

Module 1: Lesson 5 – Truth Tables

A truth table is a handy logical device that gives the truth value of a logical expression for all possible truth values of input in a logical expression. Think of the truth table as a logical calculator that figures out the output of any logical Boolean expression.

As seen in the previous lesson, here is a look at your first set of truth tables that summarize the logical behaviour of the basic logical operators.

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

A	NOT A
0	1
1	0

A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

Evaluating logical expressions using truth tables

Consider the Boolean function: $F(x, y, z) = xy + \sim z$.

What combinations of truth values for x, y, and z will make the function true? Here are the steps to create a truth table:

Step 1: how many variables does the Boolean function have? n

Step 2: create n columns.

Step 3: create 2^n rows and fill them in with all possible truth values for the n variables. To fill in the rows, you can count in binary from 0 to $2^n - 1$.

Step 4: start creating a column for each evaluation keeping in mind the order of operation.

$$F(x, y, z) = xy + \sim z$$

x	y	z	$\sim z$	xy	$xy + \sim z$
0	0	0	1	0	1
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	1	1

Exercises

1. Construct a truth table for each compound statement.

- a) $F(x, y, z) = \sim xy$
- b) $F(x, y, z) = x + yz$
- c) $F(x, y, z) = x \cdot \sim y + \sim(xyz)$
- d) $F(x, y, z) = x(yz + \sim y \cdot \sim z)$
- e) $F(x, y, z) = \sim y(xz + \sim x \cdot \sim z)$

2. Show that $P + \sim P$ is a tautology, and that $Q * \sim Q$ is a contradiction. Explain your findings.