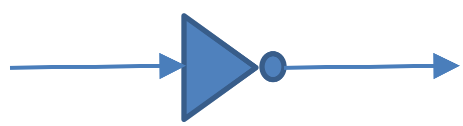
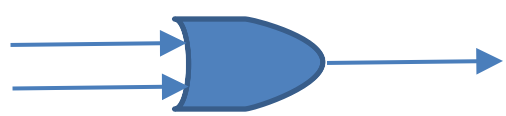
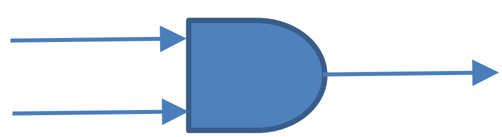
Module 1 - Lesson 06: Logic Gates & Circuits

Logic Gates

An application of Boolean algebra is to model circuits inside electronic devices. All inputs and outputs within the devices can be thought of as switches that contain a value in the set {0, 1}. The 0 represents the switch being "off" and 1 represents the switch being "on". The circuits of an electronic device can be modelled using a series of elements called gates. The following points are important to understand.

- Electronic gates require a power supply.
- Gate INPUTS are driven by voltages having two nominal values, e.g. 0V and 5V representing logic 0 and logic 1 respectively.
- The OUTPUT of a gate provides two nominal values of voltage only, e.g. 0V and 5V representing logic 0 and logic 1 respectively. In general, there is only one output to a logic gate
- There is always a time delay between an input being applied and the output responding.

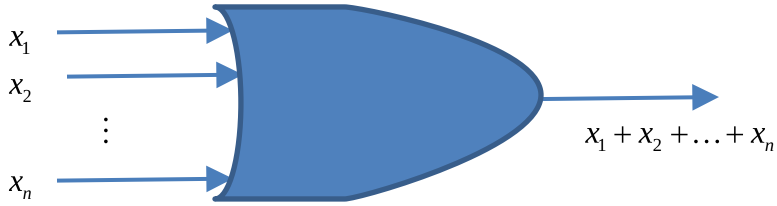
Types of Gates

	<u>**Inputs**</u>		<u>**Outputs**</u>
Inverter – The **NOT** gate	x		$\bar{x}$
OR gate	x y		$x+y$
AND gate	x y		$x \cdot y$

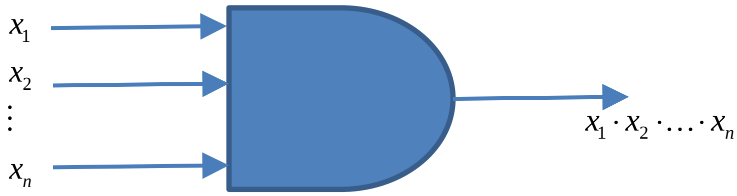
An **inverter** can only have one input and one output.
OR and **AND** gates have at least two inputs and one output.

Gates with n Inputs

OR gate



AND gate



Constructing Circuits

When constructing circuits using gates, the inputs can be shared across all gates or each gate can have a separate set of inputs. Either way is correct. The output of a gate can be chained – output can be used as input for another gate. To avoid confusion when drawing circuits, a "bump" is used whenever input lines cross.

<u>Example 1</u>: Construct circuits that produce the following outputs:

a) $x \cdot y + \bar{x} \cdot y$

b) $(x + y + z) \cdot (\bar{x} \cdot \bar{y} \cdot \bar{z})$

Sum of Products Expansion

Suppose we are given a table of all possible values of a Boolean function. Based on those values, we can construct a Boolean expression that represents the function.

Any Boolean function can be represented by a Boolean sum of Boolean products of the variables and their complements. This means that all Boolean functions can be represented using the three Boolean operators: $\cdot$, $+$, and $\bar{\ }$.

Example 1:

Find Boolean expressions that represent the functions $F(x, y, z)$ and $G(x, y, z)$, given in the table below:

x	***y***	***z***	***F***	***G***
1	1	1	0	0
1	1	0	0	1
1	0	1	1	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	1
0	0	1	0	0
0	0	0	0	0

$$F(x,y,z) = x \cdot \bar{y} \cdot z$$

$$G(x,y,z) = x \cdot y \cdot \bar{z} + \bar{x} \cdot y \cdot \bar{z}$$

Literal – a Boolean variable or its complement

Minterm – given Boolean variables $x_1, x_2, \ldots, x_n$, it is a Boolean product $y_1 y_2 \cdots y_n$ where $y_i = x_i$ or $y_i = \overline{x_i}$, where $1 \le i \le n$.

We say that a **minterm** is a product of *n* **literals**, one literal for each variable. It is also referred to as a Boolean product.

The minterm $y_1 y_2 \cdots y_n$ is 1 if and only if each y_i is 1, where $1 \le i \le n$. This occurs if and only if $x_i = 1$ when $y_i = x_i$ and $x_i = 0$ when $y_i = \overline{x_i}$

Exercises

1. Find a Boolean expression that represents the function ***F*** given the table below:

x	*Y*	*z*	*F*
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

2. Find the sum-of-products expansions of these Boolean functions:

a) $F(x, y, z) = (x + z) \cdot y$

b) $F(x, y, z) = x \cdot \bar{y}$

c) $F(x, y, z) = y \cdot \bar{z} + x$

3. Construct circuits to produce the following outputs:

a) $\bar{x} + y$

b) $\overline{(x + y)} \cdot x$

c) $x \cdot y \cdot z + \bar{x} \cdot \bar{y} \cdot \bar{z}$

d) $\overline{(\bar{x} + z)(y + \bar{z})}$