Assignment No.2

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Download all python codes from

https://github.com/Vallidevibolla/Assignment−2−1/blob/main/code.py

and latex-tikz codes from

https://github.com/Vallidevibolla/Assignment−2−1/blob/main/main.tex

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/vectors/gvv_ncert_vectors.pdf− Q.no.2.25

1 Question No.2.25

Find a point on the y-axis which is equidistant from the points A=$\begin{pmatrix}6\\5\end{pmatrix}$ and B=$\begin{pmatrix}-4\\3\end{pmatrix}$

2 Solution

Given,

$$\mathbf{A} = \begin{pmatrix}6\\5\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix}-4\\3\end{pmatrix} \tag{2.0.2}$$

Let $\underline{x}$ be the point on y-axis.Then

$$\boxed{\|\mathbf{x}-\mathbf{A}\|^2 = \|\mathbf{x}-\mathbf{B}\|^2} \tag{2.0.3}$$

$$\boxed{(\mathbf{x}-\mathbf{A})^\top(\mathbf{x}-\mathbf{A}) = (\mathbf{x}-\mathbf{B})^\top(\mathbf{x}-\mathbf{B})} \tag{2.0.4}$$

$$(\mathbf{x}-\mathbf{A})^\top(\mathbf{x}-\mathbf{A}) = \mathbf{x}^\top\mathbf{x} - \mathbf{x}^\top\mathbf{A} - \mathbf{A}^\top\mathbf{x} + \mathbf{A}^\top\mathbf{A} \tag{2.0.5}$$

$$(\mathbf{x}-\mathbf{B})^\top(\mathbf{x}-\mathbf{B}) = \mathbf{x}^\top\mathbf{x} - \mathbf{x}^\top\mathbf{B} - \mathbf{B}^\top\mathbf{x} + \mathbf{B}^\top\mathbf{B} \tag{2.0.6}$$

Consider the expressions

$$\mathbf{x}^\top\mathbf{x} = \|\mathbf{x}\|^2 \tag{2.0.7}$$

$$\mathbf{x}^\top\mathbf{A} = \mathbf{A}^\top\mathbf{x} \tag{2.0.8}$$

Final expression of (2.0.4) can be written as

$$\boxed{\|\mathbf{x}\|^2 - 2\mathbf{A}^\top\mathbf{x} + \mathbf{A}^\top\mathbf{A} = \|\mathbf{x}\|^2 - 2\mathbf{B}^\top\mathbf{x} + \mathbf{B}^\top\mathbf{B}} \tag{2.0.9}$$

$$\implies -2\mathbf{A}^\top\mathbf{x} + 2\mathbf{B}^\top\mathbf{x} = \mathbf{B}^\top\mathbf{B} - \mathbf{A}^\top\mathbf{A} \tag{2.0.10}$$

$$\implies 2\mathbf{x}(\mathbf{A}^\top - \mathbf{B}^\top) = \mathbf{A}^\top\mathbf{A} - \mathbf{B}^\top\mathbf{B} \tag{2.0.11}$$

$$2\mathbf{x}(\mathbf{A}^\top - \mathbf{B}^\top) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \tag{2.0.12}$$

x lies on the y-axis

$$\boxed{\mathbf{x} = y\begin{pmatrix}0\\1\end{pmatrix} = y\mathbf{e_2}} \tag{2.0.13}$$

Now substitute this in (2.0.12)

$$2y\mathbf{e_2}(\mathbf{A}^\top - \mathbf{B}^\top) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \tag{2.0.14}$$

$$\boxed{\mathbf{y} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2\mathbf{e_2}(\mathbf{A}^\top - \mathbf{B}^\top)}} \tag{2.0.15}$$

$$2\mathbf{e_2}(\mathbf{A}^\top - \mathbf{B}^\top) = 2\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}10 & 2\end{pmatrix} = 4 \tag{2.0.16}$$

$$\implies \mathbf{y} = \begin{pmatrix}\frac{61-25}{4}\end{pmatrix} \tag{2.0.17}$$

$$\mathbf{y} = \begin{pmatrix}\frac{36}{4}\end{pmatrix} \tag{2.0.18}$$

$$\boxed{\therefore y = 9} \tag{2.0.19}$$

Finally the desired point on y-axis equidistance from A and B is $\begin{pmatrix}0\\9\end{pmatrix}$.

See the figure generated by using python

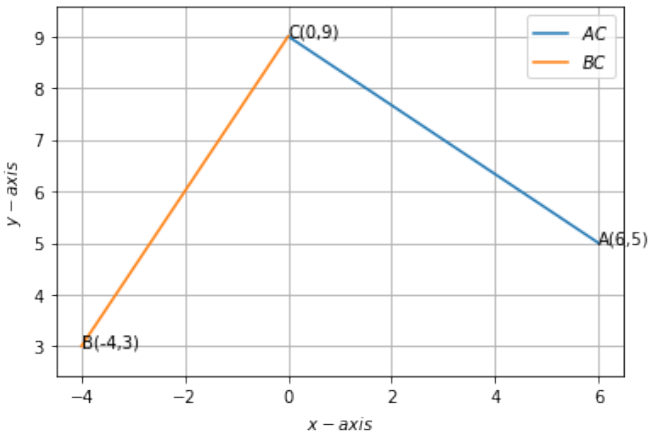

Fig. 2.1: Fig. 2.25