Assignment No.2

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Download all python codes from

https://github.com/Vallidevibolla/Assignment−2−1/blob/main/code.py

and latex-tikz codes from

https://github.com/Vallidevibolla/Assignment−2−1/blob/main/main.tex

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/vectors/gvv_ncert_vectors.pdf− Q.no.2.25

1 Question No.2.25

Find a point on the y-axis which is equidistant from the points A=$\begin{pmatrix}6\\5\end{pmatrix}$ and B=$\begin{pmatrix}-4\\3\end{pmatrix}$

2 Solution

Given,

$$\mathbf{A} = \begin{pmatrix}6\\5\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix}-4\\3\end{pmatrix} \tag{2.0.2}$$

Let $\underline{x}$ be the point on y-axis.Then

$$||\underline{x}-\mathbf{A}||^2 = ||\underline{x}-\mathbf{B}||^2 \tag{2.0.3}$$

$$\boxed{(\underline{x}-\mathbf{A})^T(\underline{x}-\mathbf{A}) = (\underline{x}-\mathbf{B})^T(\underline{x}-\mathbf{B})} \tag{2.0.4}$$

$$(\underline{x}-\mathbf{A})^T(\mathbf{x}-\mathbf{A}) = \underline{x}^T\mathbf{x}-\underline{x}^T\mathbf{A}-\mathbf{A}^T\mathbf{x}+\mathbf{A}^T\mathbf{A} \tag{2.0.5}$$

$$(\underline{x}-\mathbf{B})^T(\mathbf{x}-\mathbf{B}) = \underline{x}^T\mathbf{x}-\underline{x}^T\mathbf{B}-\mathbf{B}^T\mathbf{x}+\mathbf{B}^T\mathbf{B} \tag{2.0.6}$$

Consider the expressions

$$\mathbf{x}^T\mathbf{x} = ||\mathbf{x}||^2 \tag{2.0.7}$$

$$\mathbf{x}^T\mathbf{A} = \mathbf{A}^T\mathbf{x} \tag{2.0.8}$$

Final expression of equ.2.0.4 using this written as

$$||\mathbf{x}||^2 - 2\mathbf{A}^T\mathbf{x} + \mathbf{A}^T\mathbf{A} = ||\mathbf{x}||^2 - 2\mathbf{B}^T\mathbf{x} + \mathbf{B}^T\mathbf{B} \tag{2.0.9}$$

$$\implies -2\mathbf{A}^T\mathbf{x} + 2\mathbf{B}^T\mathbf{x} = \mathbf{B}^T\mathbf{B} - \mathbf{A}^T\mathbf{A} \tag{2.0.10}$$

$$\implies 2\mathbf{x}(\mathbf{A}^T - \mathbf{B}^T) = \mathbf{A}^T\mathbf{A} - \mathbf{B}^T\mathbf{B} \tag{2.0.11}$$

$$2\mathbf{x}(\mathbf{A}^T - \mathbf{B}^T) = ||\mathbf{A}||^2 - ||\mathbf{B}||^2 \tag{2.0.12}$$

x lies on the y-axis

x = y$\begin{pmatrix}0\\1\end{pmatrix}$= y e_2

Now substitute this in equ.2.0.12

$$2\mathbf{y}\mathbf{e_2}(\mathbf{A^T} - \mathbf{B^T}) = ||\mathbf{A}||^2 - ||\mathbf{B}||^2 \tag{2.0.13}$$

$$\boxed{\mathbf{y} = \frac{||\mathbf{A}||^2 - ||\mathbf{B}||^2}{\mathbf{2e_2}.(\mathbf{A^T} - \mathbf{B^T})}} \tag{2.0.14}$$

$$\mathbf{2e_2}.(\mathbf{A^T} - \mathbf{B^T}) = 2\begin{pmatrix}0\\1\end{pmatrix}.\begin{pmatrix}10 & 2\end{pmatrix} = 4 \tag{2.0.15}$$

$$\implies \mathbf{y} = \begin{pmatrix}\frac{61-25}{4}\end{pmatrix} \tag{2.0.16}$$

$$\implies \mathbf{y} = \begin{pmatrix}\frac{36}{4}\end{pmatrix} \tag{2.0.17}$$

$$\boxed{\therefore \text{ y=9}} \tag{2.0.18}$$

Finally the point on y-axis equidistance from A and B is $\begin{pmatrix}0, 9\end{pmatrix}$

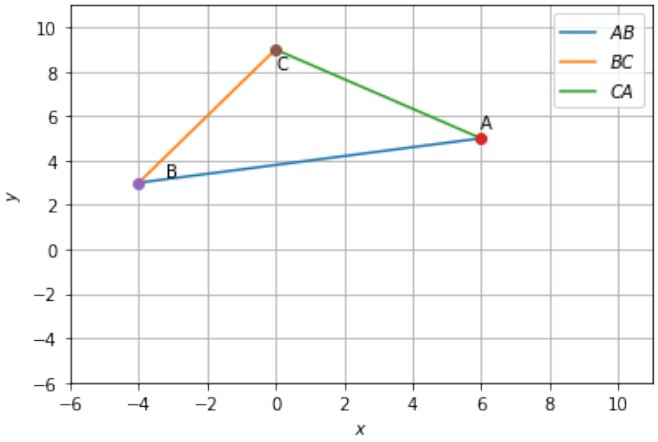

Fig. 2.1: Fig. 2.25