

Assignment 6

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Download all python codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment6/Codes>

and latex-tikz codes from

<https://github.com/ka-raja-babu/Matrix-Theory/tree/main/Assignment6>

1 QUESTION NO. 2.29

Find the coordinates of the focus, axis, the equation of the directrix and latus rectum of the parabola $y^2 = 8x$.

2 SOLUTION

Parameter	Symbol	Value	General Formula
Vertex	\mathbf{c}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix}$
Focal Length	β	2	$\frac{1}{4} \left \frac{2\eta}{\lambda_2} \right $
Focus	\mathbf{F}	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	$\mathbf{F} = \mathbf{c} + \mathbf{a}^T$
Axis		$(0 \ 1) \mathbf{x} = 0$	$\mathbf{x} = \mathbf{F} + k\mathbf{m}$
Directrix		$(1 \ 0) \mathbf{x} = -2$	$\mathbf{v}_1 \mathbf{x} = -\beta$
Latus rectum		$(1 \ 0) \mathbf{x} = 2$	$\mathbf{v}_2 \mathbf{x} = \beta$
End points of latus rectum	\mathbf{M}, \mathbf{N}	$\begin{pmatrix} 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \end{pmatrix}$	$\begin{pmatrix} \beta \\ \pm y(\beta) \end{pmatrix}$
Length of latus rectum	l	8	$\ \mathbf{M} - \mathbf{N}\ $

TABLE 2.1: Parameters of parabola $y^2 = 8x$

All parameters of parabola $y^2 = 8x$ can be summarised in table 2.1

Note : Given general formula is valid only when parabola is in standard form i.e. $|\mathbf{V}| = 0$ and $\lambda_1 = 0$

Given parabola is

$$y^2 = 8x \quad (2.0.1)$$

$$\implies y^2 - 8x = 0 \quad (2.0.2)$$

Vector form of given parabola is

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.3)$$

\therefore

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, f = 0 \quad (2.0.4)$$

$\therefore |\mathbf{V}| = 0$ and $\lambda_1 = 0$ i.e. it is in standard form
 \therefore

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.5)$$

$$\eta = \mathbf{u}^T \mathbf{p}_1 = -4 \quad (2.0.6)$$

The vertex \mathbf{c} is given by

$$\begin{pmatrix} -8 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.7)$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.8)$$

The focal length β is given by

$$\beta = \frac{1}{4} \left| \frac{2\eta}{\lambda_2} \right| = \frac{1}{4} \left| \frac{-8}{1} \right| = 2 \quad (2.0.9)$$

The focus \mathbf{F} is given by

$$\mathbf{a} = \frac{-2\eta \begin{pmatrix} 1 & 0 \end{pmatrix}}{4} = \begin{pmatrix} 2 & 0 \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{F} = \mathbf{c} + \mathbf{a}^T = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (2.0.11)$$

\therefore Axis of parabola passes through both vertex and focus.

\therefore Axis of parabola is given by

$$\mathbf{m} = \mathbf{F} - \mathbf{c} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (2.0.12)$$

$$\mathbf{x} = \mathbf{F} + k\mathbf{m} = \begin{pmatrix} 2 + 2k \\ 0 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow (0 \ 1)\mathbf{x} = 0 \quad (2.0.14)$$

\therefore Vertex of parabola is at equal distance from focus and directrix and is perpendicular to axis.

\therefore Directrix of parabola is given by

$$(0 \ 1)\mathbf{v}_1^T = 0 \quad (2.0.15)$$

$$\Rightarrow \mathbf{v}_1 = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (2.0.16)$$

So ,

$$\mathbf{v}_1\mathbf{x} = -\beta \quad (2.0.17)$$

$$\Rightarrow (1 \ 0)\mathbf{x} = -2 \quad (2.0.18)$$

\therefore Latus rectum of parabola passes through focus and is perpendicular to axis.

\therefore Latus rectum of parabola is given by

$$(0 \ 1)\mathbf{v}_2^T = 0 \quad (2.0.19)$$

$$\Rightarrow \mathbf{v}_2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (2.0.20)$$

So ,

$$\mathbf{v}_2\mathbf{x} = \beta \quad (2.0.21)$$

$$\Rightarrow (1 \ 0)\mathbf{x} = 2 \quad (2.0.22)$$

End points of latus rectum are

$$\mathbf{M} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (2.0.23)$$

$$\mathbf{N} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \quad (2.0.24)$$

So, the length of latus rectum l is

$$l = \|\mathbf{M} - \mathbf{N}\| = 8 \quad (2.0.25)$$

Plot of given parabola

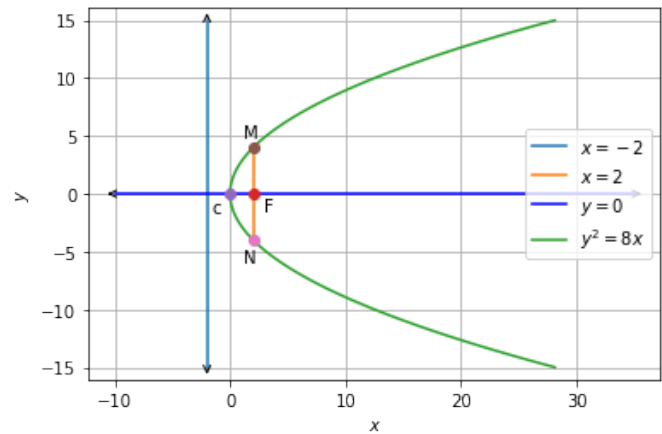


Fig. 2.1: Parabola $y^2 = 8x$