Assignment 12

K.A. Raja Babu

Download all python codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment12/Codes

and latex-tikz codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment12

1 Question No. 2.36

(Diet problem) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet ? What is the minimum amount of vitamin A ?

2 Solution

Component	P	Q	Requirement
Calcium	12 units	3 units	$\geq$ 240 units
Iron	4 units	20 units	$\geq$ 460 units
Cholesterol	6 units	4 units	$\leq$ 300 units
Vitamin A	6 units	3 units	

TABLE 2.1: Diet Requirements

Let the number of packets of food P be x and the number of packets of food Q be y such that

$$x \geq 0 \tag{2.0.1}$$
$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$12x + 3y \geq 240 \tag{2.0.3}$$
$$\implies 4x + y \geq 80 \tag{2.0.4}$$

and,

$$4x + 20y \geq 460 \tag{2.0.5}$$
$$\implies x + 5y \geq 115 \tag{2.0.6}$$

and,

$$6x + 4y \leq 300 \tag{2.0.7}$$
$$\implies 3x + 2y \leq 150 \tag{2.0.8}$$
$$\tag{2.0.9}$$

$\therefore$ Our problem is

$$\text{Minimize} : Z = 6x + 3y \tag{2.0.10}$$
$$\text{Subject to} : 4x + y \geq 80 \tag{2.0.11}$$
$$x + 5y \geq 115 \tag{2.0.12}$$
$$3x + 2y \leq 150 \tag{2.0.13}$$
$$x, y \geq 0 \tag{2.0.14}$$

After using slack and surplus variables,

$$Z - 6x - 3y = 0 \tag{2.0.15}$$
$$4x + y - s_1 = 80 \tag{2.0.16}$$
$$x + 5y - s_2 = 115 \tag{2.0.17}$$
$$3x + 2y + s_3 = 150 \tag{2.0.18}$$

Simplex tableau can be written as

$$\left(\begin{array}{cccccc} x & y & s_1 & s_2 & s_3 & c \\ 4 & 1 & -1 & 0 & 0 & 80 \\ 1 & \boxed{5} & 0 & -1 & 0 & 115 \\ 3 & 2 & 0 & 0 & 1 & 150 \\ \hline 6 & 3 & 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.19}$$

Keeping the pivot element as 5 and by using gauss-jordan elimination,

$$\left(\begin{array}{cccccc} x & y & s_1 & s_2 & s_3 & c \\ \boxed{\frac{19}{5}} & 0 & -1 & \frac{1}{5} & 0 & 57 \\ \frac{1}{5} & 1 & 0 & \frac{-1}{5} & 0 & 23 \\ \frac{13}{5} & 0 & 0 & \frac{2}{5} & 1 & 104 \\ \hline \frac{27}{5} & 0 & 0 & \frac{3}{5} & 0 & 69 \end{array}\right) \tag{2.0.20}$$

Keeping the pivot element as $\frac{19}{5}$ and by using

gauss-jordan elimination,

$$\left(\begin{array}{cccccc} x & y & s_1 & s_2 & s_3 & c \\ 1 & 0 & \frac{-5}{19} & \frac{1}{19} & 0 & 15 \\ 0 & 1 & \frac{1}{19} & \frac{-4}{19} & 0 & 20 \\ 0 & 0 & \frac{13}{19} & \frac{5}{19} & 1 & 65 \\ \hline 0 & 0 & \frac{27}{19} & \frac{6}{19} & 0 & 150 \end{array}\right) \tag{2.0.21}$$

∵All elements of the last row are non-negative .
∴ Optimal solution is given by

$$(x, y) = (15, 20) \tag{2.0.22}$$
$$Z = 6x + 3y \tag{2.0.23}$$
$$= 6(15) + 3(20) \tag{2.0.24}$$
$$= 150 \tag{2.0.25}$$

Problem can also be represented in matrix form as

$$\min_{\mathbf{x}} Z = \begin{pmatrix} 6 & 3 \end{pmatrix} \mathbf{x} \tag{2.0.26}$$
$$s.t. \quad \begin{pmatrix} 4 & 1 \\ 1 & 5 \\ -3 & -2 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 80 \\ 115 \\ -150 \end{pmatrix} \tag{2.0.27}$$
$$\mathbf{x} \geq \mathbf{0} \tag{2.0.28}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 14.99999999 \\ 20.00000001 \end{pmatrix} \tag{2.0.29}$$
$$Z = 150.00000001 \tag{2.0.30}$$

Hence , $\boxed{x = 15}$ packets of food P and $\boxed{y = 20}$ packets of food Q should be used to minimise the amount of vitamin A in the diet and the minimum amount of vitamin A is $\boxed{Z = 150}$ units .

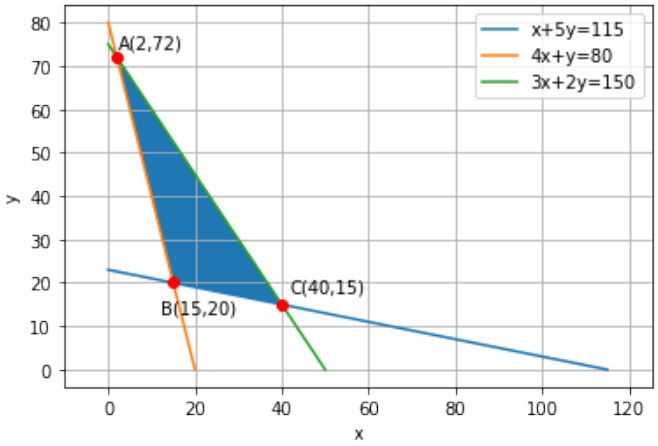

Fig. 2.1: Diet Problem