

# Assignment No.2

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Download all python codes from

<https://github.com/Vallidevibolla/Assignment-2-1/blob/main/code.py>

and latex-tikz codes from

<https://github.com/Vallidevibolla/Assignment-2-1/blob/main/main.tex>

Question taken from

[https://github.com/gadepall/ncert/blob/main/linalg/vectors/gvv\\_ncert\\_vectors.pdf](https://github.com/gadepall/ncert/blob/main/linalg/vectors/gvv_ncert_vectors.pdf)– Q.no.2.25

## 1 QUESTION No.2.25

Find a point on the y-axis which is equidistant from the points  $A = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$  and  $B = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

## 2 SOLUTION

Given,

$$\mathbf{A} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \quad (2.0.2)$$

Let  $\underline{x}$  be the point on y-axis. Then

$$\|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \quad (2.0.3)$$

$$(\mathbf{x} - \mathbf{A})^\top (\mathbf{x} - \mathbf{A}) = (\mathbf{x} - \mathbf{B})^\top (\mathbf{x} - \mathbf{B}) \quad (2.0.4)$$

$$(\mathbf{x} - \mathbf{A})^\top (\mathbf{x} - \mathbf{A}) = \mathbf{x}^\top \mathbf{x} - \mathbf{x}^\top \mathbf{A} - \mathbf{A}^\top \mathbf{x} + \mathbf{A}^\top \mathbf{A} \quad (2.0.5)$$

$$(\mathbf{x} - \mathbf{B})^\top (\mathbf{x} - \mathbf{B}) = \mathbf{x}^\top \mathbf{x} - \mathbf{x}^\top \mathbf{B} - \mathbf{B}^\top \mathbf{x} + \mathbf{B}^\top \mathbf{B} \quad (2.0.6)$$

Consider the expressions

$$\mathbf{x}^\top \mathbf{x} = \|\mathbf{x}\|^2 \quad (2.0.7)$$

$$\mathbf{x}^\top \mathbf{A} = \mathbf{A}^\top \mathbf{x} \quad (2.0.8)$$

Final expression of equ.2.0.4 using this written as

$$\|\mathbf{x}\|^2 - 2\mathbf{A}^\top \mathbf{x} + \mathbf{A}^\top \mathbf{A} = \|\mathbf{x}\|^2 - 2\mathbf{B}^\top \mathbf{x} + \mathbf{B}^\top \mathbf{B} \quad (2.0.9)$$

$$\Rightarrow -2\mathbf{A}^\top \mathbf{x} + 2\mathbf{B}^\top \mathbf{x} = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A} \quad (2.0.10)$$

$$\Rightarrow 2\mathbf{x}(\mathbf{A}^\top - \mathbf{B}^\top) = \mathbf{A}^\top \mathbf{A} - \mathbf{B}^\top \mathbf{B} \quad (2.0.11)$$

$$2\mathbf{x}(\mathbf{A}^\top - \mathbf{B}^\top) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.12)$$

$\underline{x}$  lies on the y-axis

$$\mathbf{x} = y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = y\mathbf{e}_2$$

Now substitute this in equ.2.0.12

$$2y\mathbf{e}_2(\mathbf{A}^\top - \mathbf{B}^\top) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.13)$$

$$\mathbf{y} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2\mathbf{e}_2(\mathbf{A}^\top - \mathbf{B}^\top)} \quad (2.0.14)$$

$$2\mathbf{e}_2(\mathbf{A}^\top - \mathbf{B}^\top) = 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} (10 \ 2) = 4 \quad (2.0.15)$$

$$\Rightarrow \mathbf{y} = \left( \frac{61 - 25}{4} \right) \mathbf{y} = \left( \frac{36}{4} \right) \quad (2.0.16)$$

$$\therefore y = 9 \quad (2.0.17)$$

Finally the desired point on y-axis equidistance from A and B is  $\begin{pmatrix} 0 \\ 9 \end{pmatrix}$ .

See the figure generated by using python

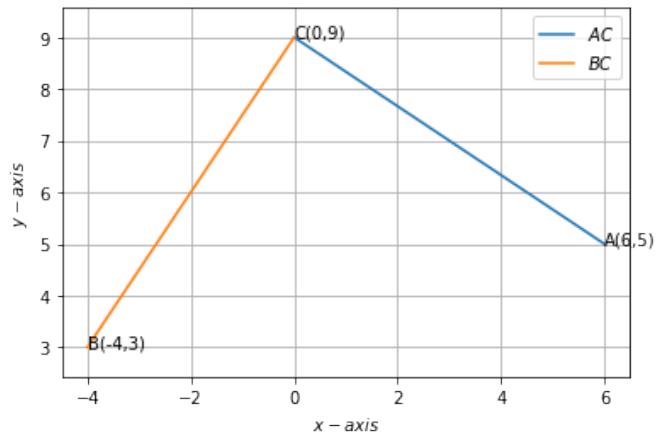


Fig. 2.1: Fig. 2.25