Assignment 9

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Download all python codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment9/Codes

and latex-tikz codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment9

1 Question No. 2.21

A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 kmh^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 ms^{-1} to hit the plane ? At what minimum altitude should the pilot fly the plane to avoid being hit ? (Take $g = 10\ ms^{-2}$) .

2 Solution

Velocity of the plane is given by

$$\mathbf{v_p} = \begin{pmatrix} 200 \\ 0 \end{pmatrix} \quad \left(\because 720kmh^{-1} = 200ms^{-1}\right) \tag{2.0.1}$$

Velocity of the bullet is given by

$$\mathbf{v_b} = 600\begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} \tag{2.0.2}$$

where θ is the angle made by $\mathbf{v_b}$ with the vertical .

Let after time t ,the bullet hits the plane such that the horizontal distance travelled by the plane and the bullet are equal.

$\therefore$

$$\mathbf{v_p}\begin{pmatrix} t & 0 \end{pmatrix} = \mathbf{v_b}\begin{pmatrix} t & 0 \end{pmatrix} \tag{2.0.3}$$

$$\implies 200t = (600\sin\theta)t \tag{2.0.4}$$

$$\implies \theta = 19.5^\circ \tag{2.0.5}$$

Acceleration of the bullet due to gravity is

$$\mathbf{g} = \begin{pmatrix} 0 \\ -10 \end{pmatrix} \tag{2.0.6}$$

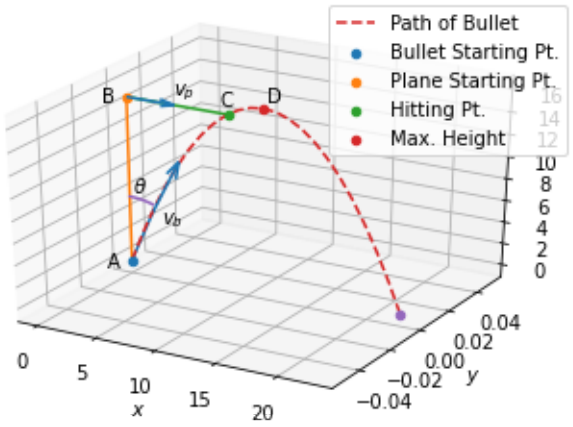

Fig. 2.1: 3D Analysis of Plane and Bullet

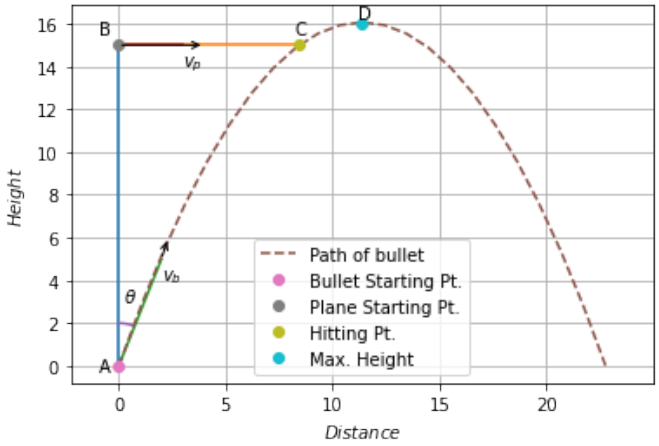

Fig. 2.2: 2D Analysis of Plane and Bullet

Velocity of the bullet at the maximum height is

$$\mathbf{v_m} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

Now,the maximum height h_{max} achieved by the bullet is given by

$$\left[\mathbf{v_m}\begin{pmatrix} 0 & 1 \end{pmatrix}\right]^2 - \left[\mathbf{v_b}\begin{pmatrix} 0 & 1 \end{pmatrix}\right]^2 = 2\mathbf{g}\begin{pmatrix} 0 & 1 \end{pmatrix}h_{max} \tag{2.0.8}$$

$$\implies -(600\cos 19.5^\circ)^2 = -20h_{max} \tag{2.0.9}$$

$$\implies h_{max} = 16km \tag{2.0.10}$$

Hence,the gun should be fired at $\boxed{\theta = 19.5^\circ}$ from the vertical to hit the plane and the pilot must fly above the maximum height of the bullet , $\boxed{h_{max} = 16km}$ to avoid being hit.