ASSIGNMENT 1

Valli Devi Bolla

Download all python codes from

```
https://github.com/Vallidevibolla/valli/blob/main/Collinear.py
```

and latex-tikz codes from

```
https://github.com/Vallidevibolla/valli/blob/main/problem14.tex
```

1 Question No.14

Find the value of k,if the points $\begin{pmatrix} k \\ 3 \end{pmatrix}$, $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ are collinear.

2 Solution

Let

$$\mathbf{A} = \begin{pmatrix} k \\ 3 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \tag{2.0.3}$$

As,given that the points are collinear,

$$\begin{pmatrix} 6 & -2 \\ -3 & 4 \\ k & 3 \end{pmatrix} \tag{2.0.4}$$

$$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ A & B & C \end{vmatrix} = 0 \tag{2.0.5}$$

$$\implies \begin{vmatrix} 1 & 1 & 1 \\ k & 6 & -3 \\ 3 & -2 & 4 \end{vmatrix} = 0 \tag{2.0.6}$$

$$\implies -6K - 9 = 0 \tag{2.0.7}$$

$$\implies k = -3/2 \tag{2.0.8}$$

3 Alternative Method

As,given that the points are collinear,

$$\begin{pmatrix} 6 & -2 \\ -3 & 4 \\ k & 3 \end{pmatrix} \tag{3.0.1}$$

$$\begin{pmatrix} k-6 & 3-(-2) \\ 6-(-3) & -2-4 \end{pmatrix}$$

$$\to \begin{pmatrix} k-6 & 5 \\ 9 & -6 \end{pmatrix} \xleftrightarrow{(R_2 \to R_1)} \begin{pmatrix} 9 & -6 \\ k-6 & 5 \end{pmatrix}$$

$$\xrightarrow{(R_1/3)} \begin{pmatrix} 3 & -2 \\ k-6 & 5 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 3 & -2 \\ 0 & 3 \times 5 - (-2 \times (k-6) \end{pmatrix}$$

$\implies$ 15 + 2K − 12=0

$\implies k = -3/2$

$\therefore$ Finally the value of k is $\frac{-3}{2}$

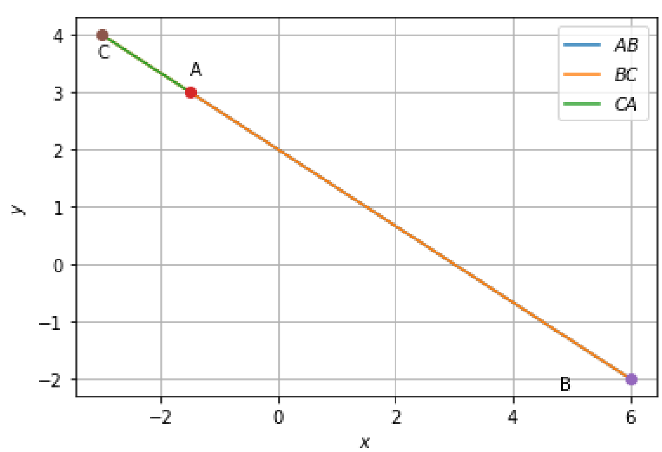

Fig. 0: collinear

3.1 prove that

These points are collinear and forms a line

$$\mathbf{A} = \begin{pmatrix} \frac{-3}{2} \\ 3 \end{pmatrix} \tag{3.1.1}$$

$$\mathbf{B} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \tag{3.1.2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \tag{3.1.3}$$

The direction vectors of AB and BC are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} \frac{15}{2} \\ -5 \end{pmatrix} \tag{3.1.4}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} \frac{-3}{2} \\ 1 \end{pmatrix} \tag{3.1.5}$$

If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a line, then, AB and AC should have the same direction vector. Hence, there exists a k such that

$$\mathbf{B} - \mathbf{A} = k(\mathbf{C} - \mathbf{B}) \tag{3.1.6}$$

$$\implies \mathbf{B} = \frac{k\mathbf{C} + \mathbf{A}}{k+1} \tag{3.1.7}$$

$$\mathbf{B} - \mathbf{A} = k(\mathbf{C} - \mathbf{A}) \tag{3.1.8}$$

$$\implies \begin{pmatrix} \frac{15}{2} \\ -5 \end{pmatrix} = k \begin{pmatrix} \frac{-3}{2} \\ 1 \end{pmatrix} \tag{3.1.9}$$

$$\implies \mathbf{k} = \text{-}5 \tag{3.1.10}$$

Since

$$\mathbf{B} - \mathbf{A} = k(\mathbf{C} - \mathbf{A}), \tag{3.1.11}$$

the points are collinear and form a line. Hence it is proved