Assignment No.7

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Download all python codes from

https://github.com/Vallidevibolla/Assignment−7/blob/main/code.py

and latex-tikz codes from

https://github.com/Vallidevibolla/Assignment−7/blob/main/main.tex

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/optimization/gvv_ncert_opt.pdf−Q.no.2.11

1 Question 2.11

Maximise

$$Z = x + y \tag{1.0.1}$$

subject to

$$x - y \le -1 \tag{1.0.2}$$

$$x + y \le 0 \tag{1.0.3}$$

$$x \ge 0 y \ge 0 \tag{1.0.4}$$

2 Solution

$$Z = x + y \tag{2.0.1}$$

$$x - y = -1 \tag{2.0.2}$$

$$-x + y = 0 \tag{2.0.3}$$

We will write the matrix forms

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \tag{2.0.4}$$

By using RREF method we will get

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2 \to \frac{R_2}{2}} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & \frac{1}{2} \end{pmatrix} \xleftrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{1}{2} \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{x} = \frac{-1}{2} \tag{2.0.7}$$

$$\mathbf{y} = \frac{1}{2} \tag{2.0.8}$$

We have determined the optimal solution to be:

$$(x, y) = \left(\frac{-1}{2}, \frac{1}{2}\right) \tag{2.0.9}$$

$$Z = x + y \tag{2.0.10}$$

$$Z = \frac{-1}{2} + \frac{1}{2} \tag{2.0.11}$$

$$Z = 0 \tag{2.0.12}$$

The given problem can be expressed in general as matrix inequality as:

$$\max_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \tag{2.0.13}$$

$$s.t. \quad \mathbf{A}\mathbf{x} \le \mathbf{b}, \tag{2.0.14}$$

$$\mathbf{x} \ge \mathbf{0} \tag{2.0.15}$$

$$\mathbf{y} \ge \mathbf{0} \tag{2.0.16}$$

where

$$\mathbf{c} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{2.0.17}$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \tag{2.0.18}$$

$$\mathbf{b} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{2.0.19}$$

and can be solved using *cvxpy*. Hence,

$$\mathbf{x} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}, Z = 0 \tag{2.0.20}$$

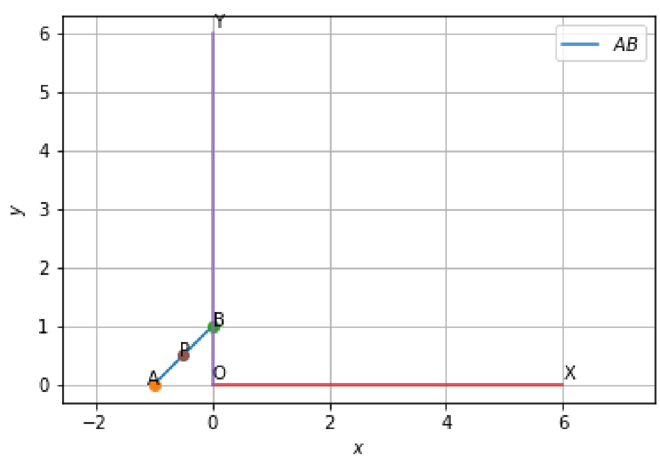

Fig. 0: Graphical solution