Module 1 - Lesson 01: Number Systems

Number Systems

In any number system, symbols are used to represent quantitative values. The **base** of any number system is equivalent to the number of distinct, admissible symbols.

Decimal Number System (Base 10)

The decimal number system is the predominant number system used in society. It consists of **ten** symbols (also known as digits) and all decimal numbers are presented as some combination of these digits.

The symbols in the decimal number system: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Binary Number System (Base 2)

There are two symbols in the binary number system: 0, 1

Why are we learning about binary in this course?

The symbols in the binary number system are known as **bits** (***bi***nary digi***ts***). Digital computers use the binary number system because the components that store data within a computer are electronic switches with two stable states (on, off). These states are referenced by the symbols 0 (off) and 1 (on).

Hexadecimal Number System (Base 16)

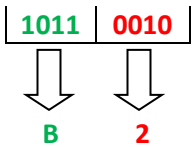
The hexadecimal number system uses sixteen admissible digits, known as **hexits**. The hexadecimal system is commonly referred to as "hex".

The symbols in the hexadecimal number system: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Why are we learning about hexadecimal in this course?

Computer memory is divided into tiny storage locations known as **bytes**. A byte consists of eight bits. Each hexit can be written as <u>four</u> bits. Therefore, a byte can be written as two hexits.

<u>Example:</u>



One byte of data (8 bits)

Hexadecimal representation of the above byte of data

Number System Notation

There are numerous ways to distinguish between hexadecimal, binary and decimal numbers. For the purposes of this course, all binary numbers will be *succeeded* by a lowercase **bin** and hexadecimal numbers will be *succeeded* by a lowercase **hex**.

Examples: 10011**bin**, 101**hex**, 7A49C**hex**, FADE**hex**

Relationship between number systems

Decimal	Binary	Hex		Decimal	Binary	Hex		Decimal	Binary	Hex
0	0	00_{hex}		16	1 0000	10_{hex}		32	10 0000	20_{hex}
1	1	01_{hex}		17	1 0001	11_{hex}		33	10 0001	21_{hex}
2	10	02_{hex}		18	1 0010	12_{hex}		34	10 0010	22_{hex}
3	11	03_{hex}		19	1 0011	13_{hex}		35	10 0011	23_{hex}
4	100	04_{hex}		20	1 0100	14_{hex}		36	10 0100	24_{hex}
5	101	05_{hex}		21	1 0101	15_{hex}		37	10 0101	25_{hex}
6	110	06_{hex}		22	1 0110	16_{hex}		38	10 0110	26_{hex}
7	111	07_{hex}		23	1 0111	17_{hex}		39	10 0111	27_{hex}
8	1000	08_{hex}		24	1 1000	18_{hex}		40	10 1000	28_{hex}
9	1001	09_{hex}		25	1 1001	19_{hex}		41	10 1001	29_{hex}
10	1010	$0A_{hex}$		26	1 1010	$1A_{hex}$		42	10 1010	$2A_{hex}$
11	1011	$0B_{hex}$		27	1 1011	$1B_{hex}$		43	10 1011	$2B_{hex}$
12	1100	$0C_{hex}$		28	1 1100	$1C_{hex}$		44	10 1100	$2C_{hex}$
13	1101	$0D_{hex}$		29	1 1101	$1D_{hex}$		45	10 1101	$2D_{hex}$
14	1110	$0E_{hex}$		30	1 1110	$1E_{hex}$		46	10 1110	$2E_{hex}$
15	1111	$0F_{hex}$		31	1 1111	$1F_{hex}$		47	10 1111	$2F_{hex}$

Can you identify any patterns?

Base 10 (Decimal) Place Values

Place value depends on the base of the number system, raised to a power dependent on position. For a decimal number, the base is 10, and is raised to a power numbered from right to left of 0, 1, 2, etc.
The place values for base 10 are as follows: 10^0 (ones), 10^1 (tens), 10^2 (hundreds), etc.

Example: The place values for the number 6,439 are:

6	4	3	9
10^3	10^2	10^1	10^0

Base 10 (Decimal) Expanded Notation

Expanded notation means taking a number and writing it based on its place values.

Example: 832

Example 2: 7012

Exercise 1:

Write the following decimal numbers in expanded notation.

a) 398

b) 19640

c) 500001

Base 2 (Binary) Place Values

Similar to the decimal system, base 2 place values are raised to a power from right to left starting at 0. The only difference is that the base is 2 instead of 10.

Example:

1	0	1	0	1
2^4	2^3	2^2	2^1	2^0

Base 2 (Binary) Expanded Notation

Example 1: 1101

Example 2: 1010011

Exercise 2:

Write the following binary numbers in expanded notation and calculate its decimal equivalent.

a) 1111

b) 10010001

c) 011010

Base 16 (Hexadecimal) Place Values

Similar to the decimal system, base 16 place values are raised to a power from right to left starting at 0. The only difference is that the base is 16 instead of 10.

Example:

B	E	7	C	A
16^4	16^3	16^2	16^1	16^0

Base 16 (Hexadecimal) Expanded Notation

Example 1: 1238_{hex}

Example 2: $40CAB_{hex}$

Exercise 3:

Write the following hexadecimal numbers in expanded notation.

a) $84A2_{hex}$

b) $3EF64_{hex}$

c) $1C0DE_{hex}$

Exercises

1. Write the following numbers in expanded notation and calculate the equivalent decimal value.

   a) $(363)_{dec}$

   b) 10110_{bin}

   c) $D2A6_{hex}$

2. Count from 10101***bin*** to 11010***bin***, writing all binary numbers in between.

3. Count from 1DA***hex*** to 1F0***hex***, writing all hexadecimal numbers in between.

4. What number comes before FD0***hex*** in hexadecimal?

5. What number comes after 10111***bin*** in binary?

6. Which of the following is the smallest value?
   110110111***bin*** 1B6***hex*** 437_{dec}