Assignment 13

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Download all python codes from

https://github.com/ka−raja−babu/Matrix−Theory/tree/main/Assignment13

and latex-tikz codes from

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1 Question No. 2.9

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact,present. However,the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then,with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

2 Solution

Let $X \in \{0, 1\}$ represent the test result and $Y \in \{0, 1\}$ represent the actual status of the disease where 0 and 1 denotes absence and presence of the disease respectively .
According to the question,

$$\Pr(X = 1|Y = 1) = \frac{99}{100} \tag{2.0.1}$$

$$\Pr(X = 1|Y = 0) = \frac{0.5}{100} = \frac{1}{200} \tag{2.0.2}$$

$$\Pr(Y = 1) = \frac{0.1}{100} = \frac{1}{1000} \tag{2.0.3}$$

$$\Pr(Y = 0) = \frac{99.9}{100} = \frac{999}{1000} \tag{2.0.4}$$

$\therefore$ Probability that a person has the disease given that his test result is positive ,is given by

$$\Pr(Y = 1|X = 1) = \frac{\Pr(X = 1|Y = 1)\Pr(Y = 1)}{\sum_{i=0}^{1} \Pr(X = 1|Y = i)\Pr(Y = i)} \tag{2.0.5}$$

$$= \frac{\frac{99}{100} \times \frac{1}{1000}}{\frac{1}{200} \times \frac{999}{1000} + \frac{99}{100} \times \frac{1}{1000}} \tag{2.0.6}$$

$$= \frac{22}{133} \tag{2.0.7}$$