Assignment 6

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Download all python codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment6/Codes
```

and latex-tikz codes from

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https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment6
```

1 Question No. 2.29

Find the coordinates of the focus, axis, the equation of the directrix and latus rectum of the parabola $y^2 = 8x$.

2 Solution

Given parabola is

$$y^2 = 8x \tag{2.0.1}$$

$$\implies y^2 - 8x = 0 \tag{2.0.2}$$

Vector form of given parabola is

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \tag{2.0.3}$$

$\therefore$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, f = 0 \tag{2.0.4}$$

$\because$ $|\mathbf{V}| = 0$ and $\lambda_1 = 0$ i.e. it is in standard form $\therefore$

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$\eta = \mathbf{u}^T \mathbf{p_1} = -4 \tag{2.0.6}$$

Now,the vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} -8 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

Now,the focal length is given by

$$\frac{1}{4} \left| \frac{2\eta}{\lambda_2} \right| = \frac{1}{4} \left| \frac{-8}{1} \right| = 2 \tag{2.0.9}$$

Now,the focus is given by

$$\mathbf{F}^T = \frac{-2\eta \begin{pmatrix} 1 & 0 \end{pmatrix}}{4} \tag{2.0.10}$$

$$\implies \mathbf{F}^T = \begin{pmatrix} 2 & 0 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{F} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.12}$$

$\because$ Axis of parabola passes through both vertex and focus .
$\therefore$ Axis of given parabola is

$$y = 0 \tag{2.0.13}$$

$\because$ Vertex of parabola is at equal distance from focus and the directrix and is perpendicular to axis.
$\therefore$ Directrix of given parabola is

$$x = -2 \tag{2.0.14}$$

$\because$ Latus rectum of parabola passes through focus and is perpendicular to axis.
$\therefore$ Latus rectum of given parabola is

$$x = 2 \tag{2.0.15}$$

End points of latus rectum are

$$\mathbf{M} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{2.0.16}$$

$$\mathbf{N} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \tag{2.0.17}$$

So,the length of latus rectum is

$$\|\mathbf{M} - \mathbf{N}\| = 8 \tag{2.0.18}$$

Plot of given parabola

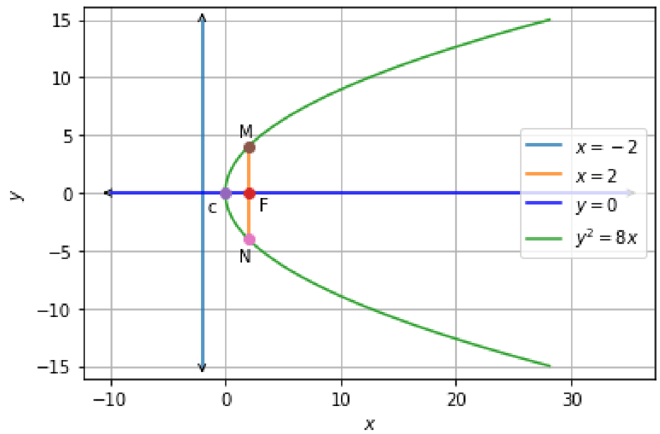

Fig. 2.1: Parabola $y^2 = 8x$