Assignment 6

K.A. Raja Babu

Download all python codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment6/Codes
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and latex-tikz codes from

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https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment6
```

1 Question No. 2.29

Find the coordinates of the focus, axis, the equation of the directrix and latus rectum of the parabola $y^2 = 8x$.

2 Solution

Parameter	Symbol	Value	General Formula
Vertex	$\mathbf{c}$	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T\\ \mathbf{V}\end{pmatrix}\mathbf{c} = \begin{pmatrix}-f\\ \eta\mathbf{p_1}-\mathbf{u}\end{pmatrix}$
Focal Length	β	2	$\frac{1}{4}\left\lvert\frac{2\eta}{\lambda_2}\right\rvert$
Focus	$\mathbf{F}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\mathbf{F} = \mathbf{c} + \mathbf{a}^T$
Axis		$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 0$	$\mathbf{x} = \mathbf{F} + k\mathbf{m}$
Directrix		$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = -2$	$\mathbf{v_1}\mathbf{x} = -\beta$
Latus rectum		$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 2$	$\mathbf{v_2}\mathbf{x} = \beta$
End points of latus rectum	$\mathbf{M}, \mathbf{N}$	$\begin{pmatrix}2\\4\end{pmatrix}, \begin{pmatrix}2\\-4\end{pmatrix}$	$\begin{pmatrix}\beta\\ \pm y(\beta)\end{pmatrix}$
Length of latus rectum	l	8	$\lVert\mathbf{M}-\mathbf{N}\rVert$

TABLE 2.1: Parameters of parabola $y^2 = 8x$

All parameters of parabola can be summarised in table 2.1

Note : Given general formula is valid only when parabola is in standard form i.e. $|\mathbf{V}| = 0$ and $\lambda_1 = 0$.

Given parabola is

$$y^2 = 8x \tag{2.0.1}$$
$$\implies y^2 - 8x = 0 \tag{2.0.2}$$

Vector form of given parabola is

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-4 & 0\end{pmatrix}\mathbf{x} + 0 = 0 \tag{2.0.3}$$

$\therefore$

$$\mathbf{V} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}, \mathbf{u} = \begin{pmatrix}-4\\0\end{pmatrix}, f = 0 \tag{2.0.4}$$

$\because$ $|\mathbf{V}| = 0$ and $\lambda_1 = 0$ i.e. it is in standard form $\therefore$

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.5}$$
$$\eta = \mathbf{u}^T\mathbf{p_1} = -4 \tag{2.0.6}$$

The vertex $\mathbf{c}$ is given by

$$\begin{pmatrix}-8 & 0\\0 & 0\\0 & 1\end{pmatrix}\mathbf{c} = \begin{pmatrix}0\\0\\0\end{pmatrix} \tag{2.0.7}$$
$$\implies \mathbf{c} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.8}$$

The focal length β is given by

$$\beta = \frac{1}{4}\left\lvert\frac{2\eta}{\lambda_2}\right\rvert = \frac{1}{4}\left\lvert\frac{-8}{1}\right\rvert = 2 \tag{2.0.9}$$

The focus $\mathbf{F}$ is given by

$$\mathbf{a} = \frac{-2\eta\begin{pmatrix}1 & 0\end{pmatrix}}{4} = \begin{pmatrix}2 & 0\end{pmatrix} \tag{2.0.10}$$
$$\mathbf{F} = \mathbf{c} + \mathbf{a}^T = \begin{pmatrix}2\\0\end{pmatrix} \tag{2.0.11}$$

$\because$ Axis of parabola passes through both vertex and focus .

$\therefore$ Axis of parabola is given by

$$\mathbf{m} = \mathbf{F} - \mathbf{c} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.12}$$

$$\mathbf{x} = \mathbf{F} + k\mathbf{m} = \begin{pmatrix} 2 + 2k \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.14}$$

$\because$ Vertex of parabola is at equal distance from focus and directrix and is perpendicular to axis.
$\therefore$ Directrix of parabola is given by

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{v_1}^T = 0 \tag{2.0.15}$$

$$\implies \mathbf{v_1} = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{2.0.16}$$

So ,

$$\mathbf{v_1}\mathbf{x} = -\beta \tag{2.0.17}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2 \tag{2.0.18}$$

$\because$ Latus rectum of parabola passes through focus and is perpendicular to axis.
$\therefore$ Latus rectum of parabola is given by

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{v_2}^T = 0 \tag{2.0.19}$$

$$\implies \mathbf{v_2} = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{2.0.20}$$

So ,

$$\mathbf{v_2}\mathbf{x} = \beta \tag{2.0.21}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \tag{2.0.22}$$

End points of latus rectum are

$$\mathbf{M} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{2.0.23}$$

$$\mathbf{N} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \tag{2.0.24}$$

So,the length of latus rectum l is

$$l = \|\mathbf{M} - \mathbf{N}\| = 8 \tag{2.0.25}$$

Plot of given parabola

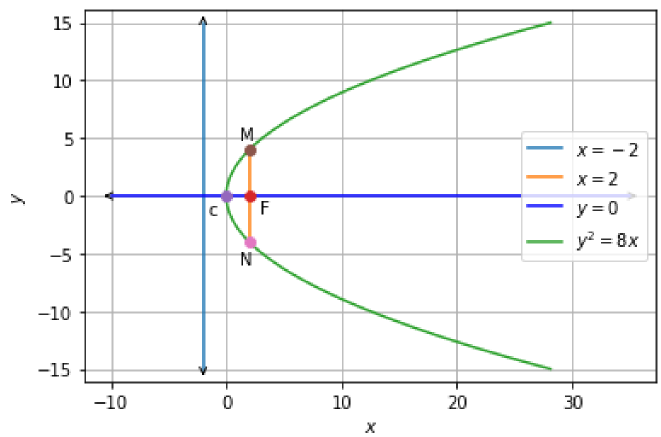

Fig. 2.1: Parabola $y^2 = 8x$