Assignment No.5

Valli Devi Bolla

Download all python codes from

```
https://github.com/Vallidevibolla/Assignment−4/
    blob/main/code.py
```

and latex-tikz codes from

```
https://github.com/Vallidevibolla/Assignment−4/
    blob/main/main.tex
```

Question taken from

```
https://github.com/gadepall/ncert/blob/main/linalg/
    quadratic_forms/gvv_ncert_quadratic_forms.
    pdf−Q.no.2.7
```

1 Question 2.7

Find the area of the region bounded by curve

$$y = x^2 \tag{1.0.1}$$

and the lines x=1,x=4 and x-axis in the first quadrant.

Solution: (1.0.1) can be expressed as

$$y^2 - x = 0 \tag{1.0.2}$$

(1.0.2)which has the form

$$ax^2 + 2bxy + cy^2 + 2dy + 2ex + f = 0 \tag{1.0.3}$$

with parameters

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}. \tag{1.0.4}$$

Thus, the given curve is a parabola. $\because$ $\mathbf{V}$ is diagonal and in standard form, Also,

$$\mathbf{Vp} = \mathbf{0} \tag{1.0.5}$$

$$\implies \mathbf{p} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.0.6}$$

Given x=1 Using (1.0.1) we get

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.0.7}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y}\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.0.8}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 1 \tag{1.0.9}$$

Similarly, x=4 we get

$$\begin{pmatrix} 4 & 0 \end{pmatrix}\mathbf{x} = 16 \tag{1.0.10}$$

The focus is 2 and the vertex $\mathbf{c}$ is

$$\begin{pmatrix} 1 & \frac{-1}{2} \\ 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{c} = \begin{pmatrix} 1 \\ 0 \\ \frac{-1}{2} \end{pmatrix} \tag{1.0.11}$$

$$\implies \begin{pmatrix} 1 & \frac{-1}{2} \\ 0 & 1 \end{pmatrix}\mathbf{c} = \begin{pmatrix} 1 \\ \frac{-1}{2} \end{pmatrix} \tag{1.0.12}$$

$$\text{or, } \mathbf{c} = \begin{pmatrix} \frac{3}{4} \\ \frac{-1}{2} \end{pmatrix} \tag{1.0.13}$$

The direction vector and normal vectors are

$$\mathbf{m} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}. \tag{1.0.14}$$

$$\kappa = \frac{\mathbf{p}_1^T\mathbf{u}}{\mathbf{p}_1^T\mathbf{n}}, \tag{1.0.15}$$

From (1.0.15), (1.0.14) and (1.0.6),

$$\kappa = 0 \tag{1.0.16}$$

which, upon substitution in (1.0.17)

$$\begin{pmatrix} \mathbf{u} + \kappa\mathbf{n}^T \\ \mathbf{V} \end{pmatrix}\mathbf{q} = \begin{pmatrix} -f \\ \kappa\mathbf{n} - \mathbf{u} \end{pmatrix} \tag{1.0.17}$$

and simplification yields the matrix equation

$$\begin{pmatrix} 0 & \frac{-1}{2} \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} \tag{1.0.18}$$

$$\implies \begin{pmatrix} 0 & \frac{-1}{2} \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} \tag{1.0.19}$$

$$\text{or, } \mathbf{q} = \begin{pmatrix} \frac{-1}{4} \\ \frac{1}{2} \end{pmatrix} \tag{1.0.20}$$

Fig. 1.1 verifies the above results.

download.png

Fig. 1.1: Tangent to parabola in (1.0.1) with slope $\frac{1}{5}$.