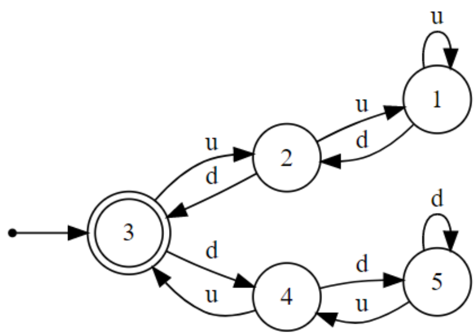
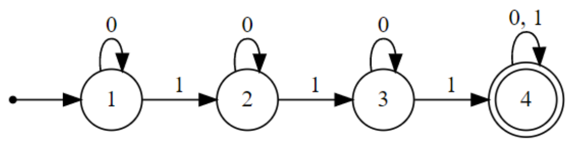
Anthony Morrell
CSIS 616

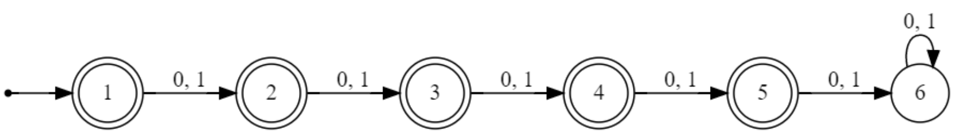
Problem (a)



Problem (b)

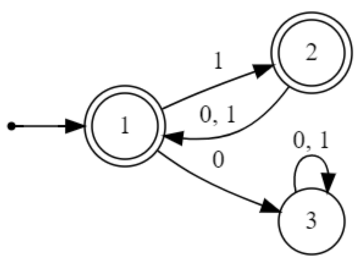


(b)

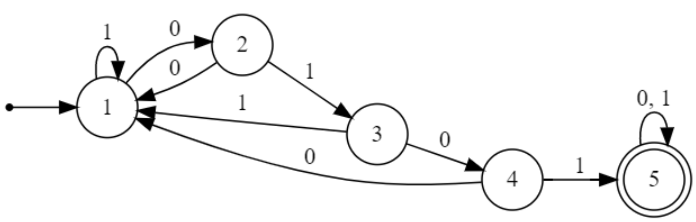


(g)

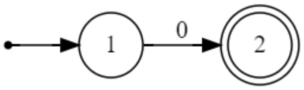
(i) Assuming the first character is at position 0, i.e. is even:



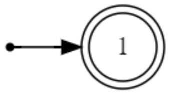
Problem (c)



(b)

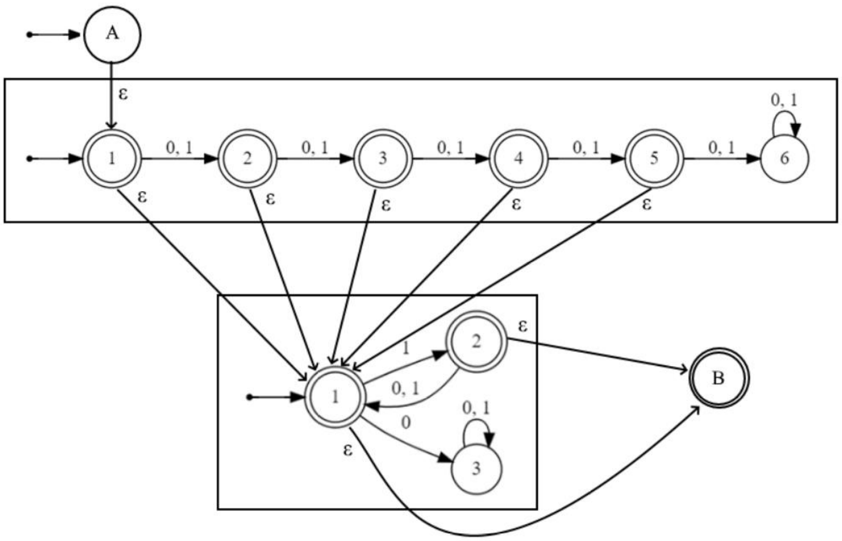


(d)



(g)

Problem (d)



Problem (e)

(b) $0^* \, 1 \, 0^* \, 1 \, 0^* \, 1 \, (0 \cup 1)^*$

(g) $(0 \cup 1 \cup \epsilon)^5$

(i) $\big((0 \cup 1)\ 1\big)^* (0 \cup 1 \cup \epsilon)$

Problem (f)

(b) Includes: ab, aab; Doesn’t include: ba, bba

(f) Includes: aba, bab; Doesn’t include: a, b

(h) Includes: aa, bba; Doesn’t include: b, ε

Problem (g)

(a) Assume that A_1 is regular. Let $s = 0^p 1^p 2^p$, where p is the pumping length given by the pumping lemma. Because $s \in A_1$ and $|s| > p$, the pumping lemma guarantees that s can be split into three parts, $s = xyz$, where (1) $xy^i z \in A_1 \ \forall\ i \geq 0$, (2) $|y| > 0$, and (3) $|xy| \leq p$.

Case 1: y contains all 0's. Then $xyyz$ would have more 0's than 1's or 2's, breaking condition (1).

Case 2: y contains letters other than 0. Then $|xy| > p$, breaking condition (3).

A contradiction is unavoidable, so A_1 must not be regular.

(b) Assume that A_2 is regular. Let $s = w_1 w_2 w_3$, where $w_1 = w_2 = w_3 = \{a, b\}^*$, $|w_1| < p/2$, and p is the pumping length given by the pumping lemma. Because $s \in A_2$ and $|s| > p$, the pumping lemma guarantees that s can be split into three parts, $s = xyz$, where (1) $xy^i z \in A_2 \ \forall\ i \geq 0$, (2) $|y| > 0$, and (3) $|xy| \leq p$.

Case 1: $y = w_2$. Then $xyyz$ would have too many repetitions of w_1, breaking condition (1).

Case 2: y contains only part of w_2. Then $xyyz$ would not contain the middle letters in the correct order, breaking condition (1).

Case 3: y contains none of w_2, but part of w_1. Same as Case 2.

Case 4: y contains none of w_2, but part of w_3. Then $|xy| > p$, which breaks condition (3).

A contradiction is unavoidable, so A_2 must not be regular.

(c) (I'm not sure that this one is correct.)

Assume that A_3 is regular. Let $s = a^{2^p}$, where p is the pumping length given by the pumping lemma. Because $s \in A_3$ and $|s| > p$, the pumping lemma guarantees that s can be split into three parts, $s = xyz$, where (1) $xy^i z \in A_2 \ \forall\ i \geq 0$, (2) $|y| > 0$, and (3) $|xy| \leq p$.

To satisfy condition (1), it must be true that $|xyyz| = 2^k$ for some integer $k > 1$. However, condition (2) requires that $|y| > 0$, so $|xyyz| > 2^p$, while condition (3) requires that $|xy| < p$, so $|xyyz| < 2^{p+1}$. There is no integer k such that $p > k > p + 1$, so $xyyz$ cannot be in A_3, breaking condition (1). Thus, A_3 must not be regular.