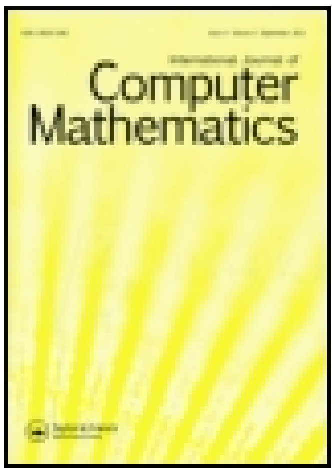

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CHEBYSHEV SOLUTION OF LAMINAR BOUNDARY LAYER FLOW

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An expansion procedure using the Chebyshev polynomials is proposed by using El-Gendi method [1], which yields more accurate results than those computed by P. M. Beckett [2] and A. R. Wadia and F. R. Payne [6] as indicated from solving the Falkner–Skan equation, which uses a boundary value technique. This method is accomplished by starting with Chebyshev approximation for the highest-order derivative and generating approximations to the lower-order derivatives through integration of the highest-order derivative.

KEY WORDS: Chebyshev approximation, non-linear equations, laminar boundary layer.

C.R. CATEGORIES: 5.17

1. INTRODUCTION

The use of Chebyshev polynomial approximations for the solution of boundary value problem in fluid mechanics has been advocated and developed by Orszag [4]. These approximations which include the Galerkin and Tau spectral methods pseudo spectral collocation method, are discussed in detail by Gottlieb and Orszag [3].

The problems of laminar or turbulent boundary layer resulting from the flow of an incompressible fluid past a semi-infinite wedge is of considerable practical and theoretical interest. Non-linear problems with semi-infinite domains are frequently encountered in the study of laminar and turbulent boundary layers. Due to the appearance of irregular boundaries, shock waves, boundary layers, derivative boundary conditions, etc., the solutions so obtained have in many cases been unsatisfactory because of poor resolution, spurious oscillations, and excessive computer time or storage.

For the past two decades finite difference numerical methods have been used extensively for the evaluation of flow and high speed, viscous and inviscid problems in fluid mechanics. Recently the similarity solutions are well studied for the testing finite difference methods. Beckett [2], Wadie and Payne [6], Rosenhead [5] and White [7] have discussed different finite difference schemes for the solutions of the Falkner–Skan equations or Blasuis equation.

In any case the farfield boundary condition is a problem, where the correct value of the unknown shear stress f''_w at the wall must be found which ensures an asymptotic approach of the velocity values f' at infinity, to unity, the well known matching condition of viscous solution (near-wall) to the inviscid solutions. Such techniques are termed as "shooting" method. In this paper we introduce a new procedure for the numerical solution based on the Chebyshev approximation using the EL-Gendi method [1] for the numerical solution of the Falkner–Skan equation. Our method defines the unknown wall shear stress f''_w directly without need of any correction interpolation method. We compare our results with values obtained by Beckett [2], Wadie and Payne [6], Resenhead [5] and White [7] as shown.

2. METHOD OF SOLUTION

Consider the Falkner–Skan equation for stagnation flows with the similarity property [5, 7].

$$f'''(\eta)+\alpha f(\eta)f''(\eta)+\beta[1-f'^2(\eta)]=0 \tag{1}$$

together with the boundary conditions:

$$f(0)=f'(0)=0 \quad \text{and} \quad f'(\eta\rightarrow\infty)=1. \tag{2}$$

Here α is assumed constant, β is a measure of the pressure gradient. The prime denotes differentiation with respect to η. The special case of the Blasius similarity relation for incompressible viscous flow along a flat plate results when $\alpha=1$ and $\beta=0$.

The domain $0\leqq\eta\leqq\eta_\infty$; where η_∞ is one end of the user specified computational domain. Using the algebraic mapping

$$x=\frac{2\eta}{\eta_\infty}-1. \tag{3}$$

The unbounded region $[0,\infty)$ is mapped into the finite domain $[-1,1]$ and the problem expressed by Eq. (1), (2) is transformed into

$$f'''+\alpha\frac{\eta_\infty}{2}ff''+\beta\left[\left(\frac{\eta_\infty}{2}\right)^3-\left(\frac{\eta_\infty}{2}\right)f'^2\right]=0 \tag{4}$$

together with the boundary conditions

$$f(-1)=f'(-1)=0 \quad \text{and} \quad f'(1)=\frac{\eta_\infty}{2} \tag{5}$$

where the prime denotes differentiation with respect to x.

Our technique is accomplished by starting with Chebyshev approximation for the highest-order derivative, f''', and generating approximation to the lower-order derivatives, f'', f' and f, through integration of the approximation of the highest-order derivative as follows:

Setting $\phi(x)=f'''(x)$ then, by integration, we get

$$f''(x)=\int_{-1}^{x}\phi(x)\,dx+C_1$$

$$f'(x)=\int_{-1}^{x}\int_{-1}^{x}\phi(x)\,dx\,dx+(x+1)C_1+C_2 \tag{6}$$

and

$$f(x)=\int_{-1}^{x}\int_{-1}^{x}\int_{-1}^{x}\phi(x)\,dx\,dx\,dx+\frac{(x+1)^2}{2}C_1+(x+1)C_2+C_3.$$

From the boundary conditions (5), we get

$$C_2=C_3=0,$$

and

$$C_1=-\frac{\eta_\infty}{4}-\tfrac{1}{2}\int_{-1}^{1}\int_{-1}^{x}\phi(x)\,dx\,dx.$$

Therefore, we can give an approximation to Eq. (6) as follows:

$$f_i=f(x_i)=\sum_{j=0}^{N}l_{ij}\phi_j+d_i,$$

$$f'_i=f'(x_i)=\sum_{j=0}^{N}l_{ij}^{(1)}\phi_j+d_i^{(1)}, \tag{7}$$

and

$$f''_i=f''(x_i)=\sum_{j=0}^{N}l_{ij}^{(2)}\phi_j+d_i^{(2)}$$

for all $i=0(1)N$, where

$$l_{ij}=b_{ij}^{(3)}-\frac{(x_i+1)^2}{4}b_{Nj}^{(2)},\quad d_i=(x_i+1)^2\frac{\eta_\infty}{8}$$

$$l_{ij}^{(1)}=b_{ij}^{(2)}-\frac{(x_i+1)}{2}b_{Nj}^{(2)},\quad d_i^{(1)}=(x_i+1)\frac{\eta_\infty}{4},$$

$$l_{ij}^{(2)}=b_{ij}-\tfrac{1}{2}b_{ij}^{(2)},\quad d_i^{(2)}=\frac{\eta_\infty}{4},$$

where

$$b_{ij}^{(3)}=\frac{(x_i-x_j)^2}{2}b_{ij},$$

$$b_{ij}^{(2)}=(x_i-x_j)b_{ij},\quad i,j=0(1)N$$

and b_{ij} are the elements of the matrix B, as mentioned in [1].

By using Eq. (7) the Eq. (4) is transformed to the following system of non-linear equations

$$\phi_i+\alpha\left(\frac{\eta_\infty}{2}\right)\left(\sum_{j=0}^{N}l_{ij}\phi_j+d_i\right)\left(\sum_{j=0}^{N}l_{ij}^{(2)}\phi_j+d_i^{(2)}\right),$$

$$+\beta\left[\left(\frac{\eta_\infty}{2}\right)^3-\left(\frac{\eta_\infty}{2}\right)\left(\sum_{j=0}^{N}l_{ij}^{(1)}\phi_j+d_i^{(1)}\right)^2\right]=0,\quad i=0(1)N \tag{8}$$

in the highest derivative and we solve it by Newton's method.

3. NUMERICAL RESULTS

The method described here-in has been used to calculate the flow for several different combinations of α and β. Table 1 shows comparison of the wall shear stress f''_w with $N=20$ for our method and the number of nodes for the other methods equal 100 and different η_∞. Table 2 illustrates the effect of the degree of approximaton for $\alpha=1$ and $\beta=0.5$ (Homann axisymmetric stagnation flow), $\alpha=1$ and $\beta=1$ (Hiemenz flow). All values of f', f''_w are in good agreement with those reported by Beckett [2], Rosenhead [5] and White [7]. Decreasing the degree of approximation, the values of the wall shear stress f''_w and the fluid velocity f' do not change after fixed degree of approximation. Table 3 lists the value of f''_w, the wall shear stress, for $\eta_\infty=\eta_{\infty 1}$, and $\eta_\infty=2\ \eta_{\infty 1}$, for two sample cases of Homann and Hiemenz flows and $\alpha=1$, $\beta=10$ (a flow with a strongly favorable pressure gradient) for a fixed degree of approximation $N=20$.

The computations were carried out on a VME2955, ICL computer.

Table 1 Comparison of the wall shear stress f''_w

η_∞	α	β	*Present method*	*Wadia & Payne [6]*	*Rosenhead [5]*	*White [7]*	*Beckett [2]*
8.0	1	−0.15	−0.1334299	−0.131999	−0.1320	—	—
8.0	1	−0.18	−0.0976934	−0.097000	−0.0970	—	—
5.6	1	−0.18	0.1289902	0.128500	0.1285	0.12864	—
5.2	1	−0.15	0.2167541	0.216101	0.2161	—	—
6.9	1	0.0	0.4696000	0.469604	0.4696	0.46960	0.4697
4.4	1	0.3	0.7747830	0.774764	—	0.77476	—
3.7	1	0.5	0.9278050	0.925999	0.9280	—	0.9277
3.5	1	1	1.2326170	1.232470	1.2330	1.23259	1.2327
3.1	1	2	1.6872260	1.686600	1.6870	1.68722	1.6874
4.6	0	1	1.1547120	1.154700	1.1547	—	—
3.6	−1	4	2.2727840	2.160200	2.2730	—	2.2730
10.0	−1	1	1.0863620	1.062800	1.0860	—	1.0870
2.0	1	10	3.6752340	3.330000	—	—	3.6757
2.0	1	15	4.4914870	3.931670	—	—	4.4923
2.0	1	20	5.1807180	4.383110	—	—	5.1811

Table 2 The effect of the degree of approximation on the fluid velocity $f', \eta_\infty = 4$

α, β	$\alpha=1$	$\beta=0.5$		$\alpha=1$	$\beta=1$	
N	*10*	*20*	*40*	*10*	*20*	*40*
η	f'	f'	f'	f'	f'	f'
0.097886	0.088414	0.088416	0.088416	0.115869	0.115869	0.115869
0.381966	0.317920	0.317912	0.317912	0.399046	0.399047	0.399047
0.824429	0.596248	0.596262	0.596262	0.698565	0.698563	0.698563
2.000000	0.942310	0.942331	0.942331	0.973229	0.973226	0.973226
2.618034	0.986080	0.986060	0.986060	0.994922	0.994925	0.994925
3.618034	0.999436	0.999428	0.999428	0.999848	0.999849	0.999849
4.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
f''_w	0.9277187	0.9277190	0.927719	1.232591	1.232591	1.232591

Table 3 The wall shear stress f''_w for different values of η_∞

α	1	1	1	1	1	1
β	1.0	1.0	0.5	0.5	10.0	10.0
η_∞	3.5	7.0	3.7	7.4	2.0	4.0
Present method	1.232617	1.232588	0.9278054	0.9276800	3.675234	3.675234
Wadia & Payne [6]	1.232173	1.23137	0.929779	0.927965	3.5548	3.55426
White [7]	1.232588	—	0.927680	—	—	—

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