Translating into mathematical expressions

Solving problems in mathematics often requires translating statements in English into mathematical forms involving variables and arithmetic operations.

Example 1: "the sum of *x* and twice *y*"

We can translate this statement into a mathematical expression by identifying the operation (addition) and the expressions that are being added (*x* and 2*y*):

$$x + 2y$$

Note that the word 'and' in the statement is where we place the mathematical symbol for addition (+).

Example 2: "the product of 4 and *x* squared plus twice *x*"

We can translate this statement into a mathematical expression by identifying the first operation (multiplication) and the expressions associated with the first operation (4 and x^2 + 2*x*):

$$4 \times \left(x^2 + 2x \right)$$

Note that the word 'and' in the statement is where we place the mathematical symbol for multiplication ($\times$).

Example 3: "the difference between *x* and *y* all squared, plus a fourth of *x*"

We can translate this statement into a mathematical expression by writing the difference of *x* and *y* (subtraction) in brackets and squaring the entire result, and then adding a fourth of *x*:

$$\left(x - y \right)^2 + \frac{1}{4}x$$

Note that the word 'and' in the statement is where we place the mathematical symbol for difference ($-$).

Exercises:

Translate each of the following statements into mathematical expressions:

a) "the product of x and y squared"

b) "the difference between one fifth of a and b"

c) "the sum of five and the tenth power of x"

d) "seven less than the square root of x"

e) "the difference between one half of x squared and three times b"

f) "the product of one third of a and c cubed"

g) "the quotient of x squared and y"

h) "the quotient of x and y all squared"

i) "the quotient of x and x squared plus 9"

j) "the quotient of x and x squared, plus 9"

Answers:

Translate each of the following statements into mathematical expressions.

a) "the product of x and y squared"

$x \times y^2$

b) "the difference between one fifth of *a* and *b*"

$\dfrac{1}{5}a - b$

c) "the sum of five and the tenth power of x"

$5 + x^{10}$

d) "seven less than the square root of *x*"

$\sqrt{x} - 7$

e) "the difference between one half of *x* squared and three times *b*"

$\dfrac{1}{2}x^2 - 3b$

f) "the product of one third of *a* and *c* cubed"

$\left(\dfrac{1}{3}a\right) \times \left(c^3\right)$

g) "the quotient of *x* squared and *y*"

$x^2 \div y$

h) "the quotient of *x* and *y* all squared"

$\left(x \div y\right)^2$

i) "the quotient of *x* and *x* squared plus 9"

$x \div \left(x^2 + 9\right)$

j) "the quotient of *x* and *x* squared, plus 9"

$x \div x^2 + 9$

Translating into equations and inequalities

Statements involving "equal to", "less than", and "greater than" can be tricky, because these terms involve constructing equations and inequalities.

Example 1:   "two more than *x* is equal to ten"

This translates into the equation: $x + 2 = 10$

Example 2:   "five is less than *x*"

This translates into the inequality: $5 < x$

Example 3:   "five less than *x*"

This does not translate into an inequality. It simply means $x - 5$

Example 4:   "three less than *x* is greater than twice *y*"

The first part translates into $x - 3$ followed by the inequality $> 2y$.

The expression is: $x - 3 > 2y$

Example 5:   "twelve greater than x is equal to two less than *y*"

Here the words "greater than" and "less than" represent addition (+) and subtraction (–).

The expression is: $x + 12 = y - 2$

Exercises:

Translate each of the following statements into mathematical expressions.

a) "*x* is less than 3 plus *y*"

b) "four less than *a* is equal to *b*"

c) "twice *x* is greater than five less than *y*"

d) "the square root of *x* is less than seven"

e) "the difference between *x* squared and *y* is equal to four"

f) "*x* is equal to the square root of ten greater than *a*"

Answers:

Translate each of the following statements into mathematical expressions.

a) "*x* is less than 3 plus *y*"

$$x < 3 + y$$

b) "four less than *a* is equal to *b*"

$$a - 4 = b$$

c) "twice *x* is greater than five less than *y*"

$$2x > y - 5$$

d) "the square root of *x* is less than seven"

$$\sqrt{x} < 7$$

e) "the difference between *x* squared and *y* is equal to four"

$$x^2 - y = 4$$

f) "*x* is equal to the square root of ten greater than *a*"

$$x = \sqrt{a + 10}$$