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CSIS 616

Chapter 5 Homework

Problem A

> **5.1** Show that EQ_{CFG} is undecidable.

EQ_{CFG} is undecidable if some other undecidable problem reduces to it. ALL_{CFG} (the problem of determining whether a CFG generates all possible strings, p. 225) is undecidable and can be reduced to EQ_{CFG}:

Assume that EQ_{CFG} and ALL_{CFG} are decidable. Let G_{ALL} be a CFG that generates all strings in its alphabet. Then we can attempt to decide if some CFG is in ALL_{CFG} by reducing it to the problem of determining if it is equivalent to G_{ALL}. However, ALL_{CFG} is undecidable, and because it reduces to EQ_{CFG}, that too must be undecidable.

Problem B

> **5.4** If $A \leq_{\mathrm{m}} B$ and B is a regular language, does that imply that A is a regular language? Why or why not?

Let a and b be (distinct) words in the languages of A and B. The following reduction maps any decidable language A to the regular language $\{a\}$:

$$f(s) : \Sigma^* \to \Sigma^* = \begin{cases} a, & s \in A \\ b, & otherwise \end{cases}$$

Therefore, reducing to a regular language does not imply regularity.

Problem C

> **5.21** Let $AMBIG_{\mathsf{CFG}} = \{\langle G \rangle |\ G \text{ is an ambiguous CFG}\}$. Show that $AMBIG_{\mathsf{CFG}}$ is undecidable. (Hint: Use a reduction from PCP. Given an instance
>
> $$P = \left\{ \left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right] \right\}$$
>
> of the Post Correspondence Problem, construct a CFG G with the rules
>
> $$\begin{aligned} S &\to T \mid B \\ T &\to t_1 T \mathtt{a}_1 \mid \cdots \mid t_k T \mathtt{a}_k \mid t_1 \mathtt{a}_1 \mid \cdots \mid t_k \mathtt{a}_k \\ B &\to b_1 B \mathtt{a}_1 \mid \cdots \mid b_k B \mathtt{a}_k \mid b_1 \mathtt{a}_1 \mid \cdots \mid b_k \mathtt{a}_k\,, \end{aligned}$$
>
> where $\mathtt{a}_1, \ldots, \mathtt{a}_k$ are new terminal symbols. Prove that this reduction works.)

I can see a relationship between $AMBIG_{CFG}$ and PCP, but I can't quite figure out the full proof. Here's what I can determine:

We can reduce PCP to $AMBIG_{CFG}$. Let G be the CFG given above. We can derive a string w_1 from T and a string w_2 from B. If we choose the strings such that their right halves (containing $a_1..a_k$) are identical, then the left halves will be a solution to PCP. In other words, we can determine if there is a solution to this instance of PCP by determining if we can derive ambiguous substrings from the grammar.