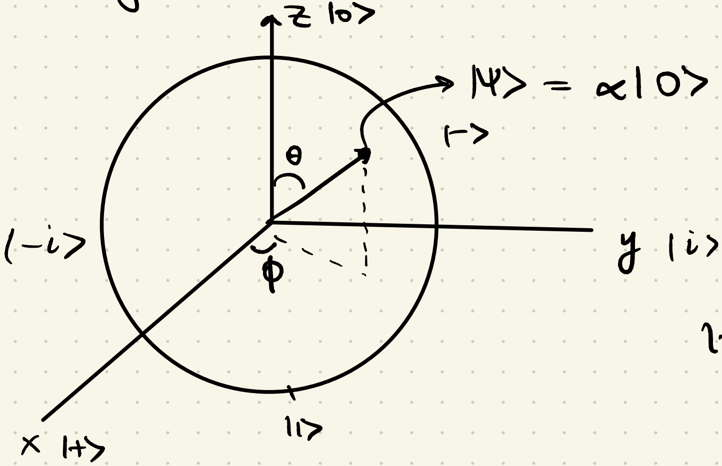
Single Qubit Gates:



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = \cos\theta|0\rangle + \underbrace{e^{i\phi}}_{\text{phase!}}\sin\theta|1\rangle$$

$X \to$ bit flip $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$Y \to$ bit + phase flip

$Z \to$ phase flip $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$H \to$ Hadamard!

$$H = \underbrace{\frac{1}{\sqrt{2}}}_{\text{normalization factor}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Multi- Qubit Gates

N - qubits - state space that is 2^N dimensions

$$1\text{-qubit} \rightarrow 2^1 \rightarrow \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$$

$$\text{Gate space} \rightarrow 2^N \times 2^N \text{ dim}$$

$$N=1 \qquad 2\times 2$$

$$X = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$

$$|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi_2\rangle = c|0\rangle + d|1\rangle$$

tensor prod / Kronecker prod

$$|\psi_{tot}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

$$\underset{2\times 1}{|v\rangle = \begin{pmatrix}v_1\\v_2\end{pmatrix}} \qquad \underset{2\times 1}{|w\rangle = \begin{pmatrix}w_1\\w_2\end{pmatrix}} \qquad v\otimes w = \begin{pmatrix}v_1\cdot w\\v_2\cdot w\end{pmatrix}$$

$$v \otimes w = \begin{pmatrix} v_1 w_1 \\ v_1 w_2 \\ v_2 w_1 \\ v_2 w_2 \end{pmatrix}$$

4×1

$$A = \begin{pmatrix} \underline{a_{11}} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$A \otimes B = \left[\begin{array}{c|c} a_{11} B & a_{12} B \\ 2\times 2 \quad & \quad 2\times 2 \\ \hline 2\times 2 \quad & \quad 2\times 2 \\ \underbrace{a_{21} B}_{} & a_{22} B \end{array}\right]$$

$\downarrow$

$$\begin{pmatrix} a_{21} b_{11} & a_{21} b_{12} \\ a_{21} b_{21} & a_{21} b_{22} \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle$$

$$|0\rangle \otimes |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = |01\rangle$$

$$|1\rangle \otimes |0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = |\underbrace{10}_{1 \otimes 0}\rangle$$

$$|1\rangle \otimes |1\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |11\rangle$$

$$|0 \ldots 0_N\rangle = 2^N \text{ dim} = \begin{pmatrix} 1 \\ \downarrow \\ 0 \end{pmatrix} 2^N \text{ dim}$$

$$|\psi_{tot}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

$$= (\alpha|0\rangle + \beta|1\rangle) \otimes (c|0\rangle + d|1\rangle)$$

$$= \alpha c|00\rangle + \alpha d|01\rangle + \beta c|10\rangle + \beta d|11\rangle$$

$$|\alpha c|^2 + |\alpha d|^2 + |\beta c|^2 + |\beta d|^2 = 1$$

Not all 2-qubit gates can be written as tensor products!!

↪ key QM- entanglement

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell state

maximal state of entanglement!

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

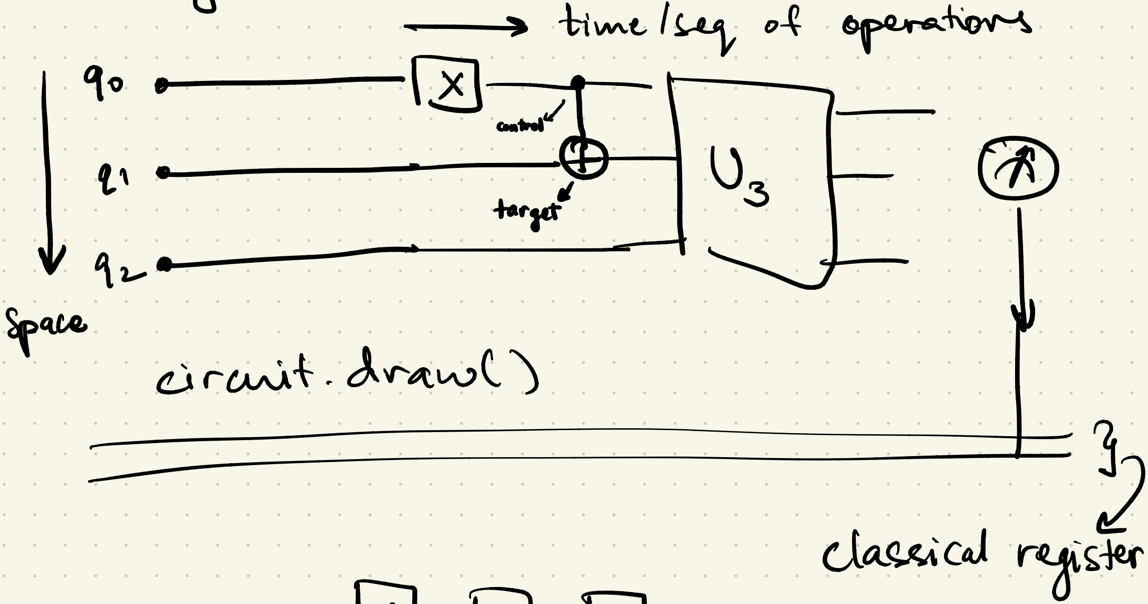
$$H|0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

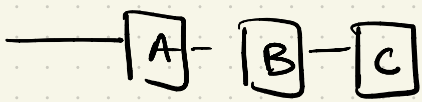
$$\text{CNOT} = \left(\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & 0 \\ \hline & & 0 \quad 1 \\ & 0 & 1 \quad 0 \end{array}\right)$$

$$\text{CNOT}\left(H|0\rangle \otimes |0\rangle\right) = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

bell = circuit.h(0).cnot(0,1)

Reading Quantum Circuits





$CBA|\psi\rangle$

$$B = HS \otimes X$$