

My Presentation

Using Beamer

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Question 2.7

Find the area of the region bounded by curve

$$y^2 = x \tag{1}$$

and the lines $x = 1$, $x = 4$ and x -axis in the first quadrant.

Solution

Given,

$$y^2 = x \quad (2)$$

can be written as

$$y^2 - x = 0 \quad (3)$$

The matrix parameters are

$$V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad u = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \quad f = 0 \quad (4)$$

Solution

Given $x = 1$

$$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (5)$$

$$(1 \ 0) x = (1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y (1 \ 0) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (6)$$

$$(1 \ 0) x = 1 \quad (7)$$

$$\text{Similarly } x = 4, \text{ we get} \quad (8)$$

$$(1 \ 0) x = 4 \quad (9)$$

The direction vector and normal vectors are

$$m = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad n = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (10)$$

Vertex of Parabola

$$\begin{pmatrix} u^T + np^T \\ V \end{pmatrix} c = \begin{pmatrix} -f \\ np - u \end{pmatrix} \quad (11)$$

$$\begin{pmatrix} \frac{-1}{4} & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} c = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} c = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (12)$$

$$k = \frac{p_1^T u}{p_1^T n} \implies \frac{-1}{2} \quad (13)$$

Point of contact q ,

$$\begin{pmatrix} u + kn^T \\ V \end{pmatrix} q = \begin{pmatrix} -f \\ kn - u \end{pmatrix} \quad (14)$$

$$\begin{pmatrix} \frac{-1}{2} & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} q = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \implies q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (15)$$

Point of intersection

$$x_i = q + \mu_i m \quad (16)$$

$$\mu_i = \frac{1}{m^T V m} (-m^T V q + u) \quad (17)$$

$$\pm \sqrt{[m^T (V q + u)]^2 - (q^T V q + 2u^T q + f)(m^T V m)} \quad (18)$$

$$\mu = 1, -1 \quad (19)$$

Substituting values for $x = 1$ we get

$$x_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \implies K_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} L_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Substituting values for $x = 4$ we get

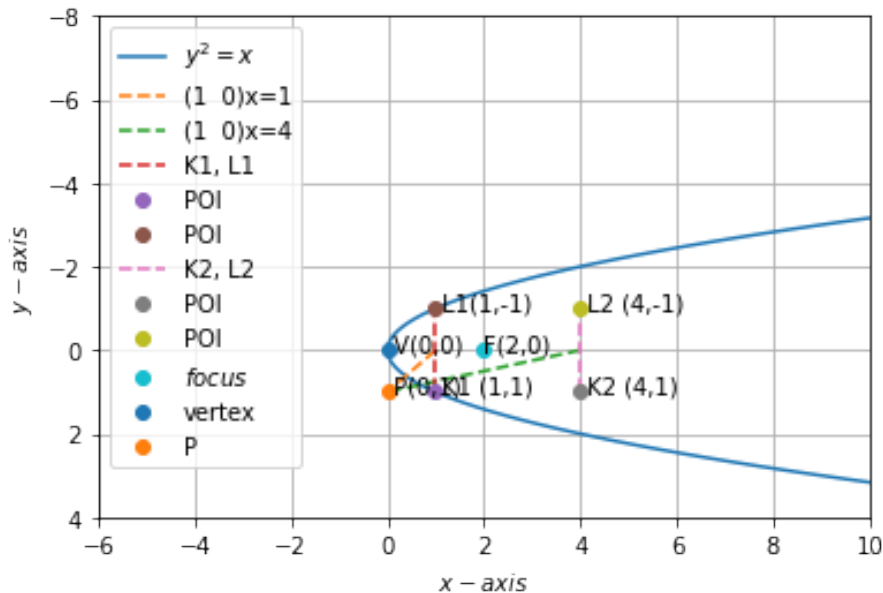
$$x_i = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \implies K_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} L_1 = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

Area enclosed by parabola

$$A = \text{Area under line}(A_1) - \text{Area under curve}(A_3)$$

$$A = -4.17, -13.17 \text{ units}$$

Output



Codes References

1 Question taken from

- https://github.com/gadepall/ncert/blob/main/linalg/quadratic_forms/gQ.no.2.7

2 Download all python codes

- <https://github.com/Vallidevibolla/Assignment-5/blob/main/code.py>

3 Download all latex-tikz codes

- <https://github.com/Vallidevibolla/Assignment-5/blob/main/main.tex>