Math assignment 4

1. a) $A+B=\begin{bmatrix}(2)+(5)\\(3)+(-1)\end{bmatrix}=\begin{bmatrix}7\\2\end{bmatrix}$

b) $CB=\begin{bmatrix}2 & 4\\1 & 3\end{bmatrix}\begin{bmatrix}5\\-1\end{bmatrix}=\begin{bmatrix}(2)(5)+(4)(-1)\\(1)(5)+(3)(-1)\end{bmatrix}=\begin{bmatrix}6\\2\end{bmatrix}$

c) $DB=\begin{bmatrix}1 & 3 & 5\\2 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\0 & 3 & 2\\1 & 1 & 2\end{bmatrix}$

$$=\begin{bmatrix}(1)(1)+(3)(0)+(5)(1) & (1)(2)+(3)(3)+(5)(1) & (1)(0)+(3)(2)+(5)(2)\\(2)(1)+(0)(0)+(1)(1) & (2)(2)+(0)(3)+(1)(1) & (2)(0)+(0)(2)+(1)(2)\end{bmatrix}$$

$$=\begin{bmatrix}6 & 16 & 16\\3 & 5 & 2\end{bmatrix}$$

d) $C^{-1}=\begin{bmatrix}2 & 4\\1 & 3\end{bmatrix}^{-1}=\frac{1}{(2\cdot3)-(1\cdot4)}\begin{bmatrix}3 & -4\\-1 & 2\end{bmatrix}=\begin{bmatrix}3/2 & -2\\-1/2 & 1\end{bmatrix}$

e) $\det(E)=1\begin{vmatrix}3 & 2\\1 & 2\end{vmatrix}-2\begin{vmatrix}0 & 2\\1 & 2\end{vmatrix}+0\begin{vmatrix}0 & 3\\1 & 1\end{vmatrix}$

$$=1(3\cdot2-1\cdot2)-2(0\cdot2-1\cdot2)+0(0\cdot1-1\cdot3)$$

$$=8$$

f) $E^{-1} = \frac{1}{\det(E)} \cdot \begin{vmatrix} (3)(2)-(1)(2) & (0)(2)-(1)(2) & (0)(1)- \\ (2)(2)-(1)(0) & (1)(2)-(1)(0) & (1)(1)-(1) \\ (2)(2)-(3)(0) & (1)(2)-(0)(6) & (1)(3)-(0) \end{vmatrix}$

$= \frac{1}{8} \begin{bmatrix} 4 & -2 & -3 \\ 4 & 2 & -1 \\ 4 & 2 & 3 \end{bmatrix}$ minors

$= \frac{1}{8} \begin{bmatrix} 4 & 2 & -3 \\ -4 & 2 & 1 \\ 4 & -2 & 3 \end{bmatrix}$ cofactors

$= \frac{1}{8} \begin{bmatrix} 4 & -4 & 4 \\ 2 & 2 & -2 \\ -3 & 1 & 3 \end{bmatrix}$ adjoint

$E^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/4 & 1/4 & -1/4 \\ -3/8 & 1/8 & 3/8 \end{bmatrix}$

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g) $A \cdot B = |A||B| \cos\theta$

$A = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$\therefore 7 = (13)(27) \cos\theta$

$A \cdot B = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \end{bmatrix}$

$\cos^{-1}\left(\frac{7}{(13 \cdot 27)}\right) = \theta$

$= 7$

$88.86 \simeq \theta$

2. $P = \begin{bmatrix} 2 & 3 \\ a & b \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $PQ = QP$

$PQ = \begin{bmatrix} (2)(1) + (3)(0) & (2)(1) + (3)(1) \\ (a)(1) + (b)(0) & (a)(1) + (b)(1) \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 \\ a & a+b \end{bmatrix}$

$QP = \begin{bmatrix} (1)(2) + (1)(a) & (1)(3) + (1)(b) \\ (0)(2) + (1)(a) & (0)(3) + (1)(b) \end{bmatrix}$

$= \begin{bmatrix} 2+a & 3+b \\ a & b \end{bmatrix}$

$\therefore 2 = 2 + a$
$a = 0$

$5 = 3 + b$
$b = 2$

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3. rotate clockwise $45° = \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix}$

stretch x-axis by $6 = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

stretch y-axis by $0.5 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$

reflect on x-axis $= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$\left.\begin{aligned} \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} &= \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 0.707 \\ -2.121 \end{bmatrix} \end{aligned}\right\} \text{rotation}$$

$$\left.\begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.707 \\ -2.121 \end{bmatrix} = \begin{bmatrix} 4.242 \\ -2.121 \end{bmatrix}\right\} \text{stretch x by 6}$$

$$\left.\begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 4.242 \\ -2.121 \end{bmatrix} = \begin{bmatrix} 4.242 \\ -1.061 \end{bmatrix}\right\} \text{stretch y by 0.5}$$

$$\left.\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 4.242 \\ -1.061 \end{bmatrix} = \begin{bmatrix} 4.242 \\ 1.061 \end{bmatrix}\right\} \text{reflect on x-axis}$$

$$\therefore A' = \begin{bmatrix} 4.242 \\ 1.061 \end{bmatrix}$$

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4. a) $3x + 2y = -2$
$x + 4y = 6$

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{det(A)}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{det(A)}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

$\therefore x = \frac{1}{10}((4)(-2) - (-2)(6))$

$= \frac{1}{10}(4)$

$x = 3/5 \simeq 0.4$

$\therefore y = \frac{1}{10}((-1)(-2) + (3)(6))$

$= \frac{1}{10}(21)$

$y \simeq 2.1$

b) $2a - c = 1$
$2a + b = 1$
$b + c = 0$

$$\begin{bmatrix} 2 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$det(A) = 2(1\cdot1 - 1\cdot0) - 0(2\cdot1 - 0\cdot0) + (-1(2\cdot1 - 0\cdot1))$

$= 2 - 0 - 2$

$= 0$

$\therefore$ because $det(A) = 0$, we know that there is no unique solution for b)