Spatial Econometrics

Wykład 6: Multi-source spatial models

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Outline

Plan prezentacji

Spatial lag distributions: parsimonious parametrisation

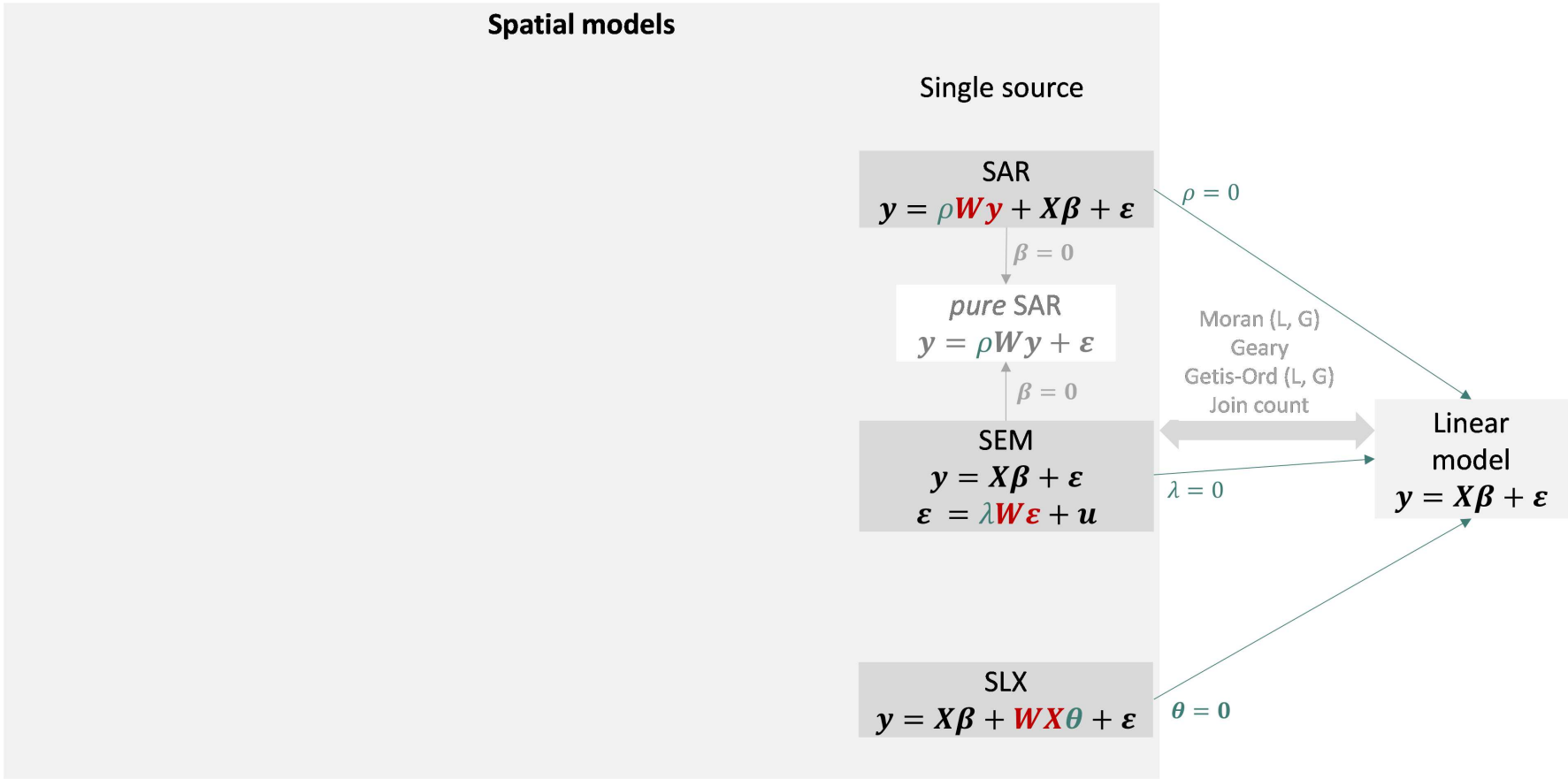
- SAR, SEM and SLX models contain only one spatial lag of the respective component, but one lag of one component may approximate the lag distribution of another:

SAR	SEM	SLX
$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$	$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + (\mathbf{I} - \lambda\mathbf{W})^{-1}\boldsymbol{\varepsilon}$	$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$
$\mathbf{y} = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{X}\boldsymbol{\beta} + (\mathbf{I} - \rho\mathbf{W})^{-1}\boldsymbol{\varepsilon}$	$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} + \lambda\mathbf{W}\boldsymbol{\varepsilon} + \lambda^2\mathbf{W}^2\boldsymbol{\varepsilon} + \ldots$	
$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \rho\mathbf{W}\mathbf{X}\boldsymbol{\beta} + \rho^2\mathbf{W}^2\mathbf{X}\boldsymbol{\beta} + \ldots + \boldsymbol{\varepsilon} + \rho\mathbf{W}\boldsymbol{\varepsilon} + \rho^2\mathbf{W}^2\mathbf{X}\boldsymbol{\varepsilon} + \ldots$		

- This is the underlying idea of combining different sources of spatial processess in one model: they allow to approximate the lags of higher order, and – in consequence – remove the spatial autocorrelation that can remain unremoved in the residuals of a single-source model.

Plan prezentacji

SARAR model – relation to other models



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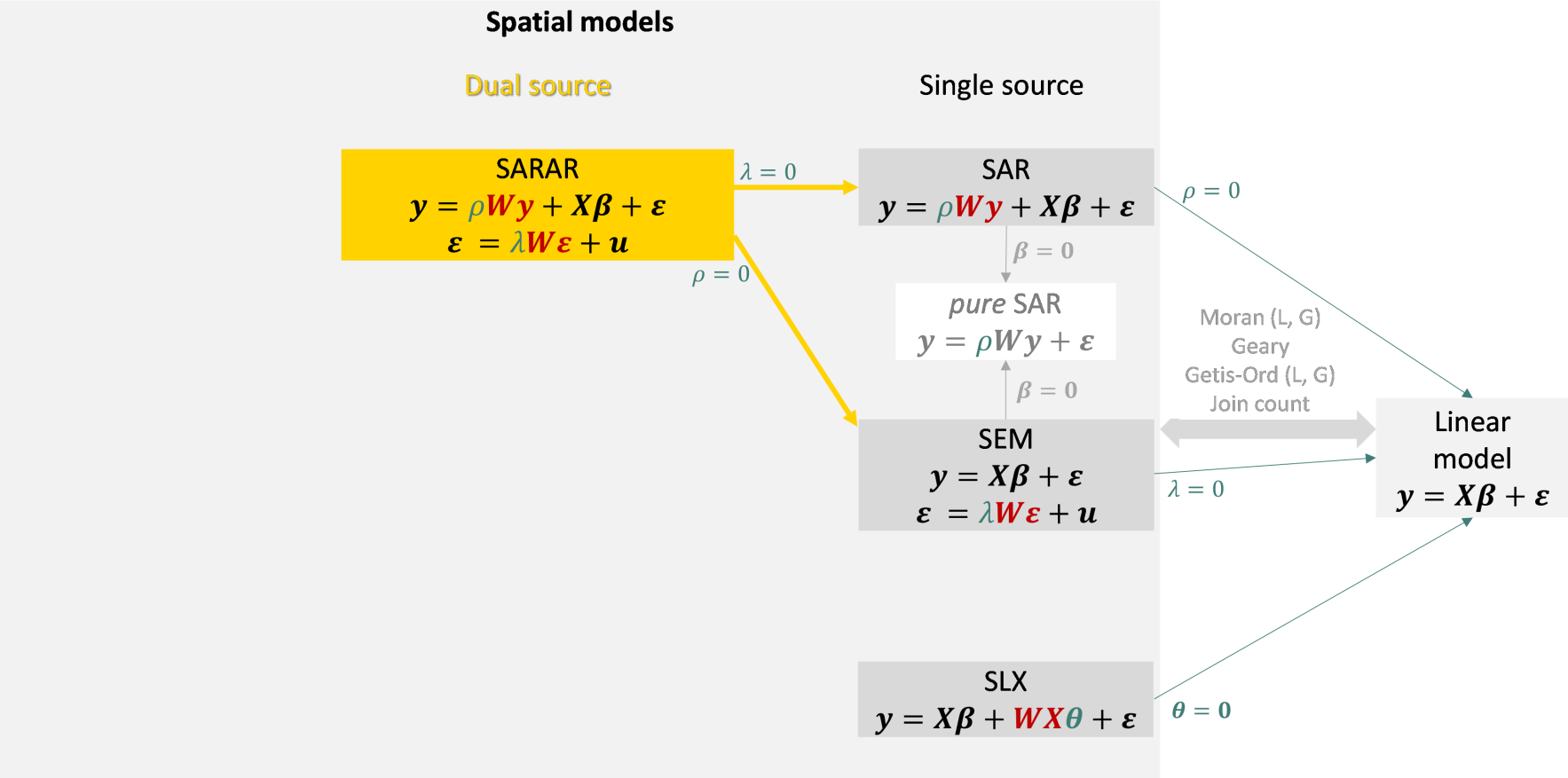

SARAR model – specification

- Also referred to as SAC (comparable to ARMA model for time series):

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} = \lambda \mathbf{W}\boldsymbol{\varepsilon} + \mathbf{u}$$

- Applied, when previously discussed methods insufficiently remove spatial autocorrelation of the residuals.
- Estimation problem: hybrid of the problems with SLM and SEM.

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SARAR model – estimation

It is possible to use both least-squares and maximum-likelihood approaches. Both procedures for SAR and SME should be accordingly combined and merged (details e.g. in ***Arbia, 2014***).

Method 1 – ML

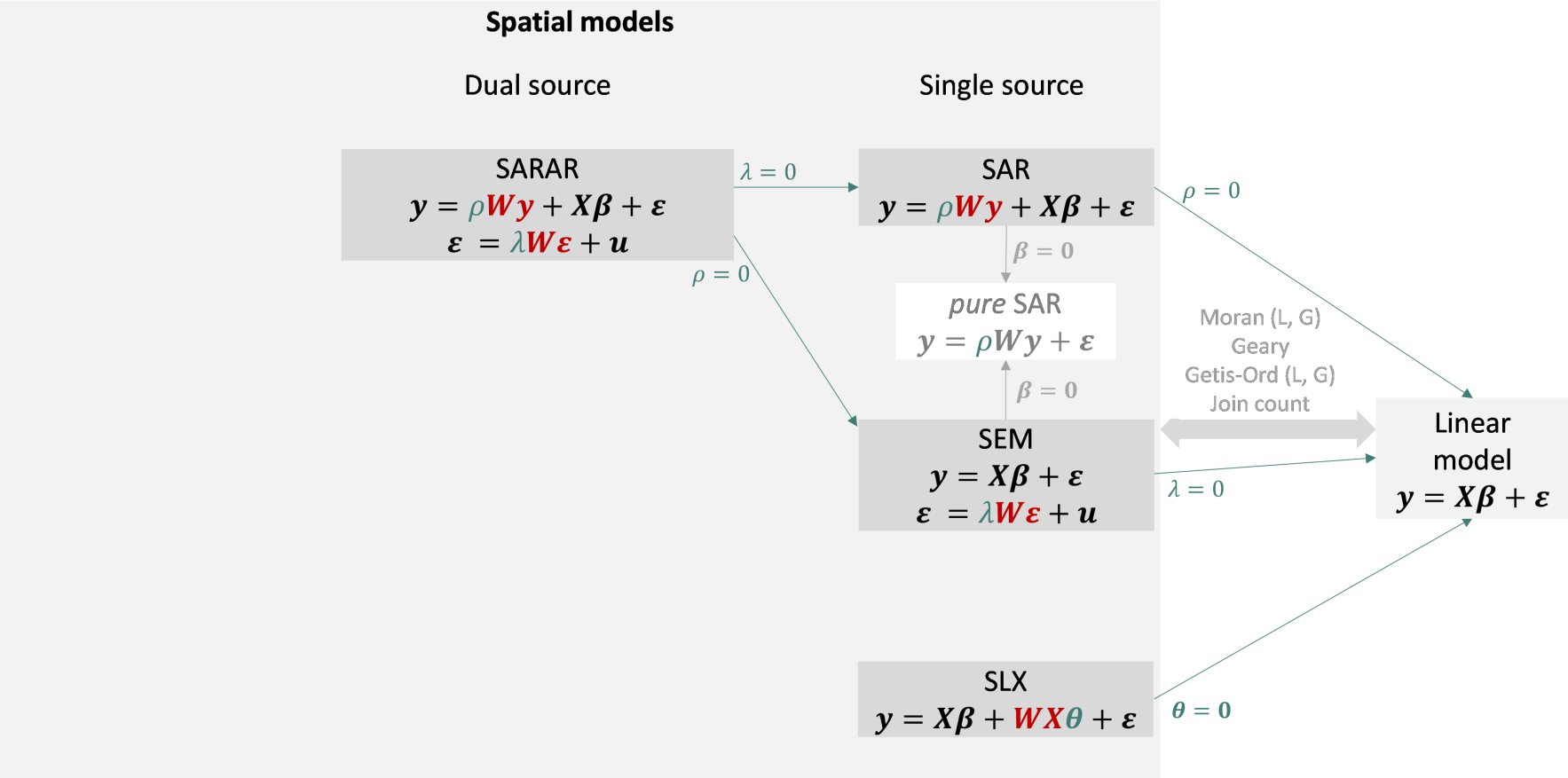
```
model <- sacsarlm(y ~ x, listw = W)
```

Method 2 – generalised spatial 2SLS

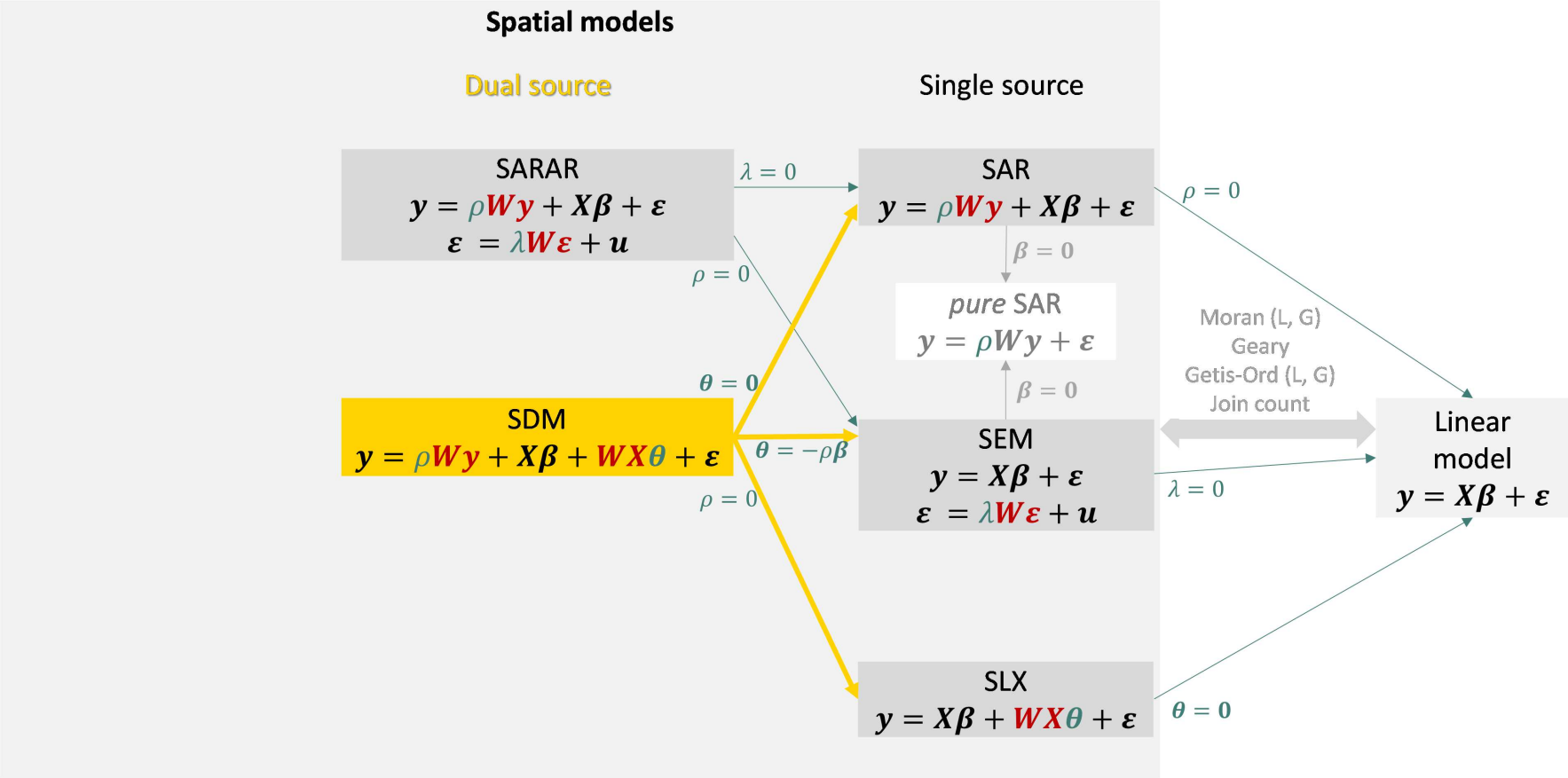
```
model <- gstsls(y ~ x, listw = W)
```

Plan prezentacji

SDM model – relation to other models



SDM model – relation to other models



SDM model

- Two sources of the spatial process: the outcome in a given region is driven by...
  - the outcome in other regions and...
  - and the cause in other regions

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

ML estimation (like SLM / pure SAR)

```
model <- lagsarlm(y ~ x, listw = W, type = "Durbin")
```

More difficult for S2SLS (matrix **WX** is a regressor, so it cannot any more serve as a separate instrument for **Wy**).

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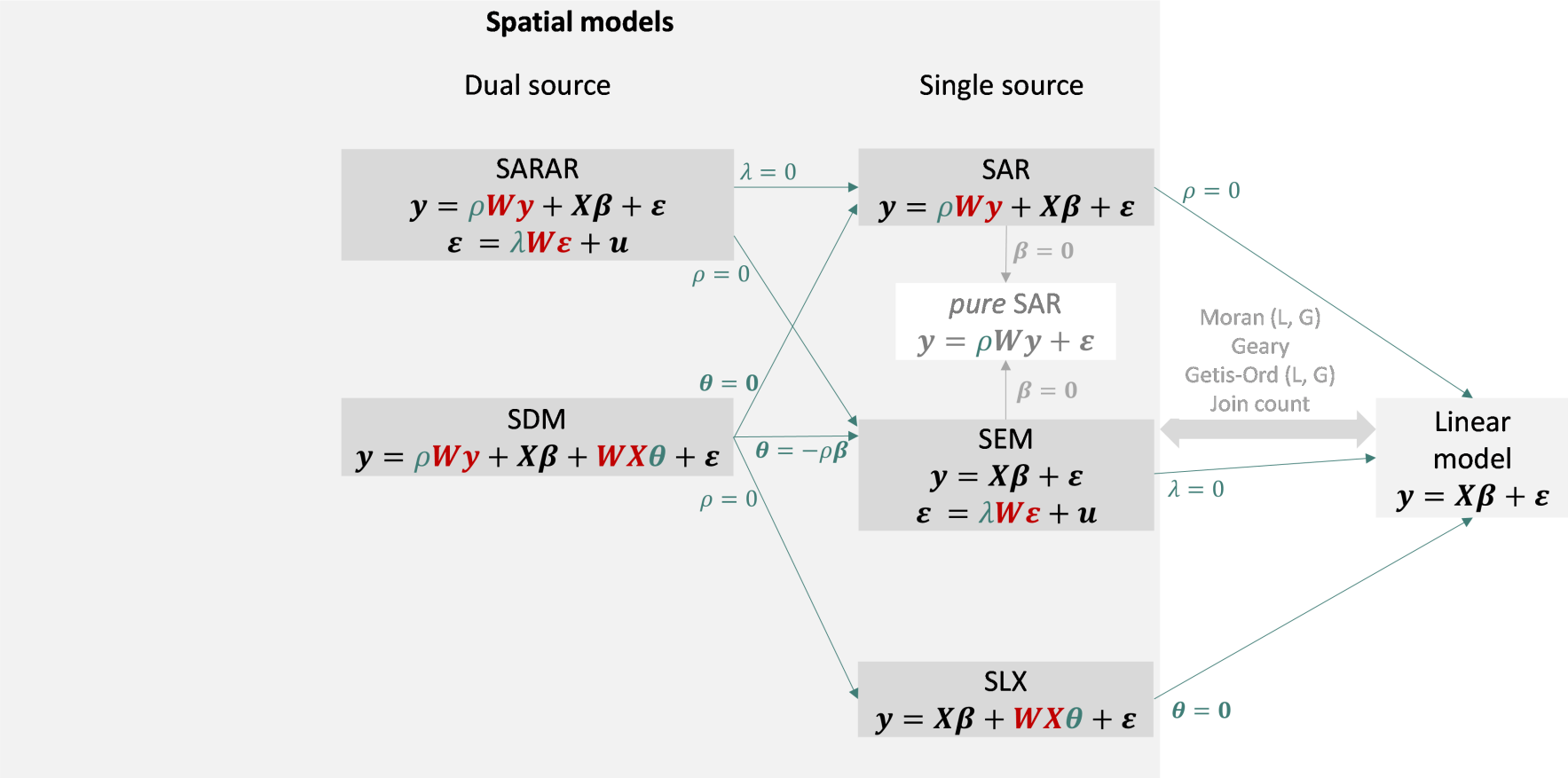
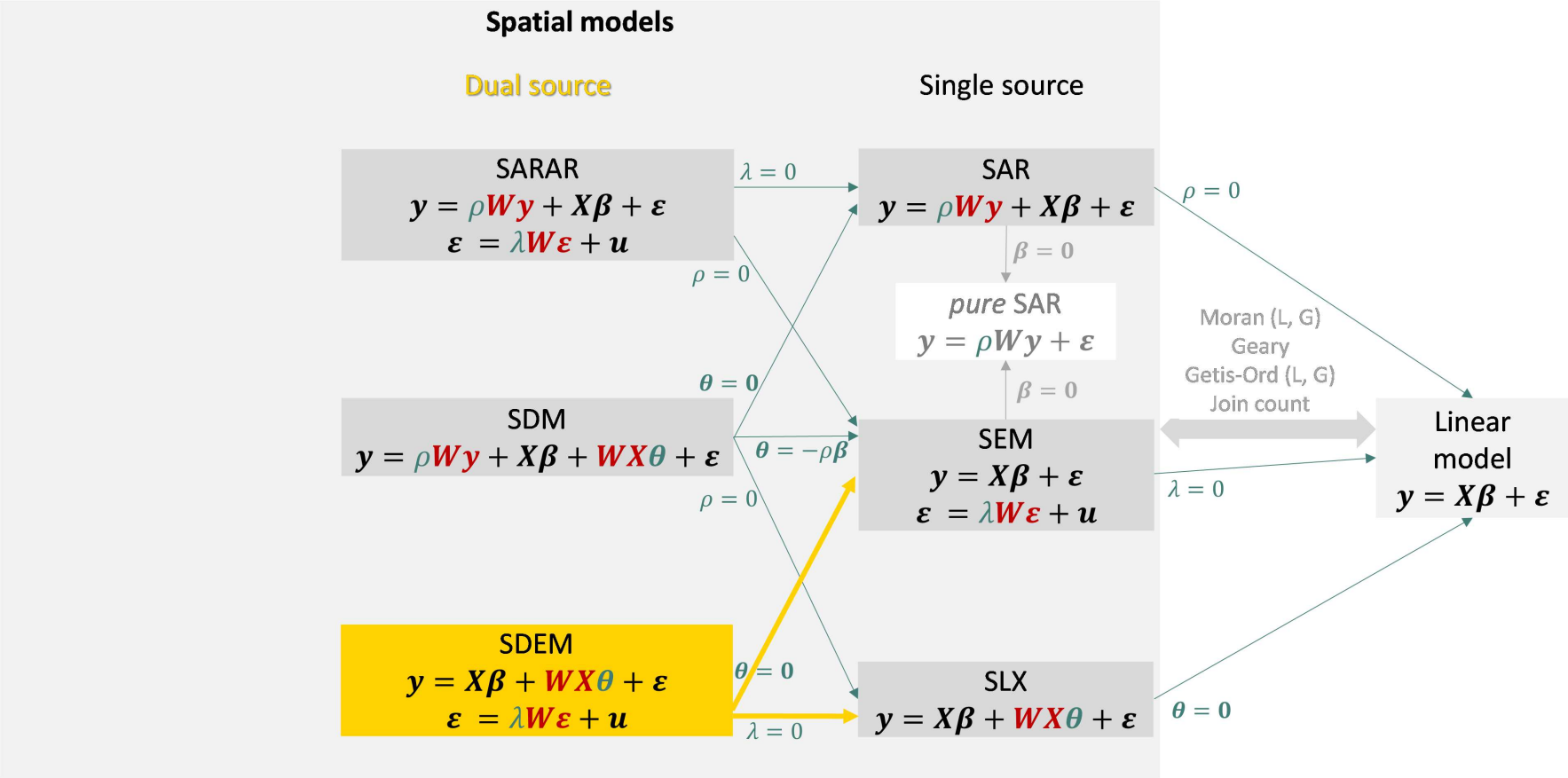
SDEM model – specification



SDEM model – specification



SDEM model – specification and estimation

- Two sources of the spatial process: the outcome in a given region is driven by...
  - the reason in other regions
  - and the unobservable factors in other regions.

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} = \lambda\mathbf{W}\boldsymbol{\varepsilon} + \mathbf{u}$$

ML estimation (like SEM, with additional regressors)

```
model <- errorsarlm(y ~ x + wx, listw = W)
```

SDEM with local interdependence of error terms

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} = \lambda\mathbf{W}\mathbf{u} + \mathbf{u}$$

Estimation: like SEM with local interdependence of error terms (but with additional regressors).

Plan prezentacji

Exercise

- Come back to the previous examples:
  - Okun's law (unemployment vs output);
  - location of Biedronka markets vs labour market characteristics.

  Is there a need to estimate a multi-source model?
- Estimate the 3 models in discussion (SARAR, SDM, SDEM) and draw appropriate conclusions.

Homework 6

Consider te applicability of SARAR, SDM and SDEM models in explaining the variable from homework 1.

- Were the previously conducted tests satisfactory?
- Do LM / LR tests reject the restriction about a single source process, or not?
- Interpret the results of the three new models.
- Compute and illustrate the spatial multipliers of the dependent variable with respect to a selected regressor in all three models under consideration.