Assignment 2

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Download all python codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment2/Codes
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and latex-tikz codes from

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https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment2
```

1 Question No. 32

Can you construct a quadrilateral $PQRS$ with $PQ = 3$,$RS = 3$,$PS = 7.5$,$PR = 8$ and $SQ = 4$?

2 Explanation

Let us assume that:

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{R} = \begin{pmatrix} c \\ d \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix} \tag{2.0.1}$$

Then,

$$\|\mathbf{Q} - \mathbf{P}\|^2 = \|\mathbf{Q}\|^2 = 3^2 = 9 \quad (\because \mathbf{P} = 0) \tag{2.0.2}$$

$$\|\mathbf{S} - \mathbf{P}\|^2 = \|\mathbf{S}\|^2 = (7.5)^2 = 56.25 \quad (\because \mathbf{P} = 0) \tag{2.0.3}$$

$$\|\mathbf{R} - \mathbf{P}\|^2 = \|\mathbf{R}\|^2 = 8^2 = 64 \quad (\because \mathbf{P} = 0) \tag{2.0.4}$$

Now,

$$\|\mathbf{Q} - \mathbf{S}\|^2 = (\mathbf{Q} - \mathbf{S})^T(\mathbf{Q} - \mathbf{S}) \tag{2.0.5}$$

$$= \mathbf{Q}^T\mathbf{Q} + \mathbf{S}^T\mathbf{S} - \mathbf{Q}^T\mathbf{S} - \mathbf{Q}^T\mathbf{S} \tag{2.0.6}$$

$$= \|\mathbf{Q}\|^2 + \|\mathbf{S}\|^2 - 2\mathbf{Q}^T\mathbf{S} \quad \left(\because \mathbf{Q}^T\mathbf{S} = \mathbf{S}^T\mathbf{Q}\right) \tag{2.0.7}$$

$$= 9 + 56.25 - 2(7.5a) \quad \left(\because \mathbf{Q}^T\mathbf{S} = 7.5a\right) \tag{2.0.8}$$

$$= 65.25 - 15a \tag{2.0.9}$$

But,

$$\|\mathbf{Q} - \mathbf{S}\|^2 = 4^2 = 16 \quad (\because Given) \tag{2.0.10}$$

Therefore,

$$65.25 - 15a = 16 \tag{2.0.11}$$

$$\implies 15a = 49.25 \tag{2.0.12}$$

$$\implies a = 3.283 \tag{2.0.13}$$

Now,

$$\|\mathbf{Q}\|^2 = a^2 + b^2 = 3^2 = 9 \tag{2.0.14}$$

$$\implies (3.283)^2 + b^2 = 9 \tag{2.0.15}$$

$$\implies b^2 = -1.778 \tag{2.0.16}$$

$$\implies b = 1.33\iota \tag{2.0.17}$$

Similarly,

$$\|\mathbf{R} - \mathbf{S}\|^2 = (\mathbf{R} - \mathbf{S})^T(\mathbf{R} - \mathbf{S}) \tag{2.0.18}$$

$$= \mathbf{R}^T\mathbf{Q} + \mathbf{S}^T\mathbf{S} - \mathbf{R}^T\mathbf{S} - \mathbf{R}^T\mathbf{S} \tag{2.0.19}$$

$$= \|\mathbf{R}\|^2 + \|\mathbf{S}\|^2 - 2\mathbf{R}^T\mathbf{S} \quad \left(\because \mathbf{R}^T\mathbf{S} = \mathbf{S}^T\mathbf{R}\right) \tag{2.0.20}$$

$$= 64 + 56.25 - 2(7.5c) \quad \left(\because \mathbf{Q}^T\mathbf{S} = 7.5c\right) \tag{2.0.21}$$

$$= 120.25 - 15c \tag{2.0.22}$$

Again,

$$\|\mathbf{R} - \mathbf{S}\|^2 = 3^2 = 9 \quad (\because Given) \tag{2.0.23}$$

Therefore,

$$120.25 - 15c = 9 \tag{2.0.24}$$

$$\implies 15c = 111.25 \tag{2.0.25}$$

$$\implies c = 7.416 \tag{2.0.26}$$

Now,

$$\|\mathbf{R}\|^2 = c^2 + d^2 = 8^2 = 64 \tag{2.0.27}$$

$$\implies (7.416)^2 + d^2 = 64 \tag{2.0.28}$$

$$\implies d^2 = 9.00 \tag{2.0.29}$$

$$\implies d = 3 \tag{2.0.30}$$

Therefore,vertices of quadrilateral $PQRS$ are as follows:

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 3.283 \\ 1.33\iota \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 7.416 \\ 3 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix} \tag{2.0.31}$$

But,value of b comes imaginary which cannot be represented in Cartesian coordinate system.
This implies that our assumption is wrong and there exists no real values of b which satisfy these conditions.
Hence,quadrilateral $PQRS$ cannot be constructed with these values.
Figure 2.1 is plotted taking b as 0 discarding its imaginary part ,which looks like a triangle not a quadrilateral.
This plot clearly concludes that construction of quadrilateral $PQRS$ is not possible with these values.
Plot of the quadrilateral $PQRS$ taking $b = 0$:

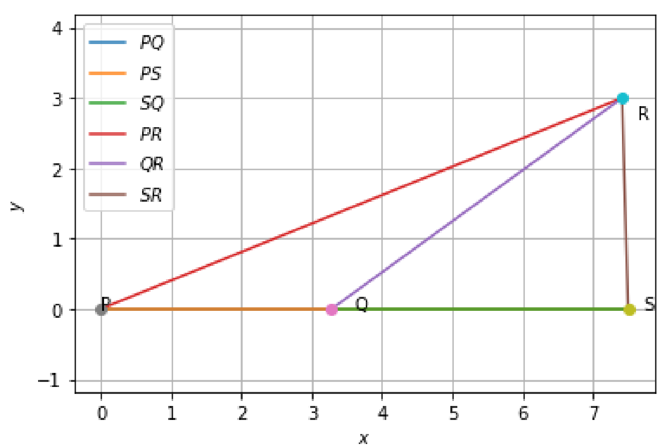

Fig. 2.1: Quadrilateral $PQRS$ when b =0