

Assignment 7

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Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment7/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment7>

1 QUESTION No. 2.99

AOBA is the part of the ellipse $\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 36$ in the first quadrant such that $OA = 2$ and $OB = 6$. Find the area between the arc AB and the chord AB .

2 SOLUTION

Given ellipse is

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 36 \quad (2.0.1)$$

On comparing it with standard form

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{D} = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 36 \quad (2.0.4)$$

$$\lambda_1 = 9 \quad (2.0.5)$$

$$\lambda_2 = 1 \quad (2.0.6)$$

\therefore Semi major and minor axes of ellipse are

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = \sqrt{\frac{36}{9}} = 2 \quad (2.0.7)$$

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = \sqrt{\frac{36}{1}} = 6 \quad (2.0.8)$$

\therefore Equation of ellipse can be written as

$$\frac{x^2}{(\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}})^2} + \frac{y^2}{(\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}})^2} = 1 \quad (2.0.9)$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{36} = 1 \quad (2.0.10)$$

Now, area of ellipse is given by

$$A = \pi \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.11)$$

$$\Rightarrow A = 12\pi \quad (2.0.12)$$

\therefore Area of a quadrant of ellipse is given by

$$A_1 = A/4 = 3\pi \quad (2.0.13)$$

Now, from fig. 2.1, AOBA is a right angled triangle whose area is given by

$$A_2 = \frac{1}{2} \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = 6 \quad (2.0.14)$$

\therefore Area between arc AB and chord AB is given by

$$A_3 = A_1 - A_2 = 3\pi - 6 \quad (2.0.15)$$

$$\Rightarrow A_3 \approx 3.424 \quad (2.0.16)$$

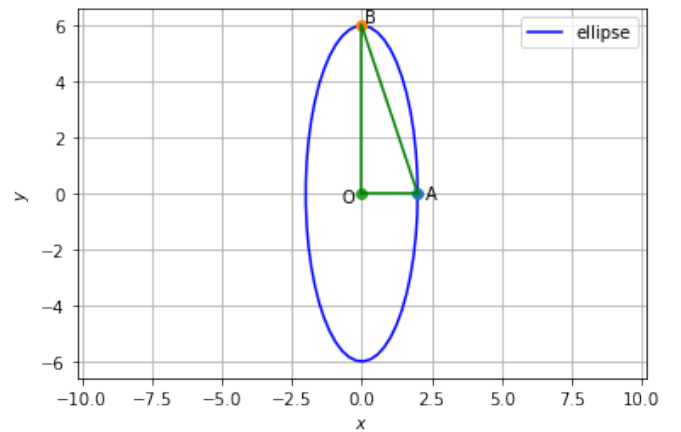


Fig. 2.1: Ellipse $\frac{x^2}{4} + \frac{y^2}{36} = 1$