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6) $P[T>10] = 1 - P[T \le 10] = 1 - \phi\left(\frac{10-\mu_T}{15}\right) = \frac{1}{2}$

$\because \phi(T) = \frac{1}{2}$ from $T=0$

$\Rightarrow \phi\left(\frac{10-\mu_T}{15}\right) = \frac{1}{2}$

So, $\frac{10-\mu_T}{15} = 0$

$10 = \mu_T$

So, $P[T>32] = 1 - P[T \le 32] = 1 - \phi\left(\frac{32-10}{15}\right)$

$= 0.074$

$P[T<0] = \phi\left(\frac{0-10}{15}\right) = 1 - (\phi(2/3))$

$P[T>60] = 1 - P[T \le 60] = 1 - \phi\left(\frac{60-10}{15}\right)$

$= Q(3.33) = 4.3 \times 10^{-4}$

7) a) A^c implies $W=0$

$\Rightarrow$ for $w \ge 0$

$F_W(w) = P[A^c] + P[A] F_{W|A}(w) = \frac{1}{2} + (\frac{1}{2}) F_X$

$\Rightarrow$ CDF of $W = \begin{cases} 0 & w < 0 \\ \frac{1}{2} + \frac{1}{2}(F_X)(w) & w \ge 0 \end{cases}$

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b) PDF of ω : $\begin{cases} \frac{1}{2}\delta\, \omega + \frac{1}{2} f_x(\omega) \\ 0 & \text{Otherwise} \end{cases}$

Since, X is non-negative, so for $x < 0$,
$f_x(x) = 0$

$\Rightarrow f_\omega(\omega) = \frac{1}{2}\delta\, \omega + \frac{1}{2} f_x(\omega)$

c) $E[\omega] = \int_{-\infty}^{\infty} \omega f_\omega(\omega) d\omega = \frac{1}{2}\int_{-\infty}^{\infty} \omega f_x(\omega) d\omega = E[x]/2$

$E[\omega^2] = \int_{-\infty}^{\infty} \omega^2 f_\omega(\omega)\, d\omega = \frac{1}{2}\int_{-\infty}^{\infty} \omega^2 f_x(\omega)\, d\omega) = E[x^2]/2$

$Var[\omega] = E[\omega^2] - (E[\omega])^2 = \frac{1}{2} Var[x] + \frac{(E[x])^2}{4}$

8) a) $F_X(x) = P[-\ln(1-U) \le x] = P[1-U \ge e^{-x}) = P[U \le 1 - e^{-x}]$

$\Rightarrow F_X(x) = F_U(1-e^{-x}) = 1 - e^{-x}$
$\because$ for $x \ge 0$, $0 \le 1 - e^{-x} \le 1$

CDF = $\begin{cases} 0 & x < 0 \\ 1 - e^{-x} & x \ge 0 \end{cases}$

b) PDF = $\begin{cases} e^{-x} & x \ge 0 \\ 0 & \text{Otherwise} \end{cases}$

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c) $\because$ X is exponential R.V. , $a = 0$, $E[x] = 1$

9) a) If $x = \frac{1}{2}$, $y = \frac{\pi r^2}{2}$

$\because$ Circumference $= 1 = 2\pi r$

$\Rightarrow r = \frac{1}{2\pi}$

$\Rightarrow y = \pi r^2 x = \frac{X}{4\pi}$

b) $CDF = P[Y \le y] = P[\frac{X}{4\pi} \le y] = P[X \le 4\pi y] = F_X(4\pi y)$

$$\Rightarrow CDF = \begin{cases} 0 & y < 0 \\ 4\pi y & 0 \le y \le 1/4\pi \\ 1 & y \geq 1/4\pi \end{cases}$$

c) $$PDF = \begin{cases} 4\pi & 0 \le y \le 1/4\pi \\ 0 & \text{otherwise} \end{cases}$$

d) $E[Y] = \int_0^{1/4\pi} 4\pi y \, dy = \frac{1}{8\pi}$