# Exercise 2: Gravitation

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| Name, First Name | |
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## Superposition of Earth's and Moon's gravitational fields

Earth and Moon can be modelled as homogeneous solid spheres with constant density as a first order approximation. Both masses are sources of the gravitational field. Visualize their superimposed gravitational field as well as the corresponding potential field in the plane that contains both mass centers.

**Task 1**: Design and implement Matlab functions for the computation of the gravitational potential $V(X,Y)$ and the vector valued gravitational attraction $\boldsymbol{a}(X,Y)$ of a homogeneous solid sphere with radius $R$, coordinates of mass center $(X_m, Y_m)$, and density $\rho$ at points with coordinates $(X,Y)$ in two dimensions. The input coordinates and the output parameters should be matrices. Possible function headers

$$\text{function } [V]\text{=V\_sphere}(X,Y,R,X_m,Y_m,\rho)$$
$$\text{function } [a_x\ a_y]\text{=a\_sphere}(X,Y,R,X_m,Y_m,\rho)$$

**Task 2**: Calculate and superimpose the potential of both Earth and Moon on a dense grid. Determine the attraction vector driven by Earth and Moon on a coarse grid.

**Task 3**: Visualize the vector field $\boldsymbol{a}(X,Y)$ using the *quiver* command. Due to the behaviour of the gravity magnitude, it is maybe necessary to normalize it.

**Task 4**: Visualize the potential field $V(X,Y)$ using the *contour* command. To avoid too dense (over crowded) contour lines close to the bodies, bigger increments can be used as compared to the far zone. Label the isolines with the command *clabel*.

## Gravitational potential and attraction of spherical shells

The basic structure of the Earth is radial: inner core, outer core, mantle, and crust. Assume the following simplified structure

$$\begin{array}{ll} \text{core } R_c = 3500\,\text{km} & \rho_c = 11\,200\,\text{kg/m}^3 \\ \text{mantle } R_m = 6400\,\text{km} & \rho_m = 4300\,\text{kg/m}^3 \end{array}$$

**Task 5**: Write down the formula to evaluate potential $V(r)$ and attraction $\boldsymbol{a}(r)$ at a point $P_c$ inside the core ($r \le R_c$), at a point $P_m$ inside the mantle ($R_c < r \le R_m$), and at a point $P_e$ outside the Earth ($r > R_m$).

**Task 6**: Design and implement Matlab functions for the computation of the potential $V$ and the attraction $\boldsymbol{a}$ of a spherical shell, which is dependent on the inner and outer radii $R_{\text{in}}$ and $R_{\text{out}}$, the density $\rho$ and the radial coordinate $r$ of the point $P$. Possible function headers can be

Hassan H. Farahani
hassan.hashemi-farahani@gis.uni-stuttgart.de

$$\text{function } [V]\text{=V\_shell}(R_{\text{in}}, R_{\text{out}}, \rho, r)$$

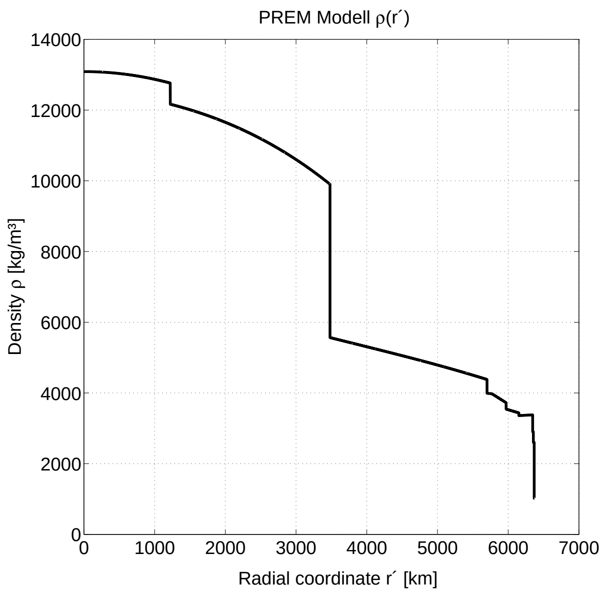
$$\text{function } [a]\text{=a\_shell}(R_{\text{in}}, R_{\text{out}}, \rho, r)$$

in which $r$ represents a vector or a matrix.

**Task 7**: Calculate the potential $V$ and the attraction $\boldsymbol{a}$ along a radial profile with $0 \leq r \leq 4R_m$ and plot them.

### PREM density model of the Earth

Detailed Earth models subclassify core, mantle and crust in further spherical shells, where each of them has a radial density distribution. Preliminary Reference Earth Model (PREM) is a model developed by A. M. Dziewonski and D. L. Anderson (1981)[1], which is determined from geodetic and seismic observations. The following figure shows the density distribution of PREM with respect to the distance $r'$ from the geocenter.




**Task 8**: Using the PREM model provided in ILIAS and the functions *V_shell* and *a_shell*, plot the density distribution $\rho$. Furthermore, compute and plot the gravitational potential $V(r)$ and the gravitational attraction $a(r)$ generated by PREM in the radial range $0 \leq r \leq 2R_{\text{E}}$.

**Task 9**: Determine the gravitational potential $V(r = R_{\text{E}})$ and the gravitational attraction $\boldsymbol{a}(r = R_{\text{E}})$ at the surface of the Earth.

**Task 10**: Identify from the figure obtained in task 8, the positions $r$ with the largest gravitational attraction $\boldsymbol{a}$ as well as the positions $r$ with the largest gravitational potential $V$. Compute the attraction $\boldsymbol{a}$ and the gravitational potential $V$ at these positions.

**Numerical values**

[1] Dziewonski, A.M., Anderson, D.L. (1981) Preliminary reference Earth model (PREM). Phys. Earth Planet. Int. 25: 297–356

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| mass of the Earth | $m_{\mathrm{E}} = 5.9736 \cdot 10^{24}\,\mathrm{kg}$ |
| radius of the Earth | $R_{\mathrm{E}} = 6371\,\mathrm{km}$ |
| coordinates of the Earth wrt. a geocentric reference system | $X_{\mathrm{E}} = 0$ km, $Y_{\mathrm{E}} = 0$ km, $Z_{\mathrm{E}} = 0$ km |
| mass of the Moon | $m_{\mathrm{M}} = 7.349 \cdot 10^{22}\,\mathrm{kg}$ |
| radius of the Moon | $R_{\mathrm{M}} = 1738\,\mathrm{km}$ |
| distance Earth–moon | $r_{\mathrm{M}} = 384\,400\,\mathrm{km}$ |
| coordinates of the moon wrt. a geocentric reference system | $X_{\mathrm{M}} = r_{\mathrm{M}} \cos(k \cdot 10^{\circ})$ km, $Y_{\mathrm{M}} = r_{\mathrm{M}} \sin(k \cdot 10^{\circ})$ km, $Z_{\mathrm{M}} = 0$ km |
| gravitational constant | $6.672\ 10^{-11}\,\mathrm{m}^3\,\mathrm{s}^{-2}\mathrm{kg}^{-1}$ |