Spatial Econometrics

Lecture 7: Specifying a spatial model. Case study (1) – wage curve

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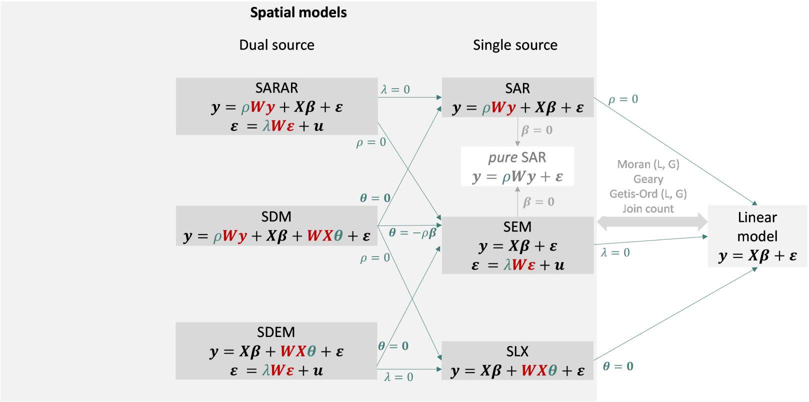
Outline

1. General Nesting Spatial (GNS) model
2. Strategy of spatial model selection
3. Case study: regional wage curve in Poland

Plan prezentacji

1. General Nesting Spatial (GNS) model
2. Strategy of spatial model selection
3. Case study: regional wage curve in Poland

GNS model – specification (1)



Spatial models

Dual source

Single source

SARAR

$$\boldsymbol{y} = \rho \boldsymbol{W y} + \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$
$$\boldsymbol{\varepsilon} = \lambda \boldsymbol{W\varepsilon} + \boldsymbol{u}$$

SDM

$$\boldsymbol{y} = \rho \boldsymbol{W y} + \boldsymbol{X\beta} + \boldsymbol{WX\theta} + \boldsymbol{\varepsilon}$$

SDEM

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{WX\theta} + \boldsymbol{\varepsilon}$$
$$\boldsymbol{\varepsilon} = \lambda \boldsymbol{W\varepsilon} + \boldsymbol{u}$$

SAR

$$\boldsymbol{y} = \rho \boldsymbol{W y} + \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$

pure SAR

$$\boldsymbol{y} = \rho \boldsymbol{W y} + \boldsymbol{\varepsilon}$$

SEM

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$
$$\boldsymbol{\varepsilon} = \lambda \boldsymbol{W\varepsilon} + \boldsymbol{u}$$

SLX

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{WX\theta} + \boldsymbol{\varepsilon}$$

Linear model

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$

Moran (L, G)
Geary
Getis-Ord (L, G)
Join count

$\lambda = 0$
$\rho = 0$
$\boldsymbol{\theta} = \boldsymbol{0}$
$\boldsymbol{\theta} = -\rho\boldsymbol{\beta}$
$\rho = 0$
$\boldsymbol{\theta} = \boldsymbol{0}$
$\lambda = 0$
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$\rho = 0$
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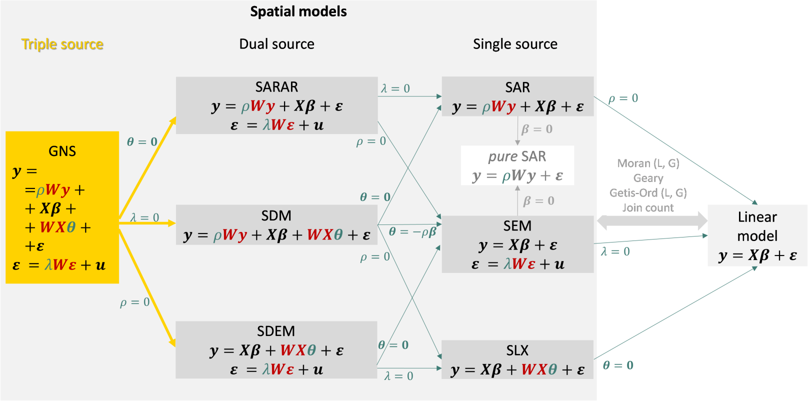
GNS model – specification (2)



Spatial models

Triple source

Dual source

Single source

GNS

$y =$
$= \rho W y +$
$+ X\beta +$
$+ WX\theta +$
$+ \varepsilon$
$\varepsilon = \lambda W \varepsilon + u$

SARAR

$y = \rho W y + X\beta + \varepsilon$
$\varepsilon = \lambda W \varepsilon + u$

SDM

$y = \rho W y + X\beta + WX\theta + \varepsilon$

SDEM

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$y = X\beta + WX\theta + \varepsilon$

Linear model

$y = X\beta + \varepsilon$

Moran (L, G)
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GNS model – specification (3)

- Specification that encompasses all the sources of spatial processes previously under consideration:

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\beta + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \varepsilon$$

$$\varepsilon = \lambda \mathbf{W}\varepsilon + \mathbf{u}$$

- Estimation: like SARAR.

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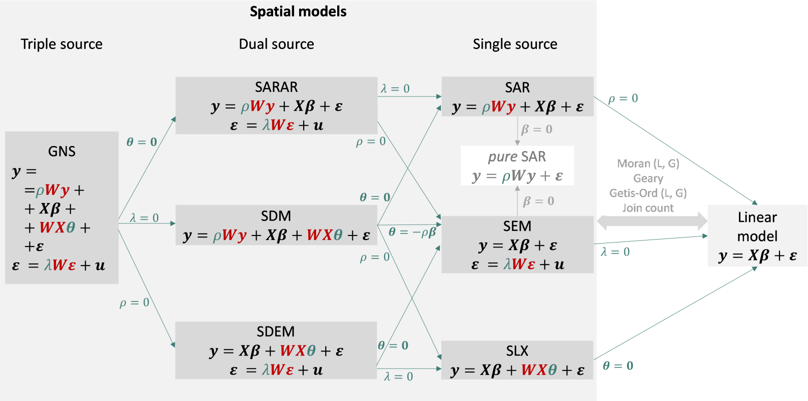
The entire set of models



Spatial models

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$$\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{W}\boldsymbol{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} = \lambda \boldsymbol{W}\boldsymbol{\varepsilon} + \boldsymbol{u}$$

SARAR
$$\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} = \lambda \boldsymbol{W}\boldsymbol{\varepsilon} + \boldsymbol{u}$$

SDM
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Linear model
$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$\boldsymbol{\theta} = \boldsymbol{0}$, $\lambda = 0$, $\rho = 0$, $\lambda = 0$, $\rho = 0$, $\boldsymbol{\theta} = \boldsymbol{0}$, $\boldsymbol{\theta} = -\rho\boldsymbol{\beta}$, $\rho = 0$, $\boldsymbol{\theta} = \boldsymbol{0}$, $\lambda = 0$, $\boldsymbol{\beta} = \boldsymbol{0}$, $\boldsymbol{\beta} = \boldsymbol{0}$, $\rho = 0$, $\lambda = 0$, $\boldsymbol{\theta} = \boldsymbol{0}$

Moran (L, G)
Geary
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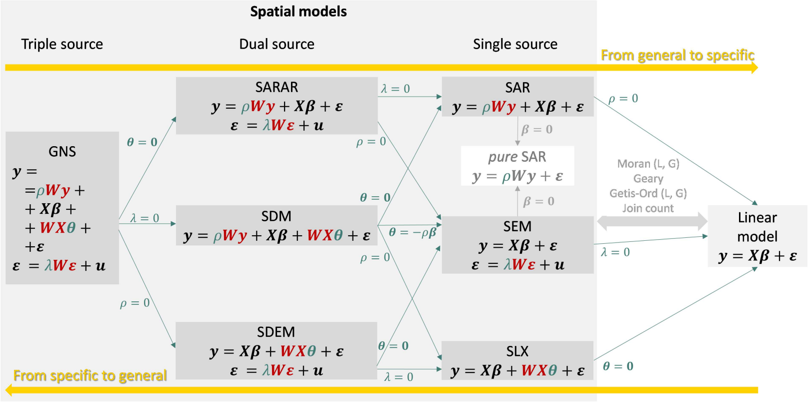
Scheme inspired by: ***Elhorst and Vega (2015)*** – with own modifications.

Order of exploration?



Spatial models

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From general to specific

GNS

$$\boldsymbol{y} = \\ = \rho \boldsymbol{W}\boldsymbol{y} + \\ + \boldsymbol{X}\boldsymbol{\beta} + \\ + \boldsymbol{W}\boldsymbol{X}\boldsymbol{\theta} + \\ + \boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} = \lambda \boldsymbol{W}\boldsymbol{\varepsilon} + \boldsymbol{u}$$

$\boldsymbol{\theta} = \boldsymbol{0}$

$\lambda = 0$

$\rho = 0$

SARAR

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SDM

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From specific to general

Scheme inspired by: ***Elhorst and Vega (2015)*** – with own modifications.

From general to specific?

From general to specific:

- Standard approach in time series econometrics (***Hendry, 1995***). We should test from left to right, looking for parsimony in parametrisation and hance higher efficiency.
- Ensures the correct sequential inference as long as no variables are omitted in the general model.
- But GNS is by no means the most general model!
  - One can add further spatial lags ($\mathbf{W}^2$, $\mathbf{W}^3$, ...).
  - Error terms can exhibit local or global spatial autocorrelation.
- Three (or more) sources of spatial process imply **weak statistical identification** of the model (***Cook et al., 2015***):
  - In practice, it is difficult to distinguish between ρ, $\boldsymbol{\theta}$ and λ (*clustering trilemma*, *label switching problem*.)
  - False decision at the beginning can get the whole inference process side-tracked.

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From specific to general?

From specific to general:

- Less popular a strategy (false partial conclusions due to omitted variable bias).
  - Over-estimating β under ommission of the spatial process can make us overlook its presence in the residuals of the linear model.
- Statistical tests frequently suggest many paths ”to the left” on the scheme (as well as many paths ”to the right” under the previously discussed strategy).
- No testing procedure to directly discriminate between models with identical count of spatial processes (e.g. between single-source SAR vs SEM).

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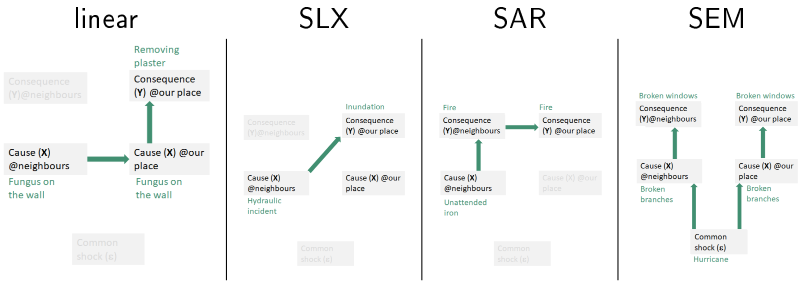
Theory?



linear

Removing plaster
Consequence (Y) @our place

Consequence (Y)@neighbours

Cause (**X**) @neighbours
Fungus on the wall

Cause (**X**) @our place
Fungus on the wall

Common shock (ε)

SLX

Inundation
Consequence (Y) @our place

Consequence (Y)@neighbours

Cause (**X**) @neighbours
Hydraulic incident

Cause (**X**) @our place

Common shock (ε)

SAR

Fire
Consequence (Y)@neighbours

Fire
Consequence (Y) @our place

Cause (**X**) @neighbours
Unattended iron

Cause (X) @our place

Common shock (ε)

SEM

Broken windows
Consequence (Y)@neighbours

Broken windows
Consequence (Y) @our place

Cause (**X**) @neighbours
Broken branches

Cause (**X**) @our place
Broken branches

Common shock (ε)
Hurricane

Get inspired by Box-Jenkins?

In time series econometrics, it holds in stationary and invertible ARMA models that the following conversions are feasible:

- AR(1) into MA(∞)
- MA(1) into AR(∞)

By the same principle, a limited number of spatial lags of $\mathbf{y}$ can replace a high number of spatial lags of $\mathbf{X}$.
All the tests discussed previously can be helpful.

Monte Carlo studies?

A very useful guide for empirical researchers written by ***Cook et al., (2015)***. Based on Monte Carlo simulations, illustrating the bias in estimation of $\boldsymbol{\beta}, \boldsymbol{\theta}, \rho$ and λ under various data generating processes, they conclude that:

- Two recommended solutions are **SARAR** and **SDEM** (unbiased estimation of β and maximum chance to avoid spurious conclusions about spatial parameters). When the spatial process is misspecified, the estimates of $\boldsymbol{\rho}$ and $\boldsymbol{\theta}$ from SDM model – heavily biased.
- Researchers that are predominantly interested in unbiased estimates of β (but not necessarily in the spatial aspects) should use **SDM**. (***Elhorst, 2011*** is much more enthusiastic towards SDM, just like ***LeSage*** in various publications).
- **GNS** is generally not recommended.
- Single-source models – only when the tests are strongly supportive.

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Choosing the estimation method

When the specification is determined, the decision is much easier here. For almost all types of models, we can choose between ML-type estimation and LS/GMM-type estimation.

- In standard cases: no recommendations.
- Large N (e.g. a few thousand): ML estimators evaluate the determinant of $N \times N$ matrix at each iteration. A lot of time and computational capacity is needed.
- LS / GMM methods are much more convenient when spatial autocorrelation is accompanied by **endogeneity of other regressors** (adding further instrumental variables – intuitive).
- ML framework implies very intuitive tools for testing nested models (e.g. test LR).
- ML sometimes ensures higher efficiency, as it consumes the knowledge of entire error distribution, instead of only error moments. We cannot be sure, however, that the usual assumption of error normality is fulfilled.
- LS / GMM methods more robust to specification errors.
- Many authors use Bayesian methods with spatial models (cf. thorough discussion in ***LeSage, Pace, 2009***). Nonetheless, they do not give answers to all the doubts listed above.

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Exercise

Using all the previously studied techniques and the dataset used in lecture 1 (for Polish poviats), investigate the impact of unemployment rate on the wages in Polish poviats.

1. Determine wheter spatial modelling is relevant here. Construct the weight matrix (look at 2-3 alternative options).
2. Find the best model.
3. Add further variables to the equation if necessary.
4. Select one poviat (but not a city-poviat) and, based on the obtained model, illustrate how a location of a foreign greenfield investment will affect the wage levels in various poviats. Assume that the investment itself will directly decrease the unemployment rate in this poviat by 0.5 p.p.