Modular Arithmetic

$a \bmod m = b$ is the mod function that gives the remainder of *a* divided by *m*.

For example $17 \bmod 5 = 2$ because 17 divided by 5 = 3, with a remainder of 2.

When we perform arithmetic operations under a modulus m, all our results will be restricted to the set of nonnegative integers less than *m*, $\mathbb{Z}_m = \{0,1,...,m-1\}$.

Under a modulus of 17, the only remainders that are possible are the integers 0,1,2,…, 15,16.

This can be written as $\mathbb{Z}_{17} = \{0,1,...,17-1\} = \{0,1,...,16\}$

Examples: determine if the following are true or false and explain your answers

$31 \bmod 5 = 1$

$10 \bmod 3 = 2$

$13 \bmod 2 = 3$

$18584 \bmod 2021 = 395$

Exercises:

1. Determine if the following are true or false and explain your answers:

a) $50 \bmod 10 = 5$

b) $121 \bmod 3 = 1$

c) $260 \bmod 16 = 4$

d) $21 \bmod 4 = 5$

2. Determine the set of valid nonnegative integers that result when performing arithmetic using a modulus of:

a) 8

b) 99

c) 2

3. If today is Tuesday, what day of the week will it be 89 days from today?

4. What day of the week will it be 1001 days after Monday?

5. If it is now 2 o'clock, what time will it be in 199 hours (ignoring AM or PM)?

6. In a card game the maximum number of cards in a deck of 52 playing cards is used and all players receive the same number of cards.

a) How many cards must be removed from the deck if there are 7 players?

b) How many cards will each player receive?

7. Consider safety deposit boxes in a bank arranged in 30 rows with 14 boxes in each row. The boxes are numbers row by row with box #001 at the left end of the top row; box #014 at the right end of the first row, box #015 at the left end of row 2, and so on. Determine the location of box #316.

8. There are 2775 containers that have to be carried down the river. Each barge can carry 120 containers. Barges are fully loaded whenever possible. How many barges are required? How many containers will be loaded on the last barge?

Answers:

1. Determine if the following are true or false and explain your answers:

a) $50 \bmod 10 = 5$ False. 50 divided by 10 = 5 with remainder 0.

b) $121 \bmod 3 = 1$ True. 121 divided by 3 = 40 with remainder 1.

c) $260 \bmod 16 = 4$ True. 260 divided by 16 = 16 with remainder 4.

d) $21 \bmod 4 = 5$ False. 21 divided by 4 = 5 with remainder 1.

2. Determine the set of valid nonnegative integers that result when performing arithmetic using a modulus of:

a) 8 0,1,2,3,4,5,6,7

b) 99 0,1,2,…,97,98

c) 2 0,1

3. If today is Tuesday, what day of the week will it be 89 days from today?
89 mod 7 = 5. 5 days past Tuesday is Sunday.

4. What day of the week will it be 1001 days after Monday?
1001 mod 7 = 0. 0 days past Monday is… Monday!

5. If it is now 2 o'clock, what time will it be in 199 hours (ignoring AM or PM)?
199 mod 12 = 7. 7 hours past 2 o'clock is 9 o'clock.

6. In a card game the maximum number of cards in a deck of 52 playing cards is used and all players receive the same number of cards.

a) How many cards must be removed from the deck if there are 7 players?
52 mod 7 = 3. 3 cards must be removed from the deck

b) How many cards will each player receive?
49 ÷ 7 = 7 cards each

7. Consider safety deposit boxes in a bank arranged in 30 rows with 14 boxes in each row. The boxes are numbers row by row with box #001 at the left end of the top row; box #014 at the right end of the first row, box #015 at the left end of row 2, and so on. Determine the location of box #316.
316 ÷ 14 = 22, with remainder 8.
The quotient of 22 indicates that there are 22 complete rows.
The location of box #316 is the 8th box in the 23rd row.

8. There are 2775 containers that have to be carried down the river. Each barge can carry 120 containers. Barges are fully loaded whenever possible. How many barges are required? How many containers will be loaded on the last barge?
2775 ÷ 120 = 23, with remainder 15.
The quotient of 23 indicates that 23 fully loaded barges must be used.
The 24th barge will have 15 containers in it.