

Assignment No.5

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Download all python codes from

<https://github.com/Vallidevibolla/Assignment-5/blob/main/code.py>

and latex-tikz codes from

<https://github.com/Vallidevibolla/Assignment-5/blob/main/main.tex>

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/quadratic_forms/gvv_ncert_quadratic_forms.pdf—Q.no.2.7

1 QUESTION 2.7

Find the area of the region bounded by curve

$$y^2 = x \quad (1.0.1)$$

and the lines $x=1, x=4$ and x -axis in the first quadrant.

2 SOLUTION

(1.0.1) can be written as

$$y^2 - x = 0 \quad (2.0.1)$$

The matrix parameters are

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \mathbf{f} = 0. \quad (2.0.2)$$

Thus, the given curve is a parabola. $\therefore \mathbf{V}$ is diagonal and in standard form, Also,

$$\mathbf{V}\mathbf{p} = \mathbf{0} \quad (2.0.3)$$

$$\Rightarrow \mathbf{p} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.4)$$

with eigen parameters

$$\lambda_1 = 0, \lambda_2 = 1 \quad (2.0.5)$$

Given $x=1$ Using (1.0.1) we get

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.6)$$

$$(1 \ 0) \mathbf{x} = (1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{y} (1 \ 0) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.7)$$

$$(1 \ 0) \mathbf{x} = 1 \quad (2.0.8)$$

Similarly, $x=4$ we get

$$(4 \ 0) \mathbf{x} = 16 \quad (2.0.9)$$

$$\Rightarrow (1 \ 0) \mathbf{x} = 4 \quad (2.0.10)$$

The direction vector and normal vectors are

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (2.0.11)$$

The equation of parabola is

$$\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f = 0 \quad (2.0.12)$$

The vertex of conic section in (2.0.12) is given by \mathbf{c} using

$$\begin{pmatrix} \mathbf{u}^T + \mathbf{n} \mathbf{p}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \mathbf{n} \mathbf{p} - \mathbf{u} \end{pmatrix} \quad (2.0.13)$$

$$\begin{pmatrix} -\frac{1}{4} & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \begin{pmatrix} -\frac{1}{4} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.15)$$

$$\text{or, } \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.16)$$

$$\kappa = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}}, \quad (2.0.17)$$

From (2.0.17), (2.0.11) and (2.0.4),

$$\kappa = \frac{-1}{2} \quad (2.0.18)$$

which, upon substitution in (2.0.19)

$$\begin{pmatrix} \mathbf{u} + \kappa \mathbf{n}^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix} \quad (2.0.19)$$

and simplification yields the matrix equation

$$\begin{pmatrix} \frac{-1}{2} & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.20)$$

$$\Rightarrow \begin{pmatrix} \frac{-1}{2} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.21)$$

$$\text{or, } \mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.22)$$

Secant: The points of intersection of the line

$$L: \mathbf{x} = \mathbf{q} + \mu \mathbf{m} \quad \mu \in \mathbb{R} \quad (2.0.23)$$

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \quad (2.0.24)$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u})]^2 - (\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f)(\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \quad (2.0.25)$$

$\therefore \mathbf{q}$ is the point of contact, \mathbf{q} satisfies parabola equation

Given the point of contact \mathbf{q} , the equation of a tangent is

$$(\mathbf{V} \mathbf{q} + \mathbf{u})^T \mathbf{x} + \mathbf{u}^T \mathbf{q} + f = 0 \quad (2.0.26)$$

From (2.0.25) we get

$$\mu_1 = 1, \mu_2 = -1$$

The lines (2.0.8), (2.0.9) can be written in parametric form in (2.0.24) we get

$$\mathbf{x}_i = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu_i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.0.27)$$

Substituting μ_1, μ_2 value in (2.0.27) we get

$$\mathbf{x}_i = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.0.28)$$

$$\Rightarrow \mathbf{K} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.0.29)$$

$$\mathbf{x}_i = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + -1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.0.30)$$

$$\Rightarrow \mathbf{L} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad (2.0.31)$$

The area enclosed by parabola and line can be given as

$A = \text{Area under line} - \text{Area under curve}$

$$\Rightarrow \boxed{A = A_1 - A_3} \quad (2.0.32)$$

Thus area under the lines (2.0.8), (2.0.9) is given by

$$A_1 = \frac{1}{2}, A_2 = \frac{-17}{2}$$

Area under the parabola (1.0.1) i.e.,

$$A_3 = \int x^{\frac{1}{2}} dx$$

$$\Rightarrow \boxed{A_3 = \frac{14}{3}}$$

Putting these values in (2.0.32) we get

$A = -4.17, -13.17$ units

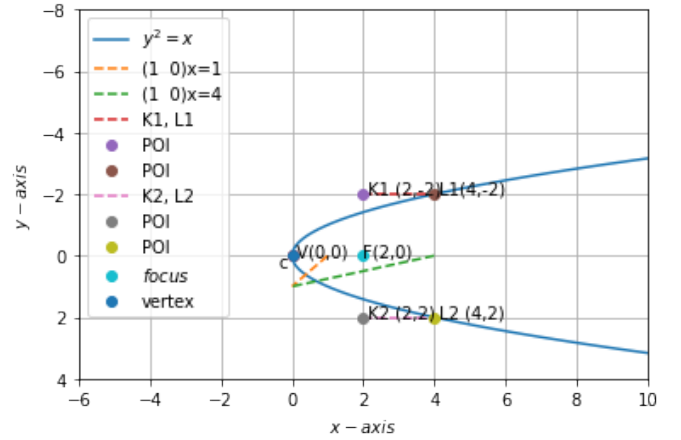


Fig. 2.1: Parabola $y^2 = x$