

Artificial Intelligence

Final Test

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Problem 1

Given the following formulas in propositional logic:

$$\rightarrow \varphi_1 : (p \rightarrow q)$$

$$\rightarrow \varphi_2 : (q \rightarrow (s \wedge t))$$

$$\rightarrow \varphi_3 : (r \rightarrow (s \wedge t))$$

$$\rightarrow \varphi_4 : (p \vee r)$$

Solution:-

By using the resolution to derive empty clause from CNF (conjunctive normal form) of $(\varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4) \wedge \neg \Phi$

where as $\varphi_1 = (p \rightarrow q) \equiv (\neg q \vee q)$ which are logically equivalents.

$$p \rightarrow q = \neg q \vee q$$

$$\begin{aligned} p \quad \varphi_2 : (q \rightarrow (s \wedge t)) &\equiv \neg q \vee (s \wedge t) \\ &\equiv (\neg q \vee s) \wedge (\neg q \vee t) \end{aligned}$$

$$\begin{aligned} \varphi_3 : (r \rightarrow (s \wedge t)) &\equiv \neg r \vee (s \wedge t) \\ &= (\neg r \vee s) \wedge (\neg r \vee t) \end{aligned}$$

$$\varphi_4 : (p \vee r)$$

where as the clauses for $\varphi_1, \varphi_2, \varphi_3, \varphi_4$

$$\begin{array}{l} 1. \{ \neg p \} \\ 2. \{ q \} \end{array} \left. \begin{array}{l} 3. \neg q \\ 4. s \\ 5. \neg q \\ 6. t \end{array} \right\} \varphi_2$$

$$\begin{array}{l} 7. \neg r \\ 8. s \\ 9. \neg r \\ 10. t \end{array} \left. \right\} \varphi_3$$

$$\begin{array}{l} 11. p \\ 12. r \end{array} \left. \right\} \varphi_4$$

$\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4$ are logically consistent because of they are
 no to. There every formula \mathcal{Q} is a logical consequence of
 them.

To satisfy $\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4$ we must chose interpretation;

$$S: \mathcal{U} \quad P^i = T \quad Q^i = F \quad r^i = T$$

To show whether \mathcal{Q} is a logical consequence of

$\mathcal{U}_1 \wedge \mathcal{U}_2 \wedge \mathcal{U}_3 \wedge \mathcal{U}_4$ we should consider the classes.

$$\neg \mathcal{Q} \equiv \neg (S \wedge T) \equiv \neg S \vee \neg T \quad \left. \begin{array}{l} \neg S \\ 14. \neg T \end{array} \right\}$$

The empty clause can be divided from classes 1 and 2
 from 4 & 5 or 8 & 9.

$\therefore \mathcal{Q}$ is logical consequence.

Problem:-5

- 1) $\forall x. \text{likes}(x, x)$
- 2) $\forall x. \exists y. \text{likes}(x, y)$
- 3) $\exists y. \forall x. \text{likes}(x, y)$
- 4) $\forall x. \forall y. \text{likes}(x, y) \rightarrow \text{likes}(y, x)$
- 5) $\forall x. \forall y. (\exists z. \text{likes}(x, z) \wedge \text{likes}(z, y)) \rightarrow \text{likes}(x, y)$

	Abby	Bess	Cody	Dana
Abby	x	-	x	x
Bess	-	x	-	-
Cody	-	-	x	-
Dana	-	x	x	-

Solution:-

$\text{likes}(\text{Abby}, \text{Abby})$
 $\sim \text{likes}(\text{Abby}, \text{Bess})$
 $\text{likes}(\text{Abby}, \text{Cody})$
 $\text{likes}(\text{Abby}, \text{Dana})$

$\sim \text{likes}(\text{Cody}, \text{Abby})$
 $\sim \text{likes}(\text{Cody}, \text{Bess})$
 $\text{likes}(\text{Cody}, \text{Cody})$
 $\sim \text{likes}(\text{Cody}, \text{Dana})$

$\sim \text{likes}(\text{Bess}, \text{Abby})$
 $\text{likes}(\text{Bess}, \text{Bess})$
 $\sim \text{likes}(\text{Bess}, \text{Cody})$
 $\text{likes}(\text{Bess}, \text{Dana})$

$\sim \text{likes}(\text{Dana}, \text{Abby})$
 $\text{likes}(\text{Dana}, \text{Bess})$
 $\text{likes}(\text{Dana}, \text{Cody})$
 $\sim \text{likes}(\text{Dana}, \text{Dana})$

1) $\forall x \text{ likes}(x, x)$
 $\therefore \forall x \Rightarrow \text{Universal}$

$x = \text{Abby, Bess, Cody, Dana}$

Likes (Abby, Abby) True

Likes (Bess, Bess) True

Likes (Cody, Cody) True

Likes (Dana, Dana) False

$\therefore \forall x \text{ likes}(x, x)$ is False

2) $\forall x \exists y \text{ likes}(x, y)$ it means everybody likes somebody.
 $\exists y \Rightarrow \text{Existential}$

$\exists y \text{ likes}(\text{Abby}, y)$ likes (Abby, Cody)

$\exists y \text{ likes}(\text{Bess}, y)$ likes (Bess, Bess)

$\exists y \text{ likes}(\text{Cody}, y)$ likes (Cody, Cody)

$\exists y \text{ likes}(\text{Dana}, y)$ likes (Dana, Cody)

$\forall x \exists y \text{ likes}(x, y)$ is true

3) $\exists y \forall x \text{ likes}(x, y)$

Directly we can declare it false because Dana doesn't like herself

$\exists y \text{ likes}(\text{Abby}, \text{Abby})$ True

$\exists y \text{ likes}(\text{Bess}, \text{Bess})$ True

$\exists y \text{ likes}(\text{Cody}, \text{Cody})$ True

$\exists y \text{ likes}(\text{Dana}, \text{Dana})$ False

4) $\forall x \forall y \text{ likes}(x, y) \rightarrow \text{likes}(y, x)$.

$\forall x \forall y$ as universes so we must check for every for instance

$\text{likes}(\text{abby}, \text{ody})$ True. But $\text{likes}(\text{ody}, \text{abby})$ false

Hence $\forall x \forall y \text{ likes}(x, y) \rightarrow \text{likes}(y, x)$ is false.

5) $\forall x \forall y (\exists z, \text{likes}(x, z) \wedge \text{likes}(z, y)) \rightarrow \text{likes}(x, y)$.

$\text{likes}(\text{abby}, \text{dana})$.

$\text{likes}(\text{dana}, \text{bess})$.

$\neg \text{likes}(\text{abby}, \text{bess})$.

$\therefore \forall x \forall y (\exists z, \text{likes}(x, z) \wedge \text{likes}(z, y)) \rightarrow \text{likes}(x, y)$.