Assignment 9

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Download all python codes from

```
https://github.com/ka-raja-babu/Matrix-Theory/
    tree/main/Assignment9/Codes
```

and latex-tikz codes from

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https://github.com/ka-raja-babu/Matrix-Theory/
    tree/main/Assignment9
```

1 Question No. 2.21

A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 kmh^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 ms^{-1} to hit the plane ? At what minimum altitude should the pilot fly the plane to avoid being hit ? (Take $g = 10\ ms^{-2}$) .

2 Solution

Velocity of the plane is given by

$$\mathbf{v_p} = \begin{pmatrix} 200 \\ 0 \end{pmatrix} \quad \left(\because 720kmh^{-1} = 200ms^{-1}\right) \tag{2.0.1}$$

Velocity of the bullet is given by

$$\mathbf{v_b} = 600 \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} \tag{2.0.2}$$

where θ is the angle made by $\mathbf{v_b}$ with the vertical
.

Let after time t ,the bullet hits the plane such that the horizontal distance travelled by the plane and the bullet are equal.
$\therefore$

$$\mathbf{v_p}\begin{pmatrix} t & 0 \end{pmatrix} = \mathbf{v_b}\begin{pmatrix} t & 0 \end{pmatrix} \tag{2.0.3}$$

$$\implies 200t = (600\sin\theta)t \tag{2.0.4}$$

$$\implies \theta = 19.5^\circ \tag{2.0.5}$$

Acceleration of the bullet due to gravity is

$$\mathbf{g} = \begin{pmatrix} 0 \\ -10 \end{pmatrix} \tag{2.0.6}$$

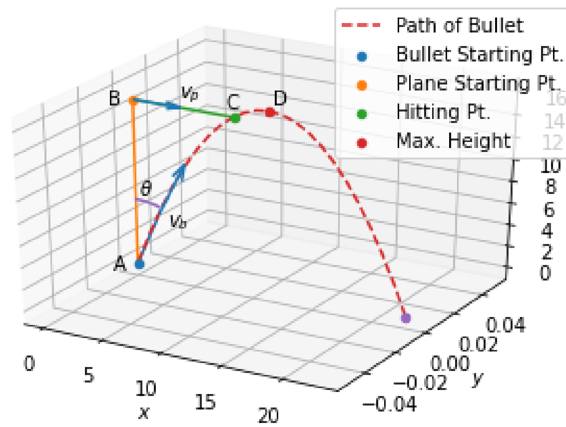

Fig. 2.1: 3D Analysis of Plane and Bullet

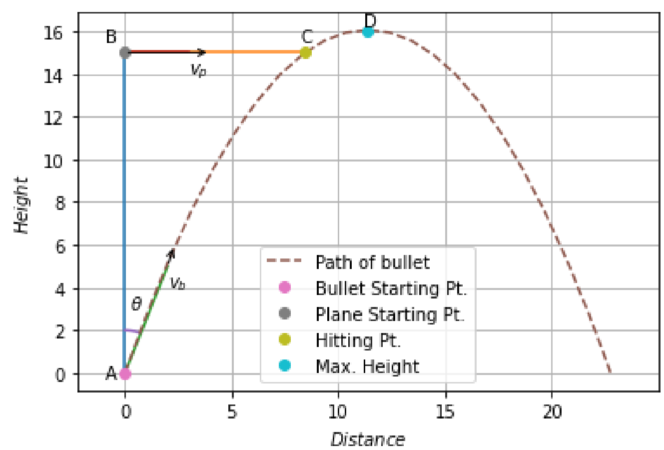

Fig. 2.2: 2D Analysis of Plane and Bullet

Position vector of the bullet's trajectory is given by

$$\mathbf{r} = \mathbf{v_b}t + \frac{1}{2}\mathbf{g}t^2 \tag{2.0.7}$$

And,velocity vector of the bullet's trajectory is given by

$$\mathbf{v} = \mathbf{v_b} + \mathbf{g}t \tag{2.0.8}$$

At the maximum height,$\theta = 0^\circ$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

$\therefore$ Time to reach the maximum height is given by

$$\begin{pmatrix}0\\0\end{pmatrix} = 600\begin{pmatrix}0\\\cos 19.5^\circ\end{pmatrix} + \begin{pmatrix}0\\-10\end{pmatrix}t_m \tag{2.0.9}$$

$$\implies 0 = 600\cos 19.5^\circ - 10t_m \tag{2.0.10}$$

$$\implies t_m = 56.558s \tag{2.0.11}$$

Substituting the value of t_m from eq.(2.0.11) into eq.(2.0.7),position vector at the maximum height is

$$\begin{pmatrix}x_m\\y_m\end{pmatrix} = 600\begin{pmatrix}\sin 19.5^\circ\\\cos 19.5^\circ\end{pmatrix}56.558 + \frac{1}{2}\begin{pmatrix}0\\-10\end{pmatrix}(56.558)^2 \tag{2.0.12}$$

$$= \begin{pmatrix}11.32\\16\end{pmatrix}km \tag{2.0.13}$$

Hence,the gun should be fired at $\boxed{\theta = 19.5^\circ}$ from the vertical to hit the plane and the pilot must fly above the maximum height of the bullet , $\boxed{y_m = 16km}$ to avoid being hit.