# Module 1 - Lesson 02: Number Conversions and Arithmetic Operations

## Converting Between Number Systems

In computer mathematics, you will be required to convert between the decimal, binary, and hexadecimal number systems. In the previous lesson you implicitly learned how convert from binary/hexadecimal to decimal by writing its expanded notation and evaluating it.

This week we will look at how to convert from:

- Decimal to Binary
- Decimal to Hexadecimal
- Hexadecimal to Binary
- Binary to Hexadecimal

## Converting from Decimal to Binary/Hexadecimal

There are a number of methods we can employ to convert a decimal number to its binary equivalent. The algorithm we will focus on is the remainder method.

**The Remainder Method**

1. Divide the decimal number by the base of the number system you want to convert to.
2. The ***remainder*** is the least significant (last) digit of the conversion.
3. Take the ***quotient*** from Step 2 and divide it by the base of the number system.
4. The ***remainder*** from Step 3 is the place value to the left of the converted equivalent.
5. Take the ***quotient*** from Step 4 and divide it by the base of the number system.
6. Repeat Steps 4 & 5 until the ***quotient*** is **zero** (0).

**Example 1:** Convert $(17)_{dec}$ to binary.

$17 \div 2 = 8, \quad Remainder\ of\ 1$
$8 \div 2 = 4, \quad Remainder\ of\ 0$
$4 \div 2 = 2, \quad Remainder\ of\ 0$
$2 \div 2 = 1, \quad Remainder\ of\ 0$
$1 \div 2 = 0, \quad Remainder\ of\ 1$

Stop when your answer is 0. Final answer is $10001_{bin.}$ Note, you start with the remainders from the bottom up and write the answer from left to write.

**Example 2:** Convert $(152)_{dec}$ to binary.

**Example 3:** Convert $(555)_{dec}$ to hexadecimal.

$555 \div 16 = 34, \quad Remainder\ of\ 11$ or B
$34 \div 16 = 2, \quad Remainder\ of\ 2$
$2 \div 16 = 0, \quad Remainder\ of\ 2$

Stop when your answer is 0. Final answer is $22B_{hex.}$ Note, you start with the remainders from the bottom up and write the answer from left to write.

**Example 4:** Convert $(4110)_{dec}$ to hexadecimal.

****Alternative Method for Converting Decimal to Binary – Subtracting Powers of 2 (OPTIONAL)**

**Example:** Convert $(25)_{dec}$ to binary.

Start with 25 and subtract the highest power of 2 possible.

$25 - 2^4 = 9$

| 1 | | | | |
|---|---|---|---|---|
| $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |

Repeat by subtracting the next highest power of 2 possible.

$9 - 2^3 = 1$

| 1 | 1 | | | |
|---|---|---|---|---|
| $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |

Repeat by subtracting the next highest power of 2 possible.

$1 - 2^0 = 0$

| 1 | 1 | | | 1 |
|---|---|---|---|---|
| $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |

Stop when you reach 0. Final answer is $11001_{bin.}$

## Converting Hexadecimal to Binary

Hexadecimal numbers are useful in computer mathematics because larger binary numbers can be represented in hexadecimal using less symbols. To convert a hexadecimal number to binary, write the four bit binary equivalent for each hexit.

**Example:** Convert A1E7**hex** to binary.

## Converting Binary to Hexadecimal

Here is the algorithm to convert a binary number to its hexadecimal equivalent:

1. Start with the right-most (least significant) bit and group into sets of four bits.
2. If the last group contains less than four bits, pad to the left with zeros until the group has four bits.
3. Convert each group of four bits to its hexadecimal equivalent.

**Example:** Convert 1011001000011111011**bin** to hexadecimal.

**Exercise 1:**

a) Convert $(2004)_{dec}$ to hexadecimal and then to binary.

b) Convert 8F2A1**hex** to binary.

c) Convert 1101010111000110111110**bin** to hexadecimal.

# Arithmetic Operations – Addition/Subtraction

## Addition in the decimal number system:

When the sum of two digits is greater than 9, a carry-out of 1 is added to the next significant digit.

**Example:** 548 + 723

## Addition in the binary number system:

- 1 + 1 = 0 plus carry-out of 1
- 1 + 1 + carry-in of 1 = 1 plus carry-out of 1
- 1 + 0 + carry-in of 1 = 0 plus carry-out of 1
- 0 + 0 + carry-in of 1 = 1 without any carry-out

**Examples:** 11011 + 11010 110011 + 100111

## Addition in the hexadecimal number system

Hex addition is similar to decimal addition with the only difference being that the sum must be greater than F (15) for a carry-out of 1 to be added to the next significant hexit.

**Examples:** D5E + 7BA 4C96 + E5A8

## Subtraction in the decimal number system

If the ***subtrahend*** (the digit you are subtracting) is greater than the ***minuend*** (the digit you are subtracting from), then you need to "borrow" (subtract 1) from the minuend of the next significant digit (digit to the left) and add 10 (the base) to the minuend. Then you can subtract:

**Example:** 349 – 165

## Subtraction in the hexadecimal number system

The concept of "borrowing" is similar to that of the decimal number system, the only difference being that you add 16 to the minuend instead of 10 (because hexadecimal is base 16!).

**Examples:** 7D3 – 29A F405 – 26E6

**Exercise 2:**

Add in binary: 101110 + 100111 0111111 + 0000001

Add in hexadecimal: 7F309 + 84BD7 FFFFF + 1

Subtract in hexadecimal: 78A4 – 5D3B BEEF – FAD FFFFF - ABCDE

## Exercises

1. Add the following binary numbers:

a)
$$\begin{array}{r} 0100110 \\ +0110111 \\ \hline \end{array}$$

b)
$$\begin{array}{r} 0111111 \\ +\quad\quad 1 \\ \hline \end{array}$$

2. Add the following hexadecimal numbers:

a)
$$\begin{array}{r} ABCD \\ +\ 4EFA \\ \hline \end{array}$$

b)
$$\begin{array}{r} FFFFF \\ +\quad\quad 1 \\ \hline \end{array}$$

c)
$$\begin{array}{r} B46 \\ +\ D29 \\ \hline \end{array}$$

d)
$$\begin{array}{r} 7B6 \\ +\ FFF \\ \hline \end{array}$$

3. Subtract the following hexadecimal numbers:

a)
$$\begin{array}{r} F1 \\ -1F \\ \hline \end{array}$$

b)
$$\begin{array}{r} DEAF \\ -CAB \\ \hline \end{array}$$

c)
$$\begin{array}{r} FFFFF \\ -ABCDE \\ \hline \end{array}$$