Assignment No.7

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Download all python codes from

```
https://github.com/Vallidevibolla/Assignment−7/
    blob/main/code.py
```

and latex-tikz codes from

```
https://github.com/Vallidevibolla/Assignment−7/
    blob/main/main.tex
```

Question taken from

```
https://github.com/gadepall/ncert/blob/main/linalg/
    optimization/gvv_ncert_opt.pdf−Q.no.2.11
```

1 Question 2.11

Maximise

$$Z = x + y \tag{1.0.1}$$

subject to

$$x - y \leq -1 \tag{1.0.2}$$

$$x + y \leq 0 \tag{1.0.3}$$

$$x \geq 0 y \geq 0 \tag{1.0.4}$$

2 Solution

$$Z - x - y = 0 \tag{2.0.1}$$

$$x - y + s_1 = -1 \tag{2.0.2}$$

$$x + y + s_2 = 0 \tag{2.0.3}$$

We will write the simplex tableau

$$\left(\begin{array}{ccccc} x & y & s_1 & s_2 & c \\ 1 & -1 & 1 & 0 & -1 \\ 1 & 1 & 0 & 1 & 0 \\ \hline -1 & -1 & 0 & 0 & 0 \end{array}\right) \tag{2.0.4}$$

Keeping the pivot element as -1, we will use gauss-jordan elimination.

$$\left(\begin{array}{ccccc} x & y & s_1 & s_2 & c \\ 1 & -1 & 1 & 0 & -1 \\ 0 & 1 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \hline 0 & -2 & 1 & 0 & -1 \end{array}\right) \tag{2.0.5}$$

Keeping the pivot element, we will use gauss-jordan elimination.

$$\left(\begin{array}{ccccc} x & y & s_1 & s_2 & c \\ 1 & 0 & \frac{1}{2} & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{-1}{2} & 0 & \frac{1}{2} \\ \hline 0 & 0 & 0 & 1 & 0 \end{array}\right) \tag{2.0.6}$$

We have determined the optimal solution to be:

$$(x, y, s_1, s_2) = \left(\frac{-1}{2}, \frac{1}{2}, 0, 0\right) \tag{2.0.7}$$

$$Z = x + y \tag{2.0.8}$$

$$Z = \frac{-1}{2} + \frac{1}{2} \tag{2.0.9}$$

$$Z = 0 \tag{2.0.10}$$

The given problem can be expressed in general as matrix inequality as:

$$\max_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \tag{2.0.11}$$

$$s.t. \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}, \tag{2.0.12}$$

$$\mathbf{x} \geq \mathbf{0} \tag{2.0.13}$$

$$\mathbf{y} \geq \mathbf{0} \tag{2.0.14}$$

where

$$\mathbf{c} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \tag{2.0.16}$$

$$\mathbf{b} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{2.0.17}$$

and can be solved using *cvxpy*. Hence,

$$\mathbf{x} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}, Z = 0 \tag{2.0.18}$$

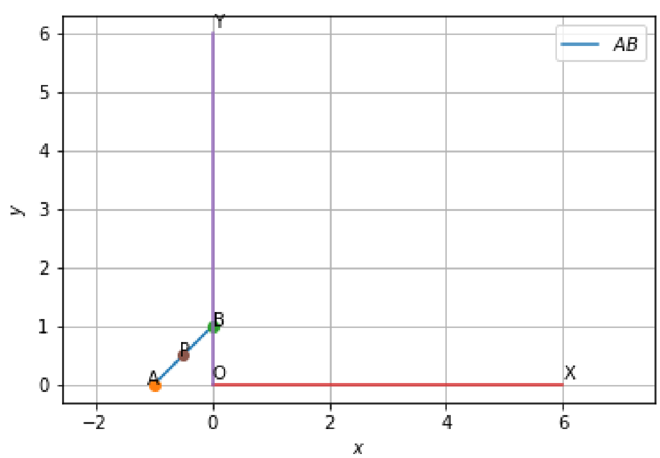

Fig. 0: Graphical solution