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Assignment No.5

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Download all python codes from

https://github.com/Vallidevibolla/Assignment-4/blob/main/code.py

and latex-tikz codes from

https://github.com/Vallidevibolla/Assignment-4/blob/main/main.tex

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/quadratic_forms/gvv_ncert_quadratic_forms.pdf-Q.no.2.7

1 Question 2.7

Find the area of the region bounded by curve

$$y = x^2 \tag{1.0.1}$$

and the lines x=1,x=4 and x-axis in the first quadrant.

2 solution

Secant: The points of intersection of the line

$$L: \quad \mathbf{x} = \mathbf{q} + \mu \mathbf{m} \quad \mu \in \mathbb{R} \tag{2.0.1}$$

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \tag{2.0.2}$$

where

$$\mu_{i} = \frac{1}{\mathbf{m}^{T} \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^{T} \left(\mathbf{V} \mathbf{q} + \mathbf{u} \right) \right.$$

$$\pm \sqrt{\left[\mathbf{m}^{T} \left(\mathbf{V} \mathbf{q} + \mathbf{u} \right) \right]^{2} - \left(\mathbf{q}^{T} \mathbf{V} \mathbf{q} + 2 \mathbf{u}^{T} \mathbf{q} + f \right) \left(\mathbf{m}^{T} \mathbf{V} \mathbf{m} \right)}$$
(2.0.3)

 \because **q** is the point of contact, **q** satisfies parabola equation and

$$\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f = 0 \tag{2.0.4}$$

Given the point of contact \mathbf{q} , the equation of a tangent is

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^T \mathbf{x} + \mathbf{u}^T \mathbf{q} + f = 0$$
 (2.0.5)

(1.0.1) can be written as

$$y^2 - x = 0 (2.0.6)$$

(2.0.6) which has the form of

$$ax^2 + 2bxy + cy^2 + 2dy + 2ex + f = 0$$
 (2.0.7)

The matrix parameters are

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}, \mathbf{f} = 0. \tag{2.0.8}$$

Thus, the given curve is a parabola. \because **V** is diagonal and in standard form, Also,

$$\mathbf{Vp} = \mathbf{0} \tag{2.0.9}$$

$$\implies \mathbf{p} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.10}$$

with eigen parameters

$$\lambda_1 = 0, \lambda_2 = 1 \tag{2.0.11}$$

Given x=1 Using (1.0.1) we get

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.0.12)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.0.13)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 1 \qquad (2.0.14)$$

Similarly, x=4 we get

$$(4 0) \mathbf{x} = 16 (2.0.15)$$

The vertex of parabola can be expressed using (2.0.14)

$$\begin{pmatrix} \mathbf{u}^{\mathrm{T}} + \mathbf{n}\mathbf{p}^{\mathrm{T}} \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \mathbf{n}\mathbf{p} - \mathbf{u} \end{pmatrix}$$
 (2.0.16)

$$\begin{pmatrix} 1 & \frac{1}{2} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ 0 \\ \frac{-1}{2} \end{pmatrix}$$
 (2.0.17)

$$\implies \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ \frac{-1}{2} \end{pmatrix} \tag{2.0.18}$$

or,
$$\mathbf{c} = \begin{pmatrix} \frac{-7}{2} \\ 5 \end{pmatrix}$$
 (2.0.19)

The direction vector and normal vectors are

$$\mathbf{m} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}. \tag{2.0.20}$$

$$\kappa = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}},\tag{2.0.21}$$

From (2.0.21), (2.0.20) and (2.0.10),

$$\kappa = 0 \tag{2.0.22}$$

which, upon substitution in (2.0.23)

$$\begin{pmatrix} \mathbf{u} + \kappa \mathbf{n}^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix}$$
 (2.0.23)

and simplification yields the matrix equation

$$\begin{pmatrix} 0 & -\frac{1}{2} \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} \tag{2.0.24}$$

$$\implies \begin{pmatrix} 0 & \frac{-1}{2} \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} \tag{2.0.25}$$

or,
$$\mathbf{q} = \begin{pmatrix} \frac{-1}{4} \\ \frac{1}{2} \end{pmatrix}$$
 (2.0.26)

From (2.0.3) we get $\mu_1 = \frac{1}{2}, \mu_2 = \frac{-1}{2}$

The lines (2.0.14), (2.0.15) can be written in parametric form in (2.0.2) we get

$$\mathbf{x}_{i1} = \begin{pmatrix} \frac{-1}{4} \\ \frac{1}{2} \end{pmatrix} + \mu_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.27}$$

$$\mathbf{x}_{i2} = \begin{pmatrix} \frac{-1}{4} \\ \frac{1}{2} \end{pmatrix} + \mu_i \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{2.0.28}$$

Substituting μ_1 and μ_2 values in (2.0.27), (2.0.28) we get

$$\mathbf{K_1} = \begin{pmatrix} \frac{-1}{4} \\ 1 \end{pmatrix}, \mathbf{L_1} = \begin{pmatrix} \frac{-1}{4} \\ 0 \end{pmatrix} \tag{2.0.29}$$

$$\mathbf{K}_2 = \begin{pmatrix} \frac{-1}{4} \\ \frac{5}{2} \end{pmatrix}, \mathbf{L}_2 = \begin{pmatrix} \frac{-1}{4} \\ \frac{-3}{2} \end{pmatrix}$$
 (2.0.30)

The area enclosed by parabola and line can be given as

A = Area under line - Area under curve

$$\implies \boxed{\mathbf{A} = \mathbf{A}_1 - \mathbf{A}_3}$$

 $Thus \overline{area under the lines} (2.0.14), (2.0.15) i.e,$

$$\mathbf{A_1} = \frac{1}{2}$$

Area under the parabola(1.0.1) i.e,

$$\int A_3 = \sqrt{x} dx$$

$$\mathbf{A_3} = \frac{14}{3}$$

Puttingthe sevalue sin(2) we get

$$\mathbf{A} = -4.17, -13.17 units$$

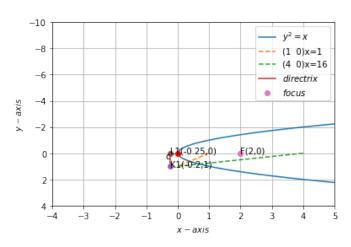


Fig. 2.1: Tangent to parabola in (1.0.1) with slope $\frac{1}{2}$.