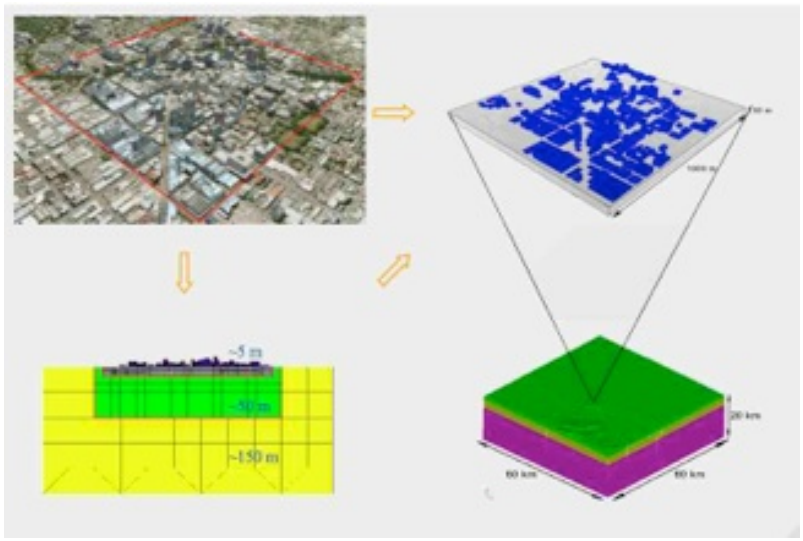


The probabilistic seismic hazard map for Italy.

ChristChurch (NZ) earthquake, 22.02.201, Mw 6.2

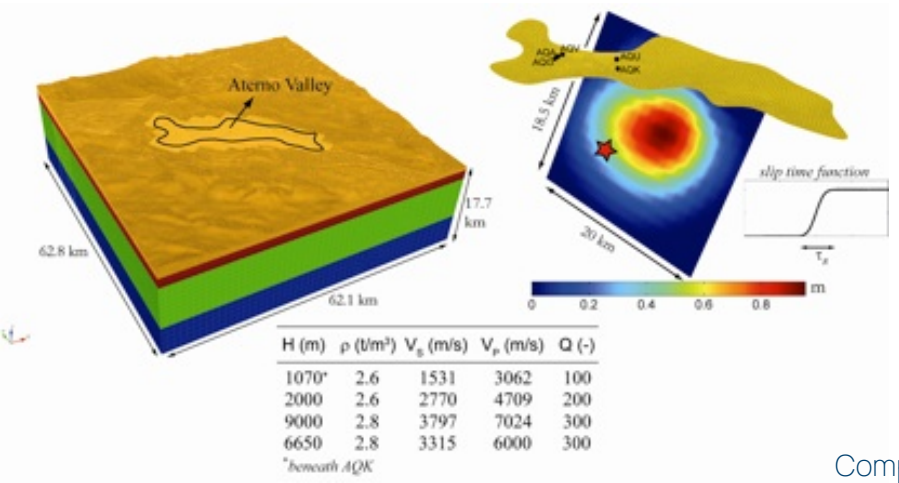






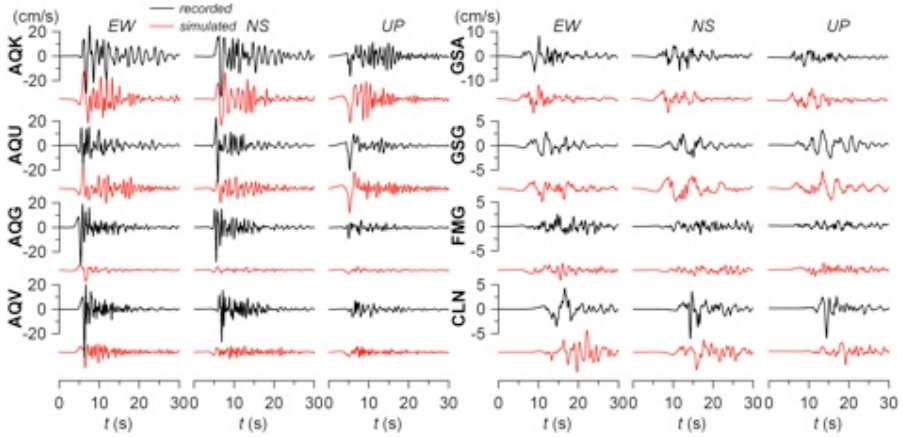
http://speed.mox.polimi.it/

L'Aquila (IT), 06.04.2009 Mw 6.3

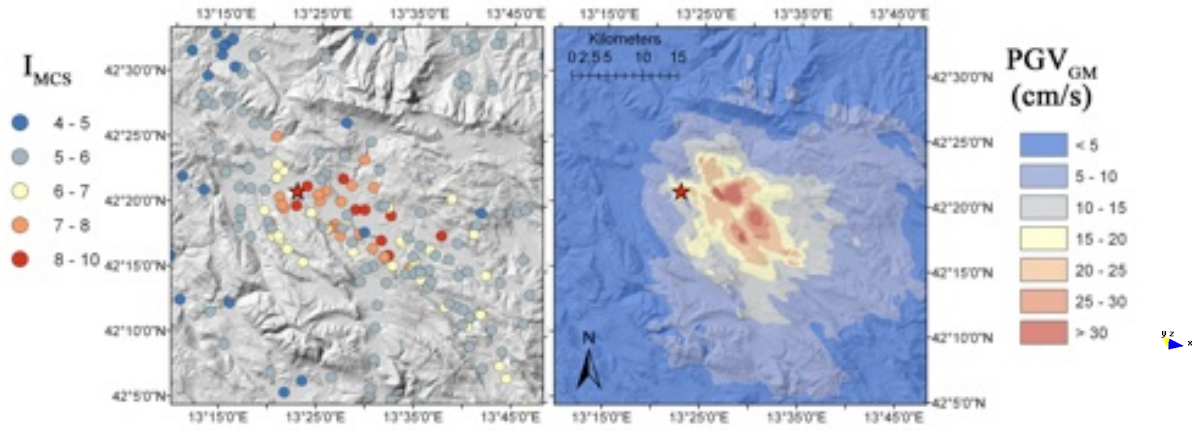


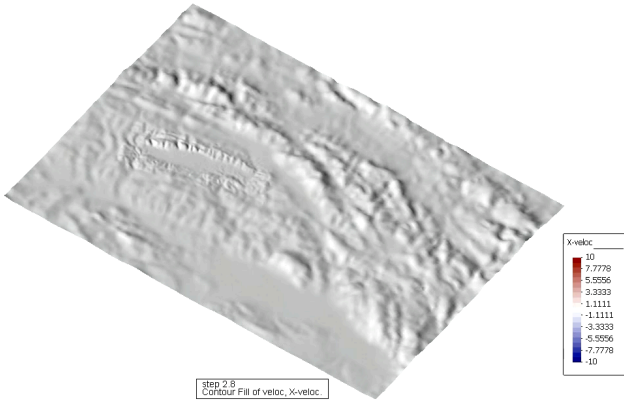
H (m)	ρ (t/m³)	V_s (m/s)	V_p (m/s)	Q (-)
1070*	2.6	1531	3062	100
2000	2.6	2770	4709	200
9000	2.8	3797	7024	300
6650	2.8	3315	6000	300

*beneath AQK



Comparison of the synthetics and strong motion recordings at 8 selected sites in terms of three component velocity time histories (0.1-2.5 Hz).





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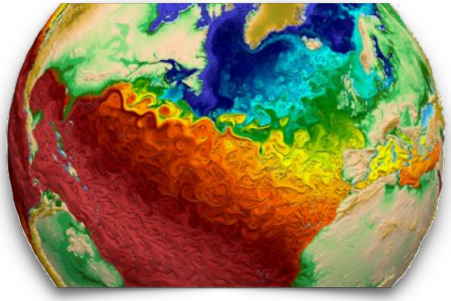
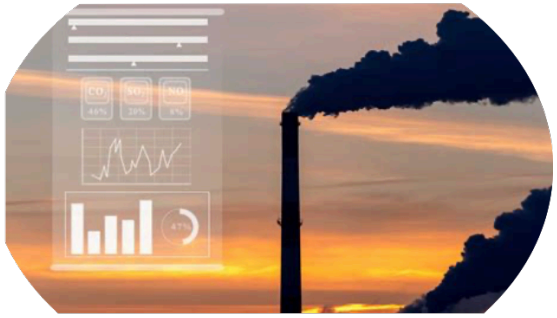
Some successful stories

Personalised medicine



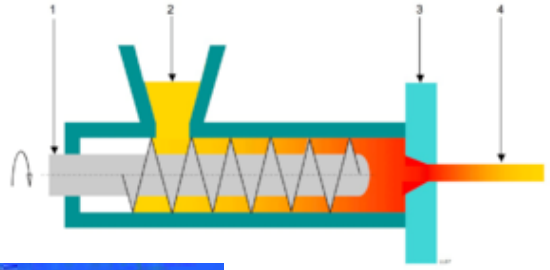
Enviromental modeling

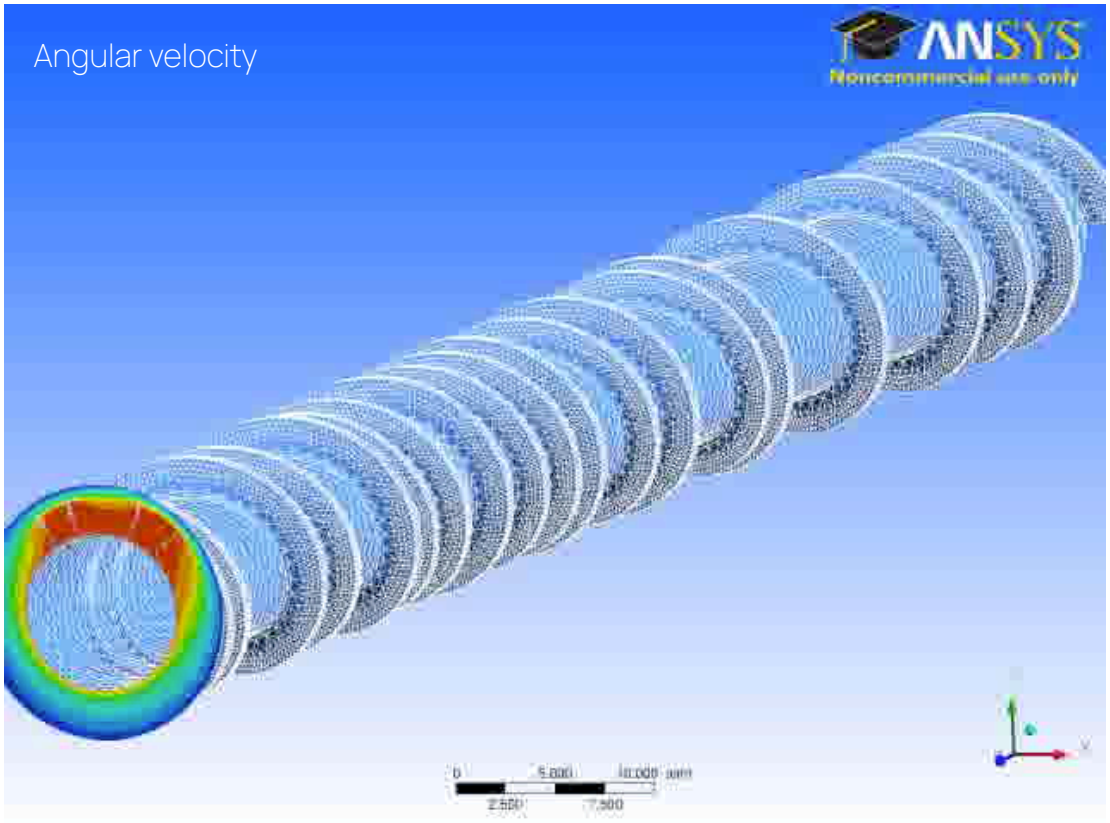


Sustainable production

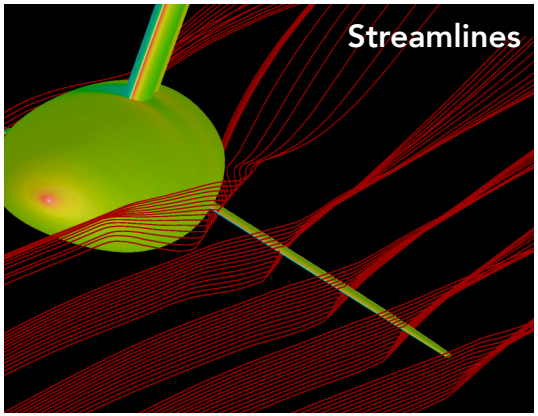
Advanced manufacturing processes

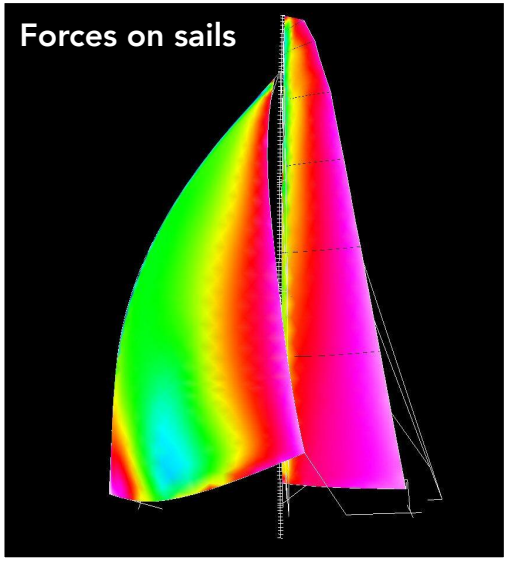
Extrusion is a process used to create objects of a fixed cross-sectional profile. A material is pushed through a die of the desired cross-section.

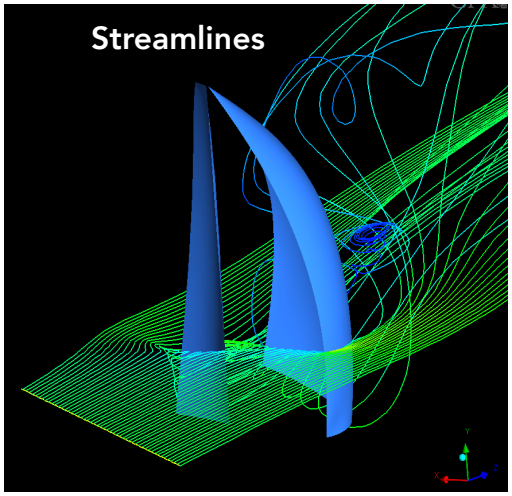


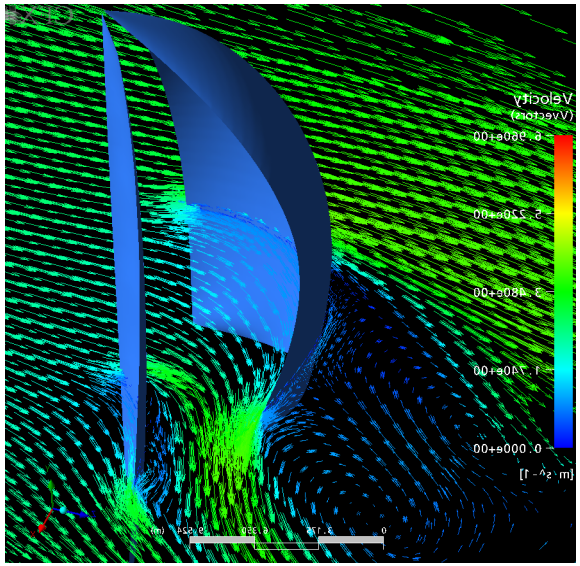


Sport competitions









velocity and flow separation