

# Assignment 9

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Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment9/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment9>

## 1 QUESTION No. 2.21

A fighter plane flying horizontally at an altitude of  $1.5 \text{ km}$  with speed  $720 \text{ kmh}^{-1}$  passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed  $600 \text{ ms}^{-1}$  to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take  $g = 10 \text{ ms}^{-2}$ ).

## 2 SOLUTION

Velocity of plane is given by

$$\mathbf{v}_p = \begin{pmatrix} 200 \\ 0 \end{pmatrix} \quad (\because 720 \text{ kmh}^{-1} = 200 \text{ ms}^{-1}) \quad (2.0.1)$$

Velocity of bullet is given by

$$\mathbf{v}_b = 600 \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} \quad (2.0.2)$$

where  $\theta$  is the angle made by  $\mathbf{v}_b$  with the vertical. Let after time  $t$ , bullet hits plane such that horizontal distance travelled by plane and bullet are equal.  
 $\therefore$

$$\mathbf{v}_p(t \ 0) = \mathbf{v}_b(t \ 0) \quad (2.0.3)$$

$$\implies 200t = (600\sin\theta)t \quad (2.0.4)$$

$$\implies \theta = 19.5^\circ \quad (2.0.5)$$

So, gun should be fired at  $\theta = 19.5^\circ$  from vertical to hit the plane.

Acceleration of the bullet due to gravity is

$$\mathbf{g} = \begin{pmatrix} 0 \\ -10 \end{pmatrix} \quad (2.0.6)$$

Velocity of the bullet at maximum height is

$$\mathbf{v}_m = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.7)$$

Now, the maximum height  $h_{max}$  achieved by the bullet is given by

$$(\mathbf{v}_m(0 \ 1))^2 - (\mathbf{v}_b(0 \ 1))^2 = 2\mathbf{g}(0 \ 1)h_{max} \quad (2.0.8)$$

$$\implies -(600\cos 19.5^\circ)^2 = -20h_{max} \quad (2.0.9)$$

$$\implies h_{max} = 16 \text{ km} \quad (2.0.10)$$

So, the pilot must fly above the maximum height of the bullet,  $h_{max} = 16 \text{ km}$  to avoid being hit.

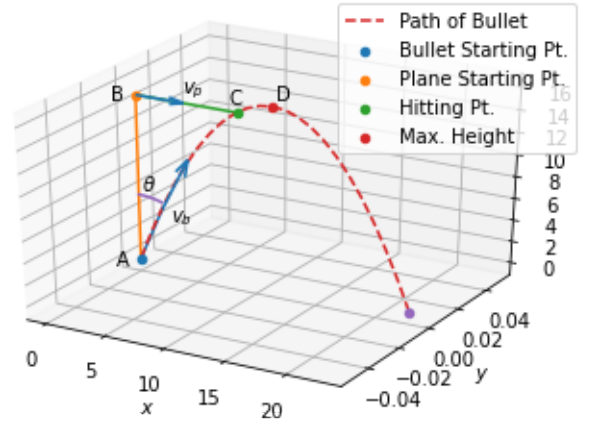


Fig. 2.1: 3D Analysis of Plane and Bullet

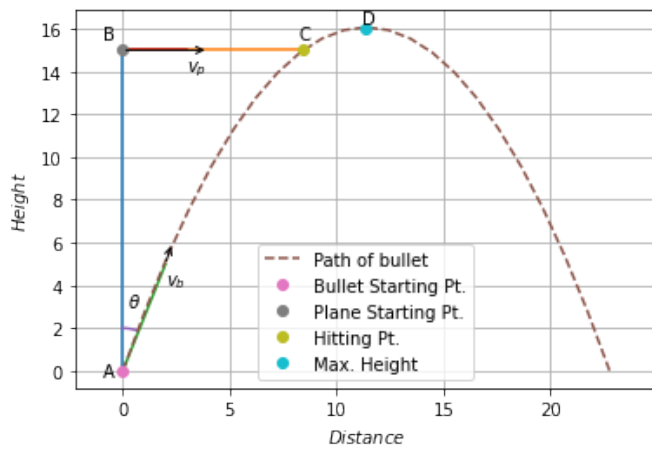


Fig. 2.2: 2D Analysis of Plane and Bullet