

Assignment No.2

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Download all python codes from

<https://github.com/Vallidevibolla/Assignment-2-1/blob/main/code.py>

and latex-tikz codes from

<https://github.com/Vallidevibolla/Assignment-2-1/blob/main/main.tex>

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/vectors/gvv_ncert_vectors.pdf– Q.no.2.25

1 QUESTION No.2.25

Find a point on the y-axis which is equidistant from the points $A = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and $B = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

2 SOLUTION

Given,

$$\mathbf{A} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \quad (2.0.2)$$

Let \underline{x} be the point on y-axis. Then

$$\|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \quad (2.0.3)$$

$$(\mathbf{x} - \mathbf{A})^T (\mathbf{x} - \mathbf{A}) = (\mathbf{x} - \mathbf{B})^T (\mathbf{x} - \mathbf{B}) \quad (2.0.4)$$

$$(\mathbf{x} - \mathbf{A})^T (\mathbf{x} - \mathbf{A}) = \mathbf{x}^T \mathbf{x} - \mathbf{x}^T \mathbf{A} - \mathbf{A}^T \mathbf{x} + \mathbf{A}^T \mathbf{A} \quad (2.0.5)$$

$$(\mathbf{x} - \mathbf{B})^T (\mathbf{x} - \mathbf{B}) = \mathbf{x}^T \mathbf{x} - \mathbf{x}^T \mathbf{B} - \mathbf{B}^T \mathbf{x} + \mathbf{B}^T \mathbf{B} \quad (2.0.6)$$

Consider the expressions

$$\mathbf{x}^T \mathbf{x} = \|\mathbf{x}\|^2 \quad (2.0.7)$$

$$\mathbf{x}^T \mathbf{A} = \mathbf{A}^T \mathbf{x} \quad (2.0.8)$$

Final expression of equ.2.0.4 using this written as

$$\|\mathbf{x}\|^2 - 2\mathbf{A}^T \mathbf{x} + \mathbf{A}^T \mathbf{A} = \|\mathbf{x}\|^2 - 2\mathbf{B}^T \mathbf{x} + \mathbf{B}^T \mathbf{B} \quad (2.0.9)$$

$$\implies -2\mathbf{A}^T \mathbf{x} + 2\mathbf{B}^T \mathbf{x} = \mathbf{B}^T \mathbf{B} - \mathbf{A}^T \mathbf{A} \quad (2.0.10)$$

$$\implies 2\mathbf{x}(\mathbf{A}^T - \mathbf{B}^T) = \mathbf{A}^T \mathbf{A} - \mathbf{B}^T \mathbf{B} \quad (2.0.11)$$

$$2\mathbf{x}(\mathbf{A}^T - \mathbf{B}^T) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.12)$$

\underline{x} lies on the y-axis

$$\mathbf{x} = y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = y\mathbf{e}_2$$

Now substitute this in equ.2.0.12

$$2y\mathbf{e}_2(\mathbf{A}^T - \mathbf{B}^T) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.13)$$

$$\mathbf{y} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2\mathbf{e}_2 \cdot (\mathbf{A}^T - \mathbf{B}^T)} \quad (2.0.14)$$

$$2\mathbf{e}_2 \cdot (\mathbf{A}^T - \mathbf{B}^T) = 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 10 & 2 \end{pmatrix} = 4 \quad (2.0.15)$$

$$\implies \mathbf{y} = \left(\frac{61 - 25}{4} \right) \mathbf{y} = \left(\frac{36}{4} \right) \quad (2.0.16)$$

$$\therefore y = 9 \quad (2.0.17)$$

Finally the desired point on y-axis equidistance from A and B is $\begin{pmatrix} 0 \\ 9 \end{pmatrix}$.

See the figure generated by using python

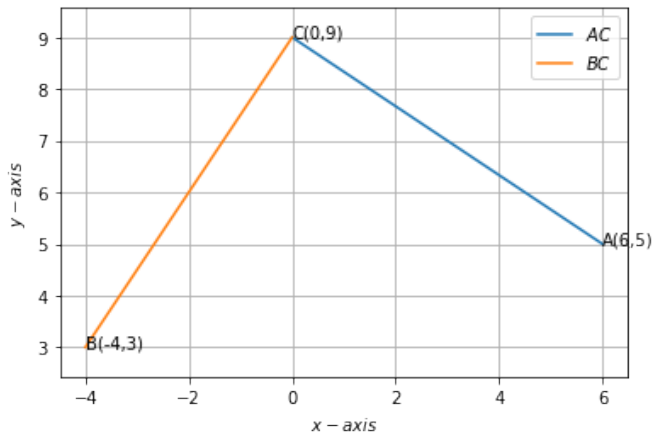


Fig. 2.1: Fig. 2.25