

Assignment No.2

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Download all python codes from

<https://github.com/Vallidevibolla/Assignment-2-1/blob/main/code.py>

and latex-tikz codes from

<https://github.com/Vallidevibolla/Assignment-2-1/blob/main/main.tex>

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/vectors/gvv_ncert_vectors.pdf– Q.no.2.25

$$\|x\|^2 - 2\mathbf{A}^T \mathbf{x} + \mathbf{A}^T \mathbf{A} = \|x\|^2 - 2\mathbf{B}^T \mathbf{x} + \mathbf{B}^T \mathbf{B} \quad (2.0.9)$$

$$\Rightarrow -2\mathbf{A}^T \mathbf{x} + 2\mathbf{B}^T \mathbf{x} = \mathbf{B}^T \mathbf{B} - \mathbf{A}^T \mathbf{A} \quad (2.0.10)$$

$$\Rightarrow 2\mathbf{x}(\mathbf{A}^T - \mathbf{B}^T) = \mathbf{A}^T \mathbf{A} - \mathbf{B}^T \mathbf{B} \quad (2.0.11)$$

$$2\mathbf{x}(\mathbf{A}^T - \mathbf{B}^T) = \|A\|^2 - \|B\|^2$$

1 QUESTION No.2.25

Find a point on the y-axis which is equidistant from the points $A = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and $B = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

2 SOLUTION

Given,

$$\mathbf{A} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \quad (2.0.2)$$

Let \mathbf{x} be the point on y-axis. Then

$$\|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \quad (2.0.3)$$

$$(\mathbf{x} - \mathbf{A})^T (\mathbf{x} - \mathbf{A}) = (\mathbf{x} - \mathbf{B})^T (\mathbf{x} - \mathbf{B}) \quad (2.0.4)$$

$$(\mathbf{x} - \mathbf{A})^T (\mathbf{x} - \mathbf{A}) = \mathbf{x}^T \mathbf{x} - \mathbf{x}^T \mathbf{A} - \mathbf{A}^T \mathbf{x} + \mathbf{A}^T \mathbf{A} \quad (2.0.5)$$

$$(\mathbf{x} - \mathbf{B})^T (\mathbf{x} - \mathbf{B}) = \mathbf{x}^T \mathbf{x} - \mathbf{x}^T \mathbf{B} - \mathbf{B}^T \mathbf{x} + \mathbf{B}^T \mathbf{B} \quad (2.0.6)$$

Consider the expressions

$$\mathbf{x}^T \mathbf{x} = \|x\|^2 \quad (2.0.7)$$

$$\mathbf{x}^T \mathbf{A} = \mathbf{A}^T \mathbf{x} \quad (2.0.8)$$

Final expression of equ.2.0.4 using this written as

\mathbf{x} lies on the y-axis

$$\mathbf{x} = y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = y \mathbf{e}_2 \quad \text{Now substitute this in equ.2.0.12}$$

$$2y \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 10 & 2 \end{pmatrix} = 61 - 25 \quad (2.0.13)$$

$$2y(0 + 2) = 36 \quad (2.0.14)$$

$$4y = 36 \quad (2.0.15)$$

$$\therefore y = 9 \quad (2.0.16)$$

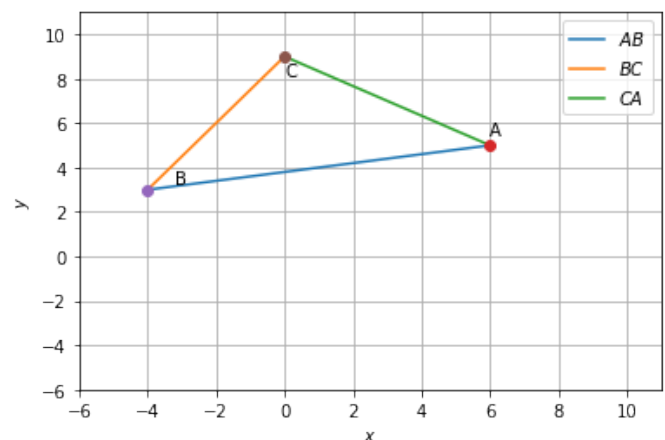


Fig. 2.1: Fig. 2.25