Axiom A4 *(Normalization).* $H(1/2, 1/2) = 1$.

Axiom A4 means that making a choice from equally probably events/outcomes gives one bit of information.

Axiom A5 *(Symmetry).* The function $\mathrm{H}(p_1, p_2, \ldots, p_n)$ is symmetric for any permutation of the arguments p_i.

Axiom A5 means that the funcion $\mathrm{H}(p_1, p_2$, dots, $p_n)$ depends only on probabilities $p_1, p_2, \ldots, p_n$ but is independent of their order.

The Axiomatic characterization is obtained in the following theorem.

Theorem 3.5.1. The only funcion that satisfies Axioms A1-A5 is the information entropy function $\mathrm{H}(p_1, p_2, \ldots, p_n)$ determined by the formula (3.2.4).

Proof. Let us consider the function $\mathrm{F}(n) = H(1/n, 1/n, \ldots, 1/n)$ and r^m events with equal probabilites of occurrence. It is possible to decompose choices from all of these events into a series of m choices from r potential events with equal probobilies of occurrence. In this situation, Axiom A3 gives us the following equality:

$$F(r^m) = mF(r)$$

Let us fix some number t. Then for any natural number n, there is number m such that

$$r^m \leq t^n < r^{m+1} \tag{3.5.1}$$

Indeed, if m is the largest number such that $r^m \leq t^n$ then $t^n < r^{m+1}$. As before, Axiom A3 gives us the following equality:

$$m \log_2 r \leq n \log_2 t < (m+1) \log_2 r \tag{3.5.2}$$

Dividing all therms in the formula (3.5.2) by $n\ log_2 r$, we have

$$m/n \leq (log_2 r) < m/n + 1/n \tag{3.5.3}$$

This gives us

$$|\ (log_2 t/log_2 r) - m/n\ | < 1/n \tag{3.5.4}$$

At the same time, Axiom A2 and inequalities (3.5.1), give us

$$F(r^m) \leq F(t^n) \leq F(r^{m+1})$$