# Assignment 6

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Download all python codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment6/Codes
```

and latex-tikz codes from

```
https://github.com/ka−raja−babu/Matrix−Theory/
    tree/main/Assignment6
```

## 1 Appendix

| Parameter | Symbol | General Formula | Value |
|---|---|---|---|
| Vertex | $\mathbf{c}$ | $\begin{pmatrix}\mathbf{u}^T+\eta\mathbf{p_1}^T\\ \mathbf{V}\end{pmatrix}\mathbf{c} = \begin{pmatrix}-f\\ \eta\mathbf{p_1}-\mathbf{u}\end{pmatrix}$ | $\begin{pmatrix}0\\0\end{pmatrix}$ |
| Focal Length | $\beta$ | $\frac{1}{2}\left\lvert\frac{(\mathbf{c}^T\mathbf{V}+\mathbf{u}^T)\mathbf{p_1}}{(\lambda_1+\lambda_2)}\right\rvert$ | 2 |
| Focus | $\mathbf{F}$ | $\mathbf{F}$ = -$\mathbf{V}^{-1}\mathbf{u}$ + $\frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f)\begin{pmatrix}1 & 0\end{pmatrix}^T}{4}$ | $\begin{pmatrix}2\\0\end{pmatrix}$ |
| Axis | | $(\mathbf{Vc}+\mathbf{u})^T\mathbf{x}=0$ | $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x}=0$ |
| Directrix | | $(\mathbf{Vc}+\mathbf{u})^T(\mathbf{x}+\beta)+\mathbf{u}^T\mathbf{c}+\mathbf{f}=0$ | $\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}=-2$ |
| Latus rectum | | $(\mathbf{Vc}+\mathbf{u})^T(\mathbf{x}-\beta)+\mathbf{u}^T\mathbf{c}+\mathbf{f}=0$ | $\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}=2$ |
| End points of latus rectum | $\kappa$ | $\mathbf{u}^T\kappa = -\frac{(\kappa^T\mathbf{V}\kappa+f)}{2}$ | $\begin{pmatrix}2\\ \pm4\end{pmatrix}$ |
| Length of latus rectum | $l$ | $\left\lVert\beta(\mathbf{Vc}+\mathbf{u})^T\right\rVert$ | 8 |

TABLE 1.1: Parameters of parabola $y^2 = 8x$

All parameters of parabola $y^2 = 8x$ can be summarised in table 1.1.

Note : Given general formula is valid only when parabola is in standard form i.e. $|\mathbf{V}| = 0$ and $\lambda_1 = 0$

**Lemma 1.1.** *General equation of a conic is given by*

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{1.0.1}$$

*and can be expressed as*

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{1.0.2}$$

*where*

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix}a & b\\ b & c\end{pmatrix} \tag{1.0.3}$$

$$\mathbf{u} = \begin{pmatrix}d & e\end{pmatrix} \tag{1.0.4}$$

**Lemma 1.2.** (1.0.2) *can be expressed as*

$$\mathbf{y}^T\mathbf{D}\mathbf{y} + 2(\mathbf{Vc}+\mathbf{u})^T\mathbf{P}\mathbf{y} + \mathbf{c}^T\mathbf{V}\mathbf{c} + 2\mathbf{u}^T\mathbf{c} + f = 0 \tag{1.0.5}$$

*where*

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c} \tag{1.0.6}$$

$$\mathbf{P}^T\mathbf{V}\mathbf{P} = \mathbf{D} \tag{1.0.7}$$

$$\mathbf{D} = \begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix} \tag{1.0.8}$$

$$\mathbf{P} = \begin{pmatrix}\mathbf{p_1} & \mathbf{p_2}\end{pmatrix} \tag{1.0.9}$$

**Lemma 1.3.** (1.0.5) *can be expressed as*

$$\mathbf{y}^T\mathbf{D}\mathbf{y} = \mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f \quad (|\mathbf{V}| \neq 0) \tag{1.0.10}$$

$$\mathbf{y}^T\mathbf{D}\mathbf{y} = -2\eta\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{y} \quad (|\mathbf{V}| = 0) \tag{1.0.11}$$

*where*

$$\eta = \mathbf{u}^T\mathbf{p_1} \tag{1.0.12}$$

**Lemma 1.4.** *Focal length of a conic is given by*

$$\beta = \frac{1}{2}\left\lvert\frac{(\mathbf{c}^T\mathbf{V}+\mathbf{u}^T)\mathbf{p}_1}{(\lambda_1+\lambda_2)}\right\rvert \quad (|\mathbf{V}| \neq 0) \tag{1.0.13}$$

$$\beta = \frac{1}{2}\left\lvert\frac{\mathbf{u}^T\mathbf{p}_1}{\lambda_2}\right\rvert \quad (|\mathbf{V}| = 0) \tag{1.0.14}$$

**Lemma 1.5.** *Vertex of a conic is given by*

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \quad (|\mathbf{V}| \neq 0) \tag{1.0.15}$$

$$\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V}\end{pmatrix}\mathbf{c} = \begin{pmatrix}-f \\ \eta\mathbf{p_1} - \mathbf{u}\end{pmatrix} \quad (|\mathbf{V}| = 0) \tag{1.0.16}$$

**Lemma 1.6.** *Focus of a conic is given by*

$$\mathbf{F} = -\mathbf{V}^{-1}\mathbf{u} + \frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)\begin{pmatrix}1 & 0\end{pmatrix}^T}{4} \quad (|\mathbf{V}| \neq 0) \tag{1.0.17}$$

$$\mathbf{F} = \begin{pmatrix}-f \\ \eta\mathbf{p_1} - \mathbf{u}\end{pmatrix}\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V}\end{pmatrix}^{-1} + \frac{-2\eta\begin{pmatrix}1 & 0\end{pmatrix}^T}{4} \quad (|\mathbf{V}| = 0) \tag{1.0.18}$$

*Proof.* From (1.0.10) and (1.0.15),focus **F** is given by

$$\mathbf{F} = \mathbf{c} + \frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)\begin{pmatrix}1 & 0\end{pmatrix}^T}{4} \quad (|\mathbf{V}| = 0) \tag{1.0.19}$$

$$\implies \mathbf{F} = -\mathbf{V}^{-1}\mathbf{u} + \frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)\begin{pmatrix}1 & 0\end{pmatrix}^T}{4} \tag{1.0.20}$$

From (1.0.11) and (1.0.16),focus **F** is given by

$$\mathbf{F} = \mathbf{c} + \frac{-2\eta\begin{pmatrix}1 & 0\end{pmatrix}^T}{4} \quad (|\mathbf{V}| = 0) \tag{1.0.21}$$

$$\implies \mathbf{F} = \begin{pmatrix}-f \\ \eta\mathbf{p_1} - \mathbf{u}\end{pmatrix}\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V}\end{pmatrix}^{-1} + \frac{-2\eta\begin{pmatrix}1 & 0\end{pmatrix}^T}{4} \tag{1.0.22}$$

□

**Lemma 1.7.** *Normal vector at any point* **q** *of a conic section is obtained as*

$$\mathbf{n} = \mathbf{V}\mathbf{q} + \mathbf{u} \tag{1.0.23}$$

**Lemma 1.8.** *Axis of a conic is given by*

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T\mathbf{x} = 0 \tag{1.0.24}$$

*Proof.* Using (1.7),Normal vector at vertex is given by

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T \tag{1.0.25}$$

So,axis is given as

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T\mathbf{x} = 0 \tag{1.0.26}$$

□

**Lemma 1.9.** *Given the point of contact* **q***,equation of tangent is given by*

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^T\mathbf{x} + \mathbf{u}^T\mathbf{q} + f = 0 \tag{1.0.27}$$

**Lemma 1.10.** *Directrix of a conic is given by*

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} + \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0 \tag{1.0.28}$$

*Proof.* Using (1.0.27),directrix is given by

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} + \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0 \tag{1.0.29}$$

□

**Lemma 1.11.** *Latus rectum of a conic is given by*

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} - \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0 \tag{1.0.30}$$

*Proof.* Using (1.0.27),latus rectum is given by

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} - \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0 \tag{1.0.31}$$

□

**Lemma 1.12.** *End points of latus rectum of a conic is given by*

$$\mathbf{u}^T\kappa = -\frac{(\kappa^T\mathbf{V}\kappa + f)}{2} \tag{1.0.32}$$

*where*

$$\kappa = \begin{pmatrix}\beta \\ \mathbf{y}\end{pmatrix} \tag{1.0.33}$$

*Proof.* Substituting $x = \kappa$ in (1.0.1),end points of latus rectum are

$$\mathbf{u}^T\kappa = -\frac{(\kappa^T\mathbf{V}\kappa + f)}{2} \tag{1.0.34}$$

□

**Lemma 1.13.** *Length of latus rectum is given by*

$$l = \left\|\beta(\mathbf{V}\mathbf{c} + \mathbf{u})^T\right\| \tag{1.0.35}$$

*Proof.* Using (1.0.30) ,length of latus rectum can be expressed as

$$l = \left\|\beta(\mathbf{V}\mathbf{c} + \mathbf{u})^T\right\| \tag{1.0.36}$$

□

## 2 Question No. 2.29

Find the coordinates of the focus, axis, the equation of the directrix and latus rectum of the parabola $y^2 = 8x$ .

## 3 solution

Given parabola is

$$y^2 = 8x \tag{3.0.1}$$

$$\implies y^2 - 8x = 0 \tag{3.0.2}$$

Vector form of given parabola is

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \tag{3.0.3}$$

$\therefore$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, f = 0 \tag{3.0.4}$$

$\because |\mathbf{V}| = 0$ and $\lambda_1 = 0$ i.e. it is in standard form $\therefore$

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.5}$$

$$\eta = \mathbf{u}^T \mathbf{p_1} = -4 \tag{3.0.6}$$

The vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} -8 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.0.7}$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.0.8}$$

The focal length $\beta$ is given by

$$\beta = \frac{1}{4}\left|\frac{2\eta}{\lambda_2}\right| = \frac{1}{4}\left|\frac{-8}{1}\right| = 2 \tag{3.0.9}$$

The focus $\mathbf{F}$ is given by

$$\mathbf{F} = \mathbf{c} + \frac{-2\eta\begin{pmatrix} 1 & 0 \end{pmatrix}^T}{4} \tag{3.0.10}$$

$$\implies \mathbf{F} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{3.0.11}$$

$$\implies \mathbf{F} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{3.0.12}$$

Axis of parabola is given by

$$k(\mathbf{V}\mathbf{c} + \mathbf{u})^T \mathbf{x} = 0 \quad (k \in \mathbb{R}) \tag{3.0.13}$$

$$\implies k\begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{3.0.14}$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{3.0.15}$$

Directrix of parabola is given by

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T (\mathbf{x} + \beta) + \mathbf{u}^T \mathbf{c} + \mathbf{f} = 0 \tag{3.0.16}$$

$$\implies \begin{pmatrix} -4 & 0 \end{pmatrix} (\mathbf{x} + \mathbf{2}) = 0 \tag{3.0.17}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2 \tag{3.0.18}$$

Latus rectum of parabola is given by

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T (\mathbf{x} - \beta) + \mathbf{u}^T \mathbf{c} + \mathbf{f} = 0 \tag{3.0.19}$$

$$\implies \begin{pmatrix} -4 & 0 \end{pmatrix} (\mathbf{x} - \mathbf{2}) = 0 \tag{3.0.20}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \tag{3.0.21}$$

End points of latus rectum are

$$\mathbf{u}^T \kappa = -\frac{(\kappa^T \mathbf{V} \kappa + f)}{2} \tag{3.0.22}$$

$$\implies \begin{pmatrix} -4 & 0 \end{pmatrix} \kappa = -\frac{\kappa^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \kappa + 0}{2} \tag{3.0.23}$$

$$\implies \kappa = \begin{pmatrix} 2 \\ \pm 4 \end{pmatrix} \tag{3.0.24}$$

Length of latus rectum $l$ is

$$l = \left\|\beta(\mathbf{V}\mathbf{c} + \mathbf{u})^T\right\| \tag{3.0.25}$$

$$\implies l = \left\|2\begin{pmatrix} -4 & 0 \end{pmatrix}\right\| \tag{3.0.26}$$

$$\implies l = 8 \tag{3.0.27}$$

Plot of given parabola

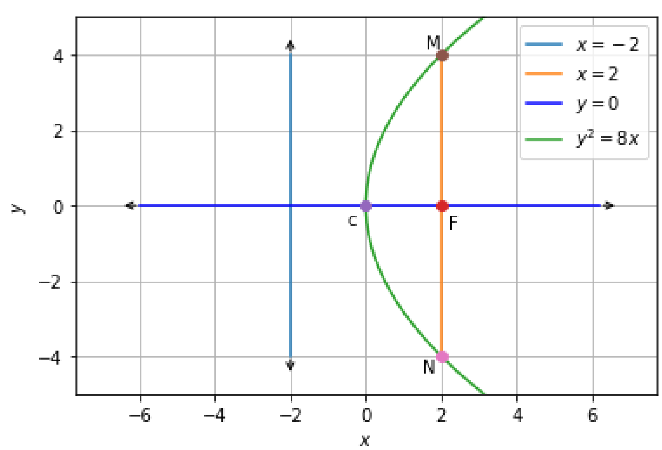


Fig. 3.1: Parabola $y^2 = 8x$

## 4 Generalisation

### 4.1 Circle

*4.1.1 Property:*

$$\mathbf{V} = \mathbf{D} = \mathbf{P} = \mathbf{I} \tag{4.1.1}$$

*4.1.2 Standard Form:* From (1.0.2),

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{4.1.2}$$

*4.1.3 Centre:* From (1.0.15),

$$\mathbf{c} = -\mathbf{u} \tag{4.1.3}$$

*4.1.4 Radius:* From (1.0.10),

$$\mathbf{r} = \sqrt{\mathbf{u}^T\mathbf{u} - f} \tag{4.1.4}$$

### 4.2 Ellipse

*4.2.1 Property:*

$$|\mathbf{V}| > 0 \tag{4.2.1}$$
$$\lambda_1 > 0, \lambda_2 < 0 \tag{4.2.2}$$

*4.2.2 Standard Form:* From (1.0.10),

$$\frac{\mathbf{y}^T\mathbf{D}\mathbf{y}}{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{4.2.3}$$

*4.2.3 Centre:* From (1.0.15),

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \tag{4.2.4}$$

*4.2.4 Axes:* From (1.0.10),

$$\begin{cases} \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} \\ \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \end{cases} \tag{4.2.5}$$

### 4.3 Hyperbola

*4.3.1 Property:*

$$|\mathbf{V}| < 0 \tag{4.3.1}$$
$$\lambda_1 > 0, \lambda_2 < 0 \tag{4.3.2}$$

*4.3.2 Standard Form:* From (1.0.10),

$$\frac{\mathbf{y}^T\mathbf{D}\mathbf{y}}{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{4.3.3}$$

*4.3.3 Centre:* From (1.0.15),

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \tag{4.3.4}$$

*4.3.4 Axes:* From (1.0.10),

$$\begin{cases} \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} \\ \sqrt{\frac{f - \mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}}{\lambda_2}} \end{cases} \tag{4.3.5}$$

### 4.4 Parabola

*4.4.1 Property:*

$$|\mathbf{V}| = 0 \tag{4.4.1}$$
$$\lambda_1 = 0 \tag{4.4.2}$$

*4.4.2 Standard Form:* From (1.0.11),

$$\mathbf{y}^T\mathbf{D}\mathbf{y} = -2\eta\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{y} \tag{4.4.3}$$

*4.4.3 Centre:* From (1.0.16),

$$\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V}\end{pmatrix}\mathbf{c} = \begin{pmatrix}-f \\ \eta\mathbf{p_1} - \mathbf{u}\end{pmatrix} \tag{4.4.4}$$

*4.4.4 Focal length:* From (1.0.14),

$$\beta = \frac{1}{2}\left|\frac{\mathbf{u}^T\mathbf{p}_1}{\lambda_2}\right| \tag{4.4.5}$$

*4.4.5 Focus:* From (1.0.18),

$$\mathbf{F} = \begin{pmatrix}-f \\ \eta\mathbf{p_1} - \mathbf{u}\end{pmatrix}\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V}\end{pmatrix}^{-1} + \frac{-2\eta\begin{pmatrix}1 & 0\end{pmatrix}^T}{4} \tag{4.4.6}$$

*4.4.6 Axis:* From (1.0.24),

$$k(\mathbf{V}\mathbf{c} + \mathbf{u})^T\mathbf{x} = 0 \tag{4.4.7}$$

*4.4.7 Directrix:* From (1.0.28),

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} + \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0 \tag{4.4.8}$$

*4.4.8 Latus Rectum:* From (1.0.30),

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} - \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0 \tag{4.4.9}$$

*4.4.9 End points of latus rectum:* From (1.0.32),

$$\mathbf{u}^T\kappa = -\frac{(\kappa^T\mathbf{V}\kappa + f)}{2} \tag{4.4.10}$$

*4.4.10 Length of latus rectum:* From (1.0.35),

$$l = \left\|\beta(\mathbf{V}\mathbf{c} + \mathbf{u})^T\right\| \tag{4.4.11}$$

| Conic | Property | Standard Form | Standard Parameters |
|---|---|---|---|
| Circle | $\mathbf{V} = \mathbf{D} = \mathbf{P} = \mathbf{I}$ | $\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0$ | 1)Centre : $\mathbf{c} = -\mathbf{u}$<br>2)Radius : $\mathbf{r} = \sqrt{\mathbf{u}^T\mathbf{u} - f}$ |
| Ellipse | $\lvert\mathbf{V}\rvert > 0$<br>$\lambda_1 > 0, \lambda_2 < 0$ | $\frac{\mathbf{y}^T\mathbf{D}\mathbf{y}}{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f} = 1$ | 1)Centre : $\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u}$<br>2)Axes : $\begin{cases} \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}} \\ \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2}} \end{cases}$ |
| Hyperbola | $\lvert\mathbf{V}\rvert < 0$<br>$\lambda_1 > 0, \lambda_2 < 0$ | $\frac{\mathbf{y}^T\mathbf{D}\mathbf{y}}{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f} = 1$ | 1)Centre : $\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u}$<br>2)Axes : $\begin{cases} \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}} \\ \sqrt{\frac{f-\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}}{\lambda_2}} \end{cases}$ |
| Parabola | $\lvert\mathbf{V}\rvert = 0$<br>$\lambda_1 = 0$ | $\mathbf{y}^T\mathbf{D}\mathbf{y} = -2\eta\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{y}$ | 1)Centre:<br>$\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V}\end{pmatrix}\mathbf{c} = \begin{pmatrix}-f \\ \eta\mathbf{p_1} - \mathbf{u}\end{pmatrix}$<br>2)Focal Length:<br>$\beta = \frac{1}{2}\left\lvert\frac{\mathbf{u}^T\mathbf{p}_1}{\lambda_2}\right\rvert$<br>3)Focus:<br>$\mathbf{F} = \begin{pmatrix}-f \\ \eta\mathbf{p_1} - \mathbf{u}\end{pmatrix}\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V}\end{pmatrix}^{-1} + \frac{-2\eta\begin{pmatrix}1 & 0\end{pmatrix}^T}{4}$<br>4)Axis:<br>$k(\mathbf{V}\mathbf{c} + \mathbf{u})^T\mathbf{x} = 0$<br>5)Directrix:<br>$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} + \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0$<br>6)Latus Rectum:<br>$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} - \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0$<br>7)End points of latus rectum :<br>$\mathbf{u}^T\kappa = -\frac{(\kappa^T\mathbf{V}\kappa + f)}{2}$<br>8)Length of latus rectum:<br>$l = \left\lVert\beta(\mathbf{V}\mathbf{c} + \mathbf{u})^T\right\rVert$ |

TABLE 4.1: Generalisation of conic