Pre-calculus

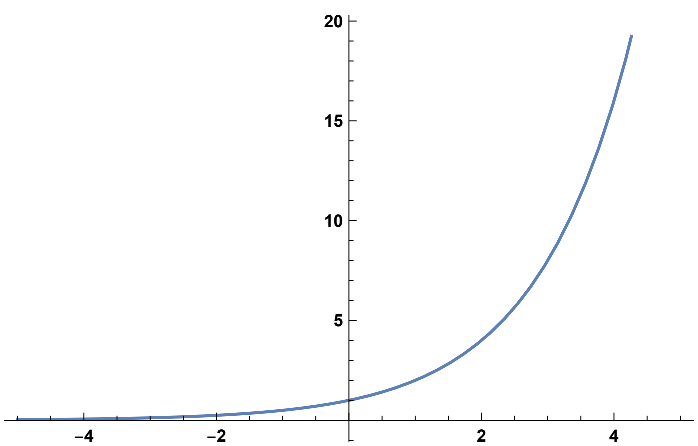
exponentials and logarithms

Not all exercises were implemented. I find it not interesting to work on items which I know that I can find a solution just by looking at them. I only do something if I do it for the first time or if it's sufficiently different from previous exercises.
This notebook is a prerequisite for learning Calculus and also a nice process for me to get my head around Mathematica.

sketch the graph of *f*

$f(x) = 2^x$

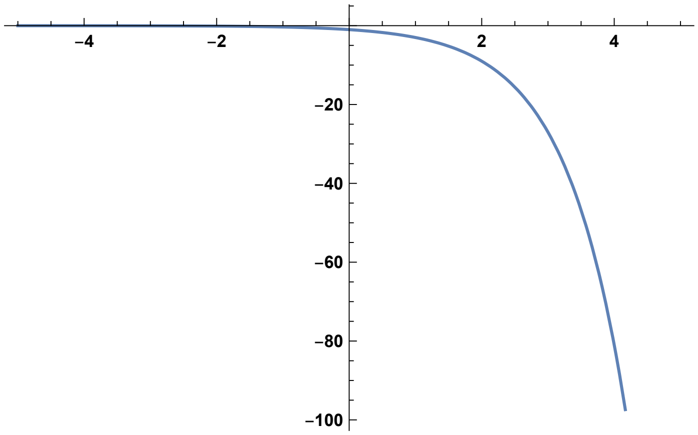
In[∘]:=
```
Plot[2^x, {x, -5, 5}]
```



$f(x) = -3^x$

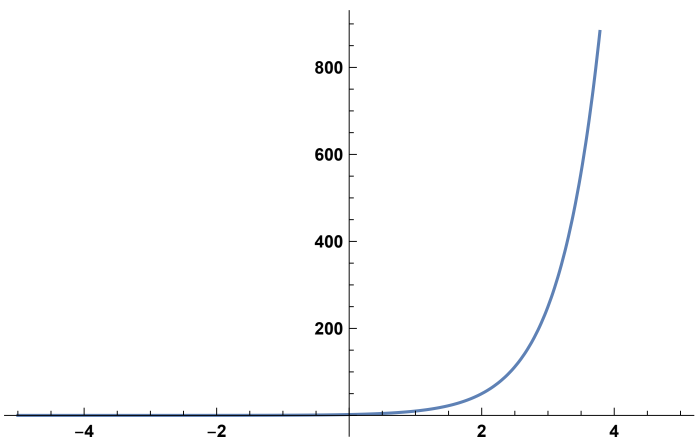
In[•]:= `Plot[-3^x, {x, -5, 5}]`

Out[•]=



$f(x) = 2(5)^x$

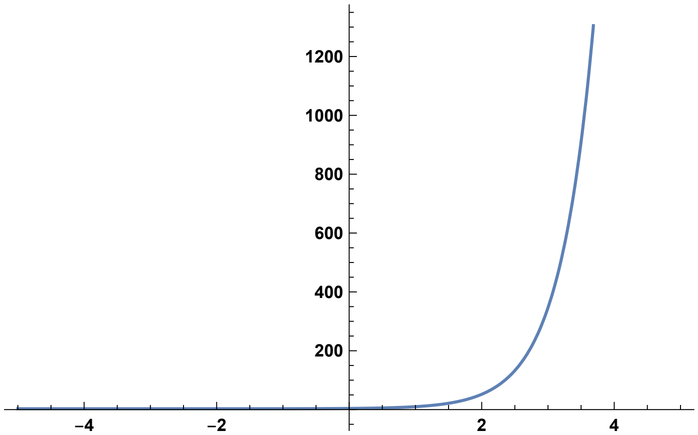
In[•]:= `Plot[2 (5)^x, {x, -5, 5}]`



$f(x) = 7^x + 3$

In[∘]:= `Plot[7^x + 3, {x, -5, 5}]`

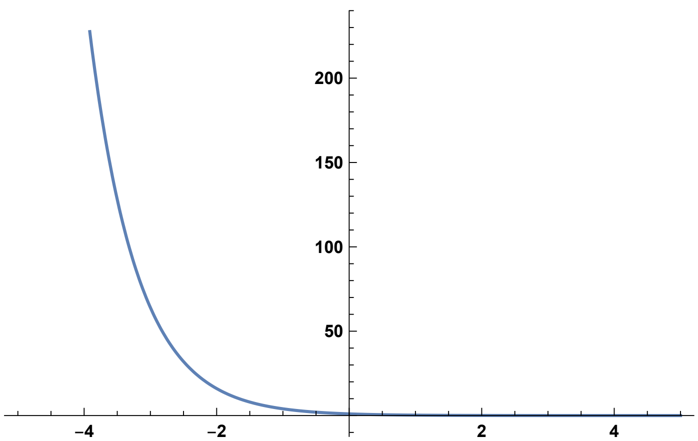
Out[∘]=



$f(x) = 4^{-x}$

In[∘]:= `Plot[4^-x, {x, -5, 5}]`

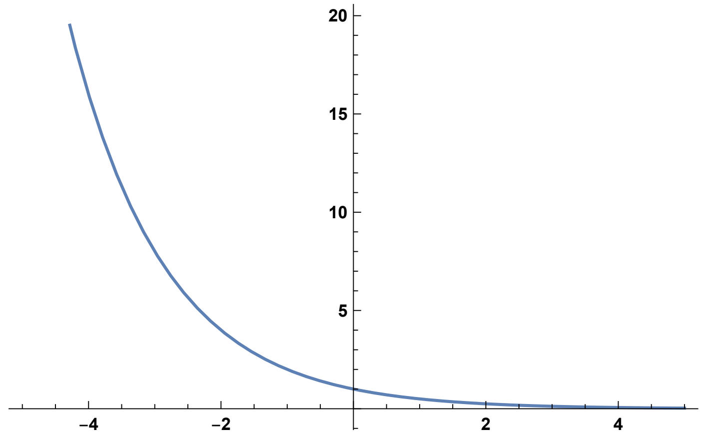
Out[∘]=



$f(x) = \left(\frac{1}{2}\right)^x$

In[∘]:=
```
Plot[(1/2)^x, {x, -5, 5}]
```

Out[∘]=



Solve the equations

$5^{x+8} = 5^{3x-2}$

$5^{x+8} = 5^{3x-2}$
$x + 8 = 3x - 2$
$x - 3x = -2 - 8$
$-2x = -10$
$2x = 10$
$x = 5$

```
Solve[5^(x + 8) == 5^(3 x - 2), x, Reals]
```

Out[∘]= {{x → 5}}

$8^{7-x} = 8^{2x+1}$

$8^{7-x} = 8^{2x+1}$
$7 - x = 2x + 1$
$6 = 3x$
$x = 2$

In[∘]:=
```
Solve[8^(7-x) == 8^(2 x+1), x, Reals]
```

Out[∘]= {{x → 2}}

$5^{(x^2)} = 5^{2x+3}$

$5^{x^2} = 5^{2x+3}$

$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x_1 = -1, \; x_2 = 3$

In[∘]:=
```
Solve[5^(x^2) == 5^(2 x+3), x, Reals]
```

Out[∘]= {{x → -1}, {x → 3}}

$25^{(x^2)} = 5^{3x+2}$

$5^{2x^2} = 5^{3x+2}$

$2x^2 = 3x + 2$

$2x^2 - 3x - 2 = 0$

I wasn't sure how to factor it, so just used Mathematica to factor

$(x - 2)(2x + 1) = 0$

$x_1 = \mathbf{2}, \; x_2 = \mathbf{-\left(\frac{1}{2}\right)}$

In[∘]:=
```
Solve[25^(x^2) == 5^(3 x + 2), x, Reals]
```

Out[∘]= $\left\{\left\{x \to -\frac{1}{2}\right\}, \{x \to 2\}\right\}$

Solve word problems

A colony of an endangered species originally numbered 1,000 was predicted to have a population N after t years given by the equation $N(t) = 1000\,(0.9)^t$.
Estimate population after:

a) 1 year

b) 5 years

c) 10 years

In[∘]:=
```
1000 (0.9)^1
1000 (0.9)^5
1000 (0.9)^10
```

Out[∘]= 900.

Out[∘]= 590.49

Out[∘]= 348.678

The number of bacteria in a certain culture increased from 600 to 1800 between

8 am and 10 am. Assuming the growth is exponential, the number *f(t)* of bacteria *t* hours after 8 am is given by $f(t) = 600\,(3)^{t/2}$.

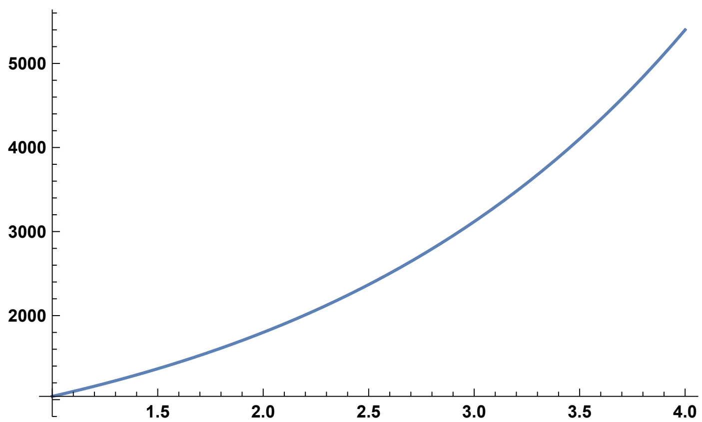
a) Estimate the number of bacteria at 9 am, 11 am, and noon.
b) Sketch the graph of *f*

In[]:=
```
600 (3)^(1/2) // N
600 (3)^(3/2) // N
600 (3)^2
Plot[600 (3)^(t/2), {t, 1, 4}]
```

Out[]= 1039.23

Out[]= 3117.69

Out[]= 5400

Out[]=


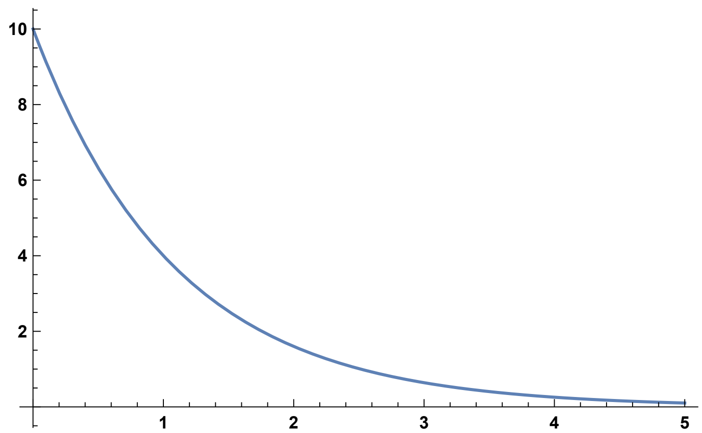
An important problem in oceanography is to determine the amount of light that can penetrate to various ocean depths. The Beer - Lambert law asserts that the exponential function given by $I(x) = I_0 c^x$ is a model for this phenomenon. For a certain location, $I(x) = 10\,(0.4)^x$ is the amount of light (in calories / cm^2 / sec) reaching a depth of x meters.

a) Find the amount of light at a depth of 2 m.

b) Sketch the graph of I for $0 \le x \le 5$.

```
In[]:= 10 (0.4)^2
       Plot[10 (0.4)^x, {x, 0, 5}]
Out[]= 1.6
```

Out[]=



Change to logarithmic form

$5^3 = 125$

$x = \log_5 125$

$3^x = 7 + t$

$x = \log_3(7 + t)$

$5^{-3} = \frac{1}{125}$

$x = \log_5\left(\frac{1}{125}\right)$

Change to exponential form

$\log_2 32 = 5$

$2^5 = 32$

$\log_7 m = 5x + 3$

$7^{5x+3} = m$

$\log_2\left(\frac{1}{64}\right) = -6$

$2^{-6} = \frac{1}{64}$

Solve for *t* using logarithms with base *a*

$2a^{t/5} = 5$

$\frac{t}{5} = \log_{2a} 5$

$t = 5\log_{2a} 5$

Let’s check the solution for $a = 4$

In[ ]:=
```
t = 5 Log[8, 5] // N;
8^(t/5) == 5
```

Out[ ]=
```
True
```

$5a^{3t} = 63$

$3t = \log_{5a}(63)$

$t = \frac{\log_{5a}(63)}{3}$

$A = \mathrm{Ba}^{\mathrm{Ct}} + D$

$\mathrm{Ba}^{\mathrm{Ct}} = A - D$

$\mathrm{Ct} = \mathrm{Log}_{\mathrm{Ba}}(A - D)$

$t = \frac{\mathrm{Log}_{\mathrm{Ba}}(A - D)}{C}$

Find the numbers, if possible

$\mathrm{Log}_9(-3)$

Yes, it's possible, LOL.

In[ ]:=
```
Log[9, -3]
```

Out[ ]=
$$\frac{i\pi + \mathrm{Log}[3]}{\mathrm{Log}[9]}$$

Solve the logarithmic equation

$\log_4 x = \log_4(8 - x)$

$x = 8 - x$
$2x = 8$
$x = 4$

Word problems II

The loudness of a sound, as experienced by the human ear, is based on its intensity level. The intensity level α (in decibels) that corresponds to a sound intensity $\boldsymbol{I}$ is $\alpha = 10\log\left(\frac{I}{I_0}\right)$, where $\boldsymbol{I_0}$ is a special value of $\boldsymbol{I}$ agreed to be the weakest sound that can be detected by the human ear under certain conditions. Find α if
a) $\boldsymbol{I}$ is 10 times as great as $\boldsymbol{I_0}$
b) $\boldsymbol{I}$ is 1000 times as great as $\boldsymbol{I_0}$

a) $\alpha = 10\log 10$; $\alpha = 10$;
b) $\alpha = 10\log 1000$; $\alpha = 30$;

A sound intensity level of 140 decibels produces pain in the average human ear. Approximately how many times greater than I_0 must I be in order for α to reach this level?

$\alpha = 10\log\left(\frac{I}{I_0}\right)$
$140 = 10\log\left(\frac{I}{I_0}\right)$
$\log\left(\frac{I}{I_0}\right) = 14$
$\left(\frac{I}{I_0}\right) = 10^{14}$

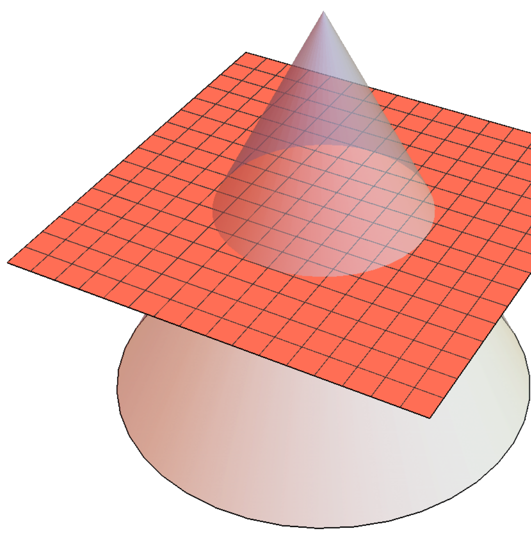
Conic sections

First of all, some conic section plots

Circle section

In[•]:=
```
gcone = Graphics3D[{Opacity[0.5], Cone[]}, Boxed → False];
gplane = ContourPlot3D[0 x + 0 y + 10 z == 0,
   {x, -1, 1}, {y, -1, 1}, {z, -1, 1}, ContourStyle → {Pink}];
Show[gcone, gplane]
```

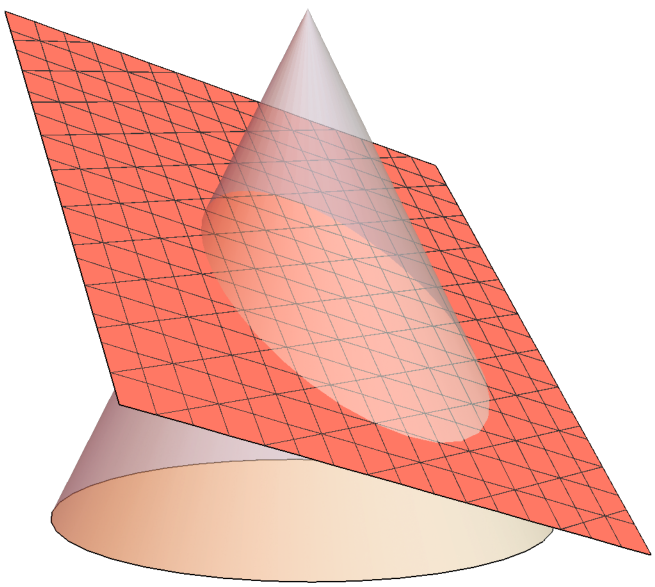
Out[•]=



Ellipse section

In[ ]:=
```
gcone = Graphics3D[{Opacity[0.5], Cone[]}, Boxed → False];
gplane = ContourPlot3D[-1 x + 2 y - 3 z == 0,
   {x, -1, 1}, {y, -1, 1}, {z, -1, 1}, ContourStyle → {Pink}];
Show[gcone, gplane]
```

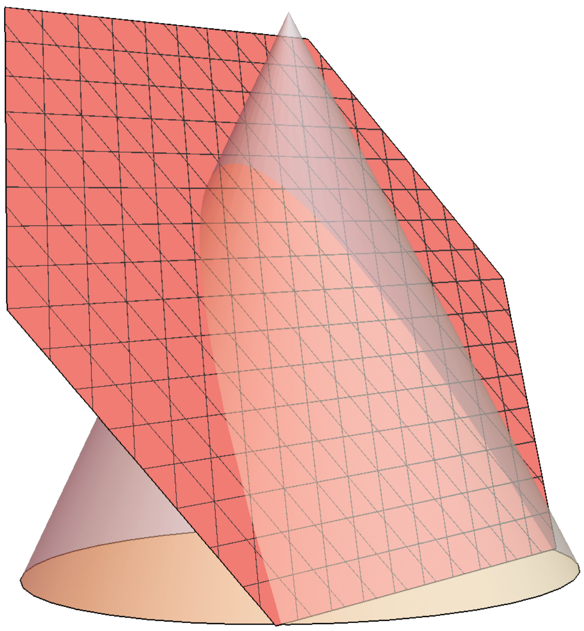
Out[ ]=



Parabola section

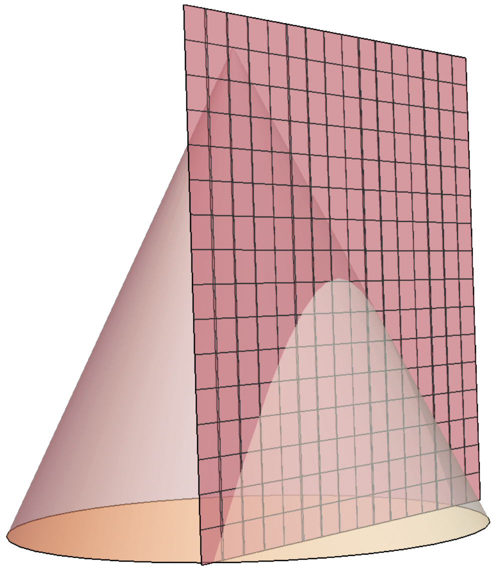
In[ ]:=
```
gcone = Graphics3D[{Opacity[0.5], Cone[]}, Boxed → False];
gplane = ContourPlot3D[-1 x + y - 0.8 z == 0,
   {x, -1, 1}, {y, -1, 1}, {z, -1, 1}, ContourStyle → {Pink}];
Show[gcone, gplane]
```

Out[ ]=



Hyperbola section

In[ ]:=
```
gcone = Graphics3D[{Opacity[0.5], Cone[]}, Boxed → False];
gplane = ContourPlot3D[x - 0.16 y - 0 z == 0.5, {x, -1, 1},
   {y, -1, 1}, {z, -1, 1}, ContourStyle → {Pink, Opacity[0.7]}];
Show[gcone, gplane]
```



Find the vertex, the focus and the directrix of the parabola. Sketch its graph, showing the focus and the directrix

$y = -\frac{1}{12} x^2$

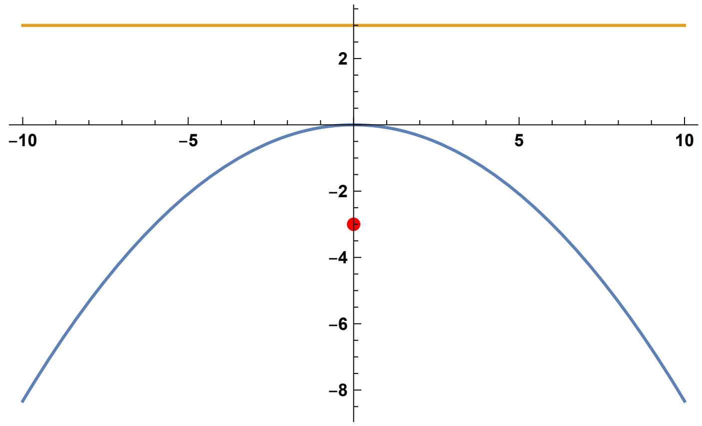
This equation has the form $y = ax^2$ with $a = -\frac{1}{12}$.

$a = \frac{1}{4p}$ so $p = \frac{1}{4a}$; $p = \frac{1}{4\left(-\frac{1}{12}\right)} = \frac{1}{-\frac{4}{12}} = \frac{1}{-\frac{1}{3}} = -3$;

Vertex = $(0, 0)$; Focus = $(0, -3)$; Directrix is $y = 3$;

```
In[]:= focus = Graphics[{PointSize[Large], Red, Point[{0, -3}]}];
parabola = Plot[{-1/12 x^2, y = 3}, {x, -10, 10}, AspectRatio → Automatic];
Show[parabola, focus]
```

Out[]=



$2y^2 = -3x$

We can rewrite this as $-3x = 2y^2$

Divide both ends by -3

$x = -\frac{2}{3}y^2$

This equation has the form $x = ay^2$ with $a = -\frac{2}{3}$

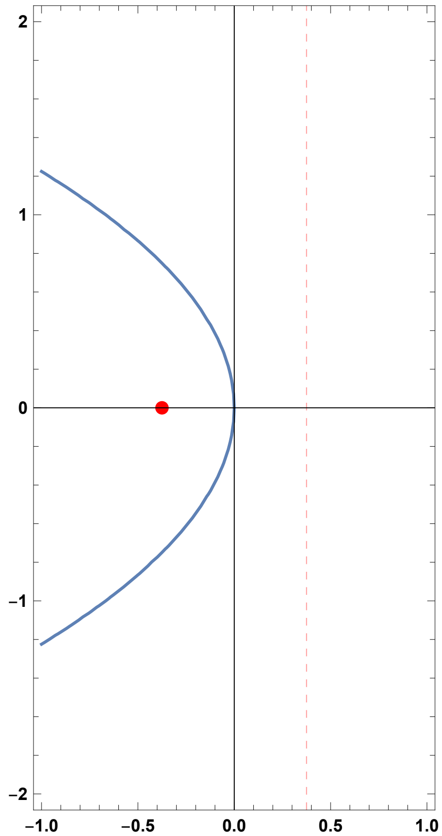
$a = \frac{1}{4p}$, so $p = \frac{1}{4a}$

$p = \frac{1}{4\left(-\frac{2}{3}\right)} = \frac{1}{-\frac{8}{3}} = -\frac{3}{8}$

Vertex = (0, 0); Focus = $\left(-\frac{3}{8},\ 0\right)$; Directrix is $x = \frac{3}{8}$

In[•]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{-3/8, 0}]}];
parabola = ContourPlot[{2 y^2 == -3 x}, {x, -1, 1}, {y, -2, 2}, Axes → True,
   GridLines → {{{3/8, {Red, Dashed}}}, None}, AspectRatio → Automatic];

Show[parabola, focus]
```

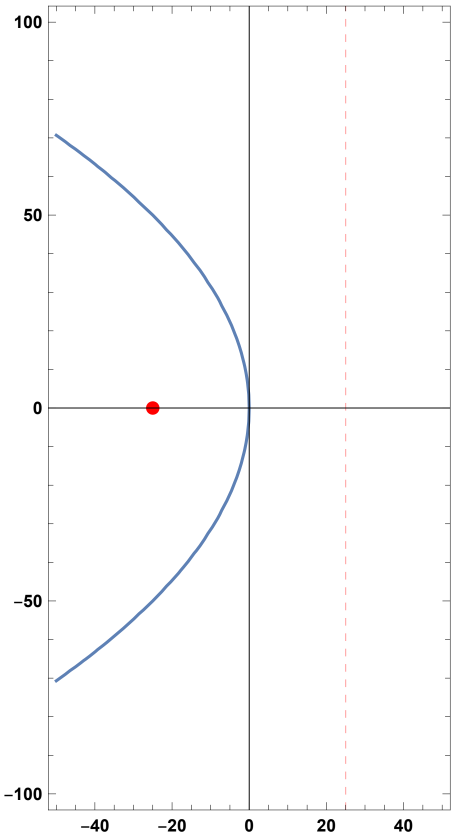
Out[•]=


$y^2 = -100\,x$

We can rewrite this as $-100\,x = y^2$
Divide both ends by -100
$x = -\frac{1}{100}y^2$
This equation has the form $x = ay^2$ with $a = -\frac{1}{100}$
$a = \frac{1}{4\,p}$, so $p = \frac{1}{4\,a}$
$p = \frac{1}{4\left(-\frac{1}{100}\right)} = \frac{1}{-\frac{4}{100}} = \frac{-100}{4} = -25$
Vertex = $(0,\ 0)$; Focus = $(-25,\ 0)$; Directrix is $x = 25$

In[ ]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{-25, 0}]}];
parabola =
  ContourPlot[{y^2 == -100 x}, {x, -50, 50}, {y, -100, 100}, Axes → True,
   GridLines → {{{25, {Red, Dashed}}}, None}, AspectRatio → Automatic];

Show[parabola, focus]
```

Out[ ]=


Find the vertex and the focus of the parabola. Sketch its graph, showing the focus.

$y = x^2 - 4x + 2$

Completing the square for $x^2 - 4x + 2$:

$(x^2 - 4x + 4) + 2 - 4$

$(x^2 - 4x + 4) - 2$

$(x - 2)^2 - 2 = y$

$(x - h)^2 = 4p(y - k)$

$(x - 2)^2 = y + 2$

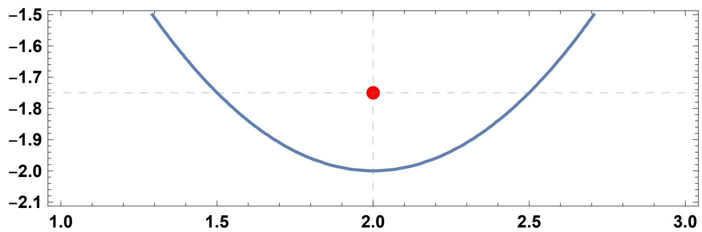
$(x - 2)^2 = 4\left(\frac{1}{4}\right)(y - (-2))$

$h = 2, \; k = -2, \; p = \frac{1}{4}$

Vertex = (2, -2), F = $(2, -2 + \frac{1}{4}) = \left(2, \; -1\,\frac{3}{4}\right)$

In[•]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{2, -1.75}]}];
parabola = ContourPlot[{y == x^2 - 4 x + 2}, {x, 1, 3}, {y, -2.1, -1.5}, Axes → True,
   GridLines → {{{2, Dashed}}, {{-1.75, Dashed}}}, AspectRatio → Automatic];

Show[parabola, focus]
```

Out[•]=


$y = 8x^2 + 16x + 10$

Completing the square for $\frac{y}{8} = x^2 + 2x + \frac{5}{4}$:

$(x^2 + 2x + 1) + \frac{5}{4} - 1$

$(x^2 + 2x + 1) + \frac{1}{4}$

$(x + 1)^2 + \frac{1}{4} = \frac{y}{8}$

$(x + 1)^2 = \frac{y}{8} - \frac{1}{4}$

$(x - h)^2 = 4p(y - k)$

$(x - (-1))^2 = \frac{y-2}{8} = \frac{1}{8}(y - 2)$

$(x - (-1))^2 = 4\left(\frac{1}{32}\right)(y - 2)$

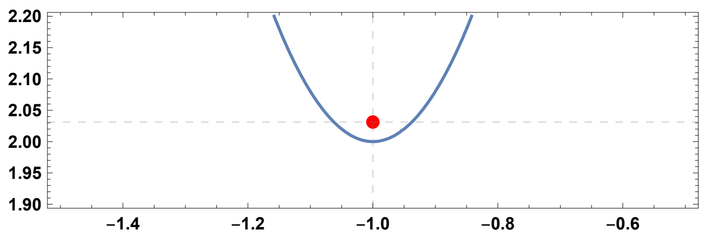
$h = -1, \; k = 2, \; p = \frac{1}{32}$

Vertex = $(-1, 2)$; Focus = $\left(-1, \; 2\,\frac{1}{32}\right)$

In[∘]:=
```
(* some helper method to use a mixed fraction input *)
(* https://mathematica.stackexchange.com/a/184782/54824 *)
CurrentValue[EvaluationNotebook[], {InputAliases, "mf"}] =
  TemplateBox[{"■", "□", "□"}, "MixedFraction",
   DisplayFunction → (RowBox[{#1, FractionBox[#2, #3]}] &),
   InterpretationFunction → (RowBox[{#1, "+", FractionBox[#2, #3]}] &)];
focus = Graphics[{PointSize[Large], Red, Point[{-1, 2 1/32}]}];
parabola =
  ContourPlot[{y == 8 x^2 + 16 x + 10}, {x, -1.5, -0.5}, {y, 1.9, 2.2}, Axes → True,
   GridLines → {{{-1, Dashed}}, {{2 1/32, Dashed}}}, AspectRatio → Automatic];

Show[parabola, focus]
```

Out[∘]=

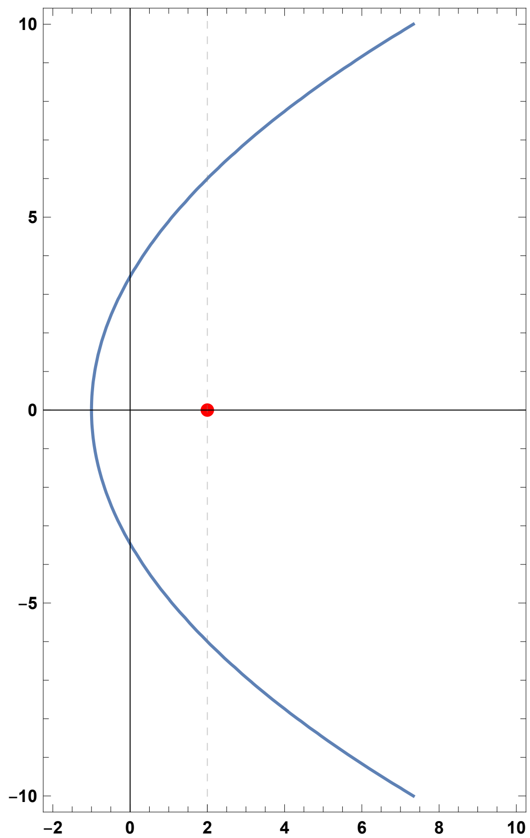


$y^2 - 12 = 12x$

$y^2 = 12x + 12$
$(y-k)^2 = 4p(x-h)$
$(y-0)^2 = 12x + 12$
$12x + 12 = 12(x+1) = 4*3(x+1)$
$(y-0)^2 = 4*3(x-(-1))$
$k = 0,\ h = -1,\ p = 3$
$\text{Vertex} = (-1,\ 0),\ \text{Focus} = (2,\ 0)$

```
In[]:= focus = Graphics[{PointSize[Large], Red, Point[{2, 0}]}];
parabola = ContourPlot[{y² == 12 x + 12}, {x, -2, 10}, {y, -10, 10},
   Axes → True, GridLines → {{{2, Dashed}}, None}, AspectRatio → Automatic];
Show[parabola,
 focus]
```

Out[]=



$y^2 - 20y + 100 = 6x$

$6x = y^2 - 20y + 100$

Completing the square for $y^2 - 20y + 100$

$(y^2 - 20y) + 100$

$(y^2 - 20y + 100)$

$(y - 10)^2 = 6x$

$(y - k)^2 = 4p(x - h)$

$4p(x - h) = 4\left(\frac{3}{2}\right)(x - 0)$

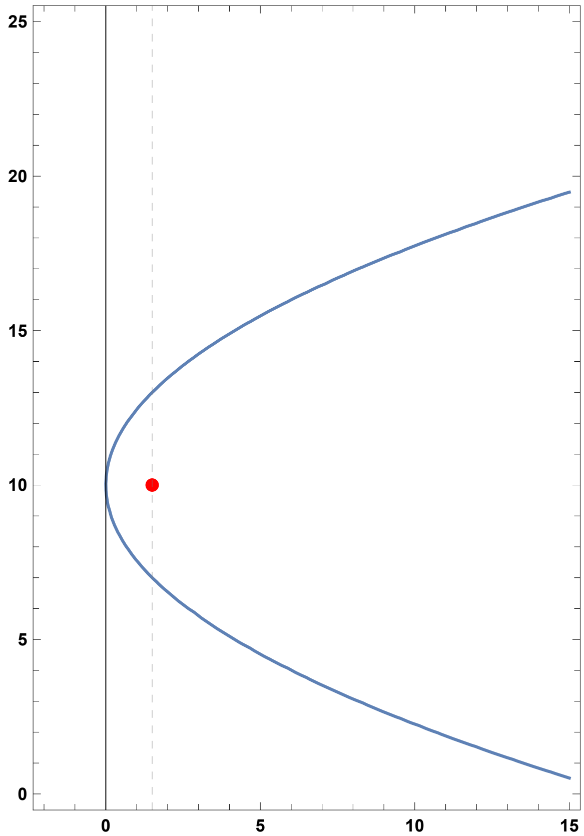
$(y - 10)^2 = 4\left(\frac{3}{2}\right)(x - 0)$

$k = 10, \; h = 0, \; p = \frac{3}{2}$

Vertex $= (0, 10)$, Focus $= \left(\frac{3}{2}, 10\right)$

In[•]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{3/2, 10}]}];
parabola = ContourPlot[{y^2 - 20 y + 100 == 6 x}, {x, -2, 15}, {y, 0, 25},
   Axes → True, GridLines → {{{3/2, Dashed}}, None}, AspectRatio → Automatic];
Show[parabola,
 focus]
```

Out[•]=



$y^2 - 4y - 2x - 4 = 0$

$2x = y^2 - 4y - 4$

Completing the square for $y^2 - 4y - 4$

$(y^2 - 4y) - 4$

$(y^2 - 4y + 4) - 4 - 4$

$(y - 2)^2 - 8 = 2x$

$(y - k)^2 = 4p(x - h)$

$(y - 2)^2 = 2x + 8$

$(2x + 8) = 2(x + 4)$

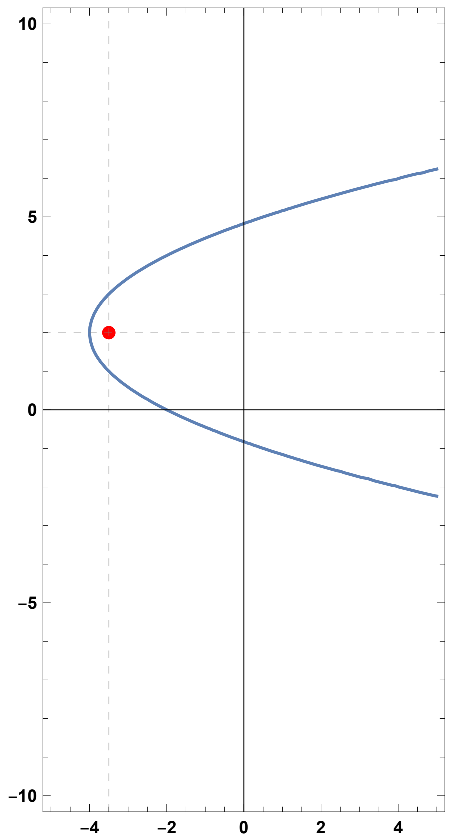
$(y - 2)^2 = 2\left(\frac{1}{2}\right)(x - (-4))$

$k = 2,\ h = -4,\ p = \frac{1}{2}$

Vertex = $(-4, 2)$, Focus = $(-3.5, 2)$

In[∘]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{-3.5, 2}]}];
parabola = ContourPlot[{2 x == y^2 - 4 y - 4}, {x, -5, 5}, {y, -10, 10}, Axes → True,
   GridLines → {{{-3.5, Dashed}}, {{2, Dashed}}}, AspectRatio → Automatic];
Show[parabola,
 focus]
```

Out[∘]=



$y^2 + 14\,y + 4\,x + 45 = 0$

$-4\,x = y^2 + 14\,y + 45$

Completing the square for $y^2 + 14\,y + 45$

$(y^2 + 14\,y + 49) + 45 - 49$

$(y^2 + 14\,y + 49) - 4$

$(y + 7)^2 - 4 = -4\,x$

$(y + 7)^2 = -4\,x + 4$

$(y - k)^2 = 4\,p(x - h)$

$(y - (-7))^2 = -4\,(x - 1)$

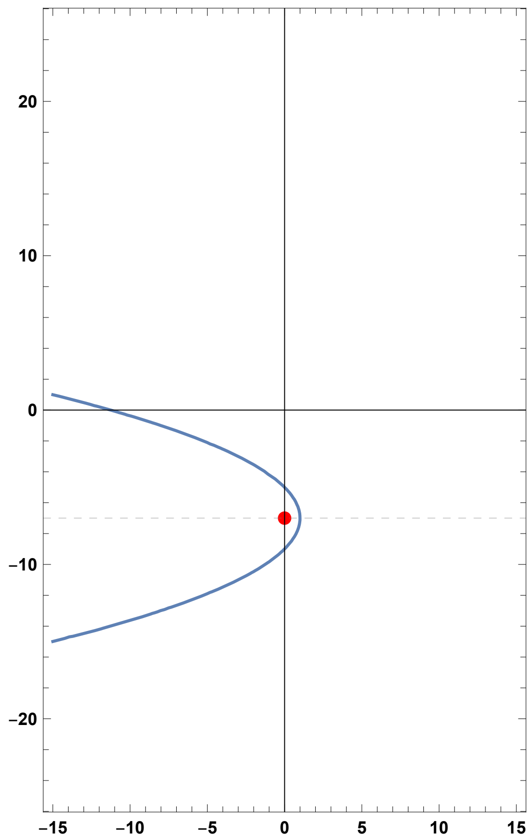
$(y - (-7))^2 = 4\,(-1)\,(x - 1)$

$k = -7,\; h = 1,\; p = -1$

Vertex = $(1, -7)$, Focus$(0, -7)$

In[ ]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{0, -7}]}];
parabola = ContourPlot[{y^2 + 14 y + 4 x + 45 == 0}, {x, -15, 15}, {y, -25, 25},
   Axes → True, GridLines → {None, {{-7, Dashed}}}, AspectRatio → Automatic];
Show[parabola,
 focus]
```

Out[ ]=



$4x^2 + 40x + y + 106 = 0$

$-y = 4x^2 + 40x + 106$

Completing the square for $4x^2 + 40x + 106$

$4(x^2 + 10x) + 106$

$4(x^2 + 10x + 25) + 106 - 100$

$4(x+5)^2 + 6 = -y$

$4(x+5)^2 = -y - 6$

$(x+5)^2 = -\frac{1}{4}y - \frac{3}{2}$

$(x-h)^2 = 4p(y-k)$

$(x-(-5))^2 = -\frac{1}{4}(y+6)$

$(x-(-5))^2 = 4\left(-\frac{1}{16}\right)(y-(-6))$

$h = -5,\ k = -6,\ p = -\frac{1}{16}$

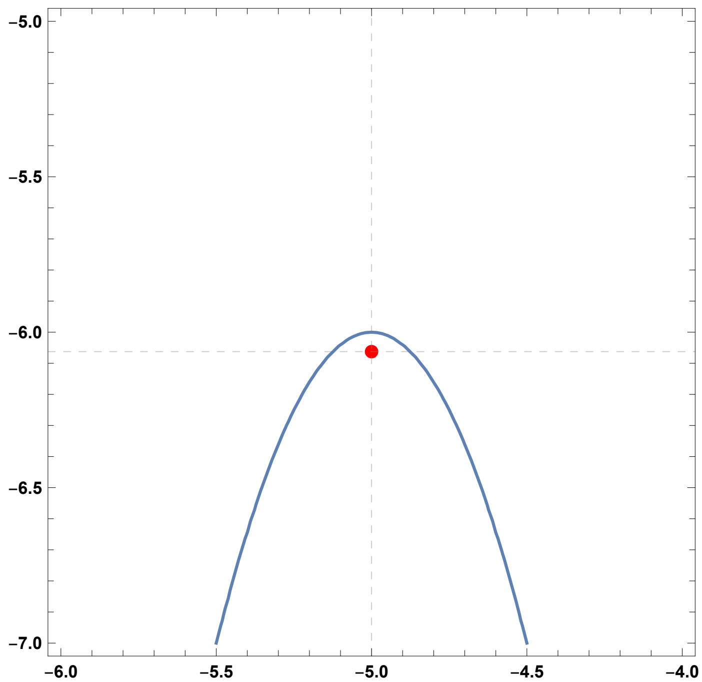
Vertex $= (-5,\ -6)$, Focus $= \left(-5,\ -6\frac{1}{16}\right)$

```
In[∘]:= focus = Graphics[{PointSize[Large], Red, Point[{-5, -6 1/16}]}];

parabola =
  ContourPlot[{-y == 4 x^2 + 40 x + 106}, {x, -6, -4}, {y, -5, -7}, Axes → True,
   GridLines → {{{-5, Dashed}}, {{-97/16, Dashed}}}, AspectRatio → Automatic];

Show[parabola,
 focus]
```

Out[∘]=



$y = 40x - 97 - 4x^2$

$y = -4x^2 + 40x - 97$

Completing the square for $-4x^2 + 40x - 97$

$-4(x^2 - 10x) - 97$

$-4(x^2 - 10x + 25) + 3$

$-4(x-5)^2 + 3 = y$

$-4(x-5)^2 = y - 3$

$(x-5)^2 = -\frac{1}{4}(y-3)$

$(x-h)^2 = 4p(y-k)$

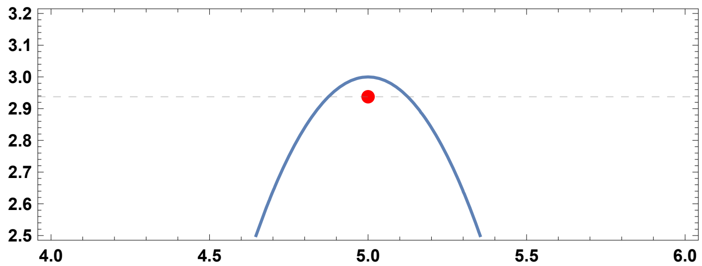
$(x-5)^2 = 4\left(-\frac{1}{16}\right)(y-3)$

$h = 5,\ y = 3,\ p = -\frac{1}{16}$

Vertex $= (5,\ 3)$, Focus $= \left(5,\ 2\frac{15}{16}\right)$

```
In[•]:= focus = Graphics[{PointSize[Large], Red, Point[{5, 2 15/16}]}];
parabola = ContourPlot[{y == 40 x - 97 - 4 x^2}, {x, 4, 6}, {y, 2.5, 3.2},
   Axes → True, GridLines → {None, {{2 15/16, Dashed}}}, AspectRatio → Automatic];
Show[parabola,
 focus]
```

Out[•]=

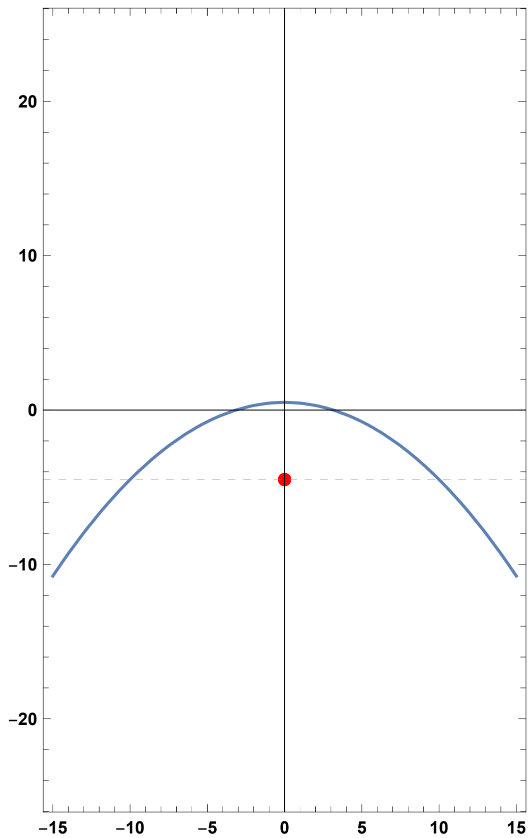


$x^2 + 20y = 10$

$$(x-h)^2 = 4p(y-k)$$
$$(x-0)^2 = -20y + 10$$
$$(x-0)^2 = -20\left(y - \tfrac{1}{2}\right)$$
$$(x-0)^2 = 4(-5)\left(y - \tfrac{1}{2}\right)$$
$$h = 0,\ k = \tfrac{1}{2},\ p = -5$$
$$\text{Vertex} = \left(0,\ \tfrac{1}{2}\right),\ \text{Focus} = (0,\ -4.5)$$

In[∘]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{0, -4.5}]}];
parabola = ContourPlot[{x^2 + 20 y == 10}, {x, -15, 15}, {y, -25, 25}, Axes → True,
   GridLines → {None, {{-4.5, Dashed}}}, AspectRatio → Automatic];
Show[parabola,
 focus]
```

Out[∘]=



$4x^2 + 4x + 4y + 1 = 0$

$4x^2 + 4x + 1 = -4y$

Completing the square for $4x^2 + 4x + 1$

$4(x^2 + x) + 1$

$4(x^2 + x + 1/4) + 1 - 1$

$4\left(x + \frac{1}{2}\right)^2$

$4\left(x + \frac{1}{2}\right)^2 = -4y$

$\left(x + \frac{1}{2}\right)^2 = -y$

$\left(x + \frac{1}{2}\right)^2 = -1(y)$

$\boldsymbol{(x - h)^2 = 4p(y - k)}$

$\left(x + \frac{1}{2}\right)^2 = 4\left(-\frac{1}{4}\right)(y - 0)$

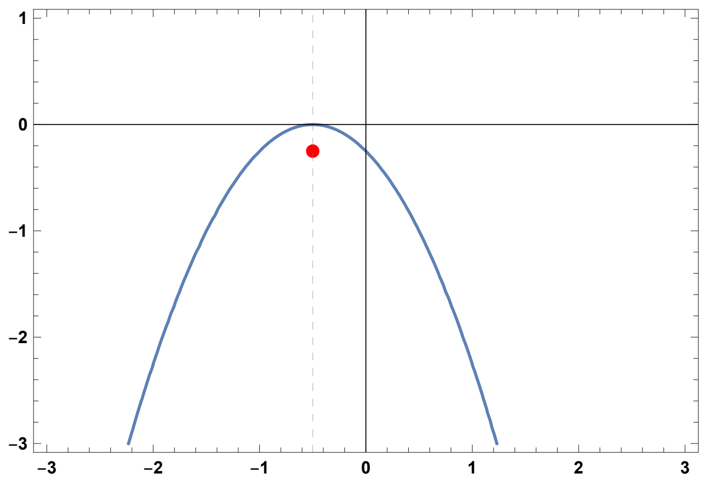
$\left(x - \left(-\frac{1}{2}\right)\right)^2 = 4\left(-\frac{1}{4}\right)(y - 0)$

$h = -\frac{1}{2},\ k = 0,\ p = -\frac{1}{4}$

Vertex$\left(-\frac{1}{2},\ 0\right)$, Focus$\left(-\frac{1}{2},\ -\frac{1}{4}\right)$

In[]:=
```
focus = Graphics[{PointSize[Large], Red, Point[{-1/2, -1/4}]}];
parabola = ContourPlot[{4 x^2 + 4 x + 4 y + 1 == 0}, {x, -3, 3}, {y, -3, 1},
   Axes → True, GridLines → {{{-0.5, Dashed}}, None}, AspectRatio → Automatic];
Show[parabola,
 focus]
```

Out[]=



Find an equation of the parabola that satisfies the given conditions.

Focus $F(2,\ 0)$; Directrix $x\ =\ -2$

Focus is always 2p from the directrix. This parabola opens to the right and axis is on X.

$p = |(-2-2)/2| = 2,$ vertex $= (0, 0)$
$(y-k)^2 = 4p(x-h)$
$(y-0)^2 = 4\,(2)\,(x-0)$
$y^2 = 8x$

Focus $F(0,\ -4)$; Directrix $y\ =\ 4$

$p = -4,$ vertex $= (0, 0)$
$(x-h)^2 = 4p(y-k)$
$x^2 = -16y$

Vertex $V(3,\ -5)$; Directrix $x = 2$

$p = 1$
$(y-k)^2 = 4p(x-h)$
$(y-(-5))^2 = 4\,(x-3)$

$(y+5)^2 = 4x - 12$
$(y+5)(y+5) = 4x - 12$
$y^2 + 10y + 25 = 4x - 12$
$y^2 + 10y + 37 = 4x$

Vertex V(-2, 3); Directrix x = 5

p=-7
$(y-k)^2 = 4p(x-h)$
$(y-3)^2 = 4(-7)(x+2)$
$(y-3)(y-3) = -28(x+2)$
$y^2 - 6y + 9 = -28x - 56$
$y^2 - 6y + 65 = -28x$

Vertex V(-1, 0); Focus F (-4, 0)

$p = -3$
$(y-k)^2 = 4p(x-h)$
$y^2 = 4(-3)(x+1)$
$y^2 = -12x - 12$

Vertex V(1, -2); Focus F(1, 0)

$p = 2$
$(x-h)^2 = 4p(y-k)$
$(x-1)^2 = 8y + 16$
$x^2 - 2x + 1 = 8y + 16$
$x^2 - 2x - 15 = 8y$

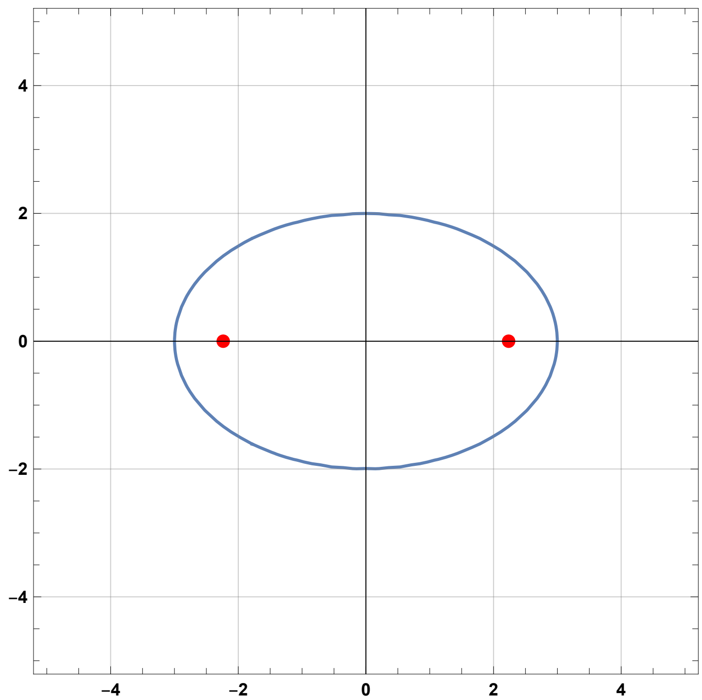
Find the vertices and the foci of the ellipse. Sketch its graph, showing the foci.

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
$a = 3, \; b = 2$
Vertices $= (\pm 3, \; 0)$
$c = \sqrt{a^2 - b^2}$; $c = \sqrt{9-4} = \sqrt{5}$
foci $= \left(\pm\sqrt{5}, \; 0\right)$

In[ ]:=
```
ellipse = ContourPlot[{x^2/9 + y^2/4 == 1},
    {x, -5, 5}, {y, -5, 5}, GridLines → Automatic, Axes → True];
focus1 = Graphics[{PointSize[Large], Red, Point[{-√5, 0}]}];
focus2 = Graphics[{PointSize[Large], Red, Point[{√5, 0}]}];
Show[ellipse, focus1, focus2]
```

Out[ ]=



$4x^2 + y^2 = 16$

$\frac{x^2}{4} + \frac{y^2}{16} = 1$

$a = 4, \ b = 2$

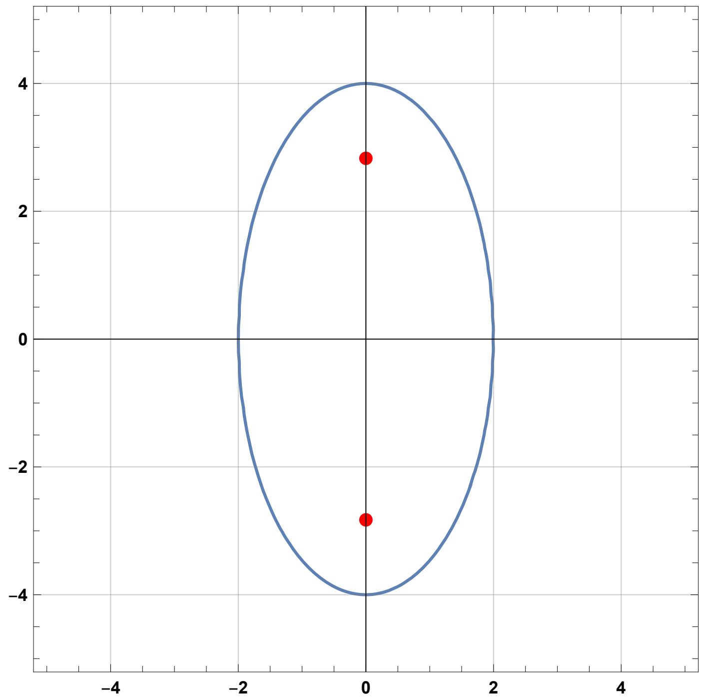
Vertices = $(0, \ \pm 4)$

$c = \sqrt{16 - 4} = 2\sqrt{2}$

foci = $\left(0, \ \pm 2\sqrt{2}\right)$

In[∘]:=
```
ellipse = ContourPlot[{4 x^2 + y^2 == 16},
    {x, -5, 5}, {y, -5, 5}, GridLines → Automatic, Axes → True];
focus1 = Graphics[{PointSize[Large], Red, Point[{0, -2 √2}]}];
focus2 = Graphics[{PointSize[Large], Red, Point[{0, 2 √2}]}];
Show[ellipse, focus1, focus2]
```

Out[∘]=



$5x^2 + 2y^2 = 10$

$\frac{x^2}{2} + \frac{y^2}{5} = 1$

$a = \sqrt{5}$, $b = \sqrt{2}$

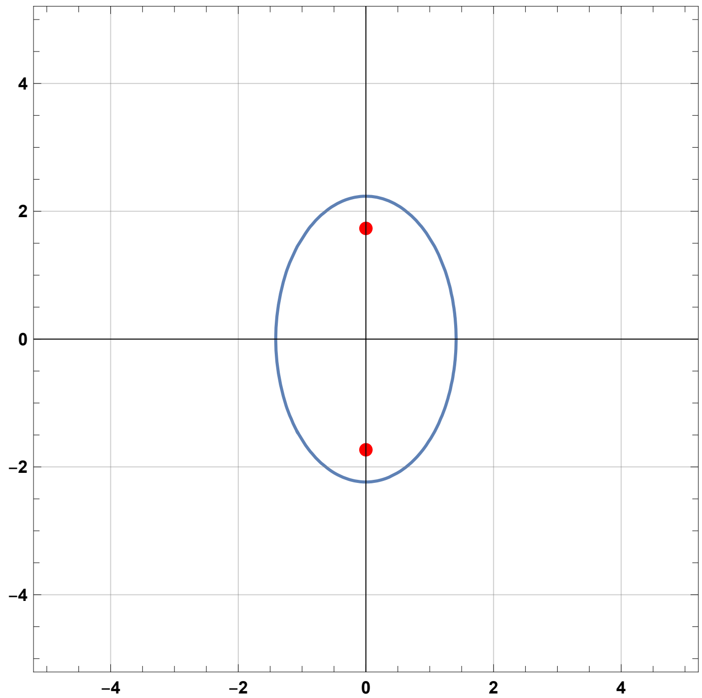
Vertices = $\left(0, \pm\sqrt{5}\right)$

$c = \sqrt{5-2} = \sqrt{3}$

foci = $\left(0, \pm\sqrt{3}\right)$

In[ ]:=
```
ellipse = ContourPlot[{5 x^2 + 2 y^2 == 10},
   {x, -5, 5}, {y, -5, 5}, GridLines → Automatic, Axes → True];
focus1 = Graphics[{PointSize[Large], Red, Point[{0, -√3}]}];
focus2 = Graphics[{PointSize[Large], Red, Point[{0, √3}]}];
Show[ellipse, focus1, focus2]
```

Out[ ]=



$4x^2 + 25y^2 = 1$

The graph is an ellipse with center at the origin.

To find x intercepts, let $y = 0$

$4x^2 = 1;\ x^2 = \frac{1}{4};\ x = \sqrt{\frac{1}{4}}\ ;\ x = \pm\frac{1}{2};$

To find y intercepts, let $x = 0$

$25y^2 = 1;\ y = \pm\frac{1}{5}$

$\frac{1}{2} > \frac{1}{5}$ hence major axis is a X axis

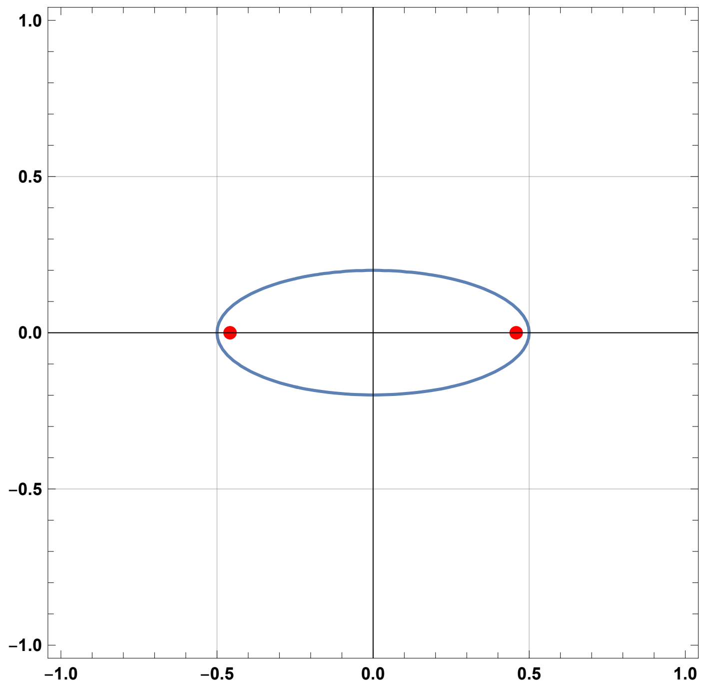
$c^2 = a^2 - b^2 = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{5}\right)^2 = \frac{1}{4} - \frac{1}{25} = \frac{25-4}{100} = \frac{21}{100}$

$c = \sqrt{\frac{21}{100}} = \frac{\sqrt{21}}{10}$

Vertices $= \left(\pm\frac{1}{2},\ 0\right)$, Foci $= \left(\pm\frac{\sqrt{21}}{10},\ 0\right)$

```
In[•]:= ellipse = ContourPlot[{4 x^2 + 25 y^2 == 1}, {x, -1, 1}, {y, -1, 1},
    GridLines → Automatic, Axes → True, AspectRatio → Automatic];
  focus1 = Graphics[{PointSize[Large], Red, Point[{-Sqrt[21]/10, 0}]}];
  focus2 = Graphics[{PointSize[Large], Red, Point[{Sqrt[21]/10, 0}]}];
  Show[ellipse, focus1, focus2]
```

Out[•]=



$10y^2 + x^2 = 5$

$x = \pm\sqrt{5}$, $y^2 = \frac{1}{2}$, $y = \pm\sqrt{\frac{1}{2}}$, $y \approx 0.707$

$a = x$, $b = y$

$c^2 = a^2 - b^2$, $c^2 = 5 - \frac{1}{2} = \frac{9}{2}$

$c = \pm\sqrt{\frac{9}{2}} = \pm\frac{3}{\sqrt{2}}$

Vertices = $\left(\pm\sqrt{5}, 0\right)$; Foci = $\left(\pm\frac{3}{\sqrt{2}}, 0\right)$

$4x^2 + 9y^2 - 32x - 36y + 64 = 0$

$4(x^2 - 8x) + 9(y^2 - 4y) = -64$

$4(x^2 - 8x + 16) + 9(y^2 - 4y + 4) = -64 + 64 + 36$

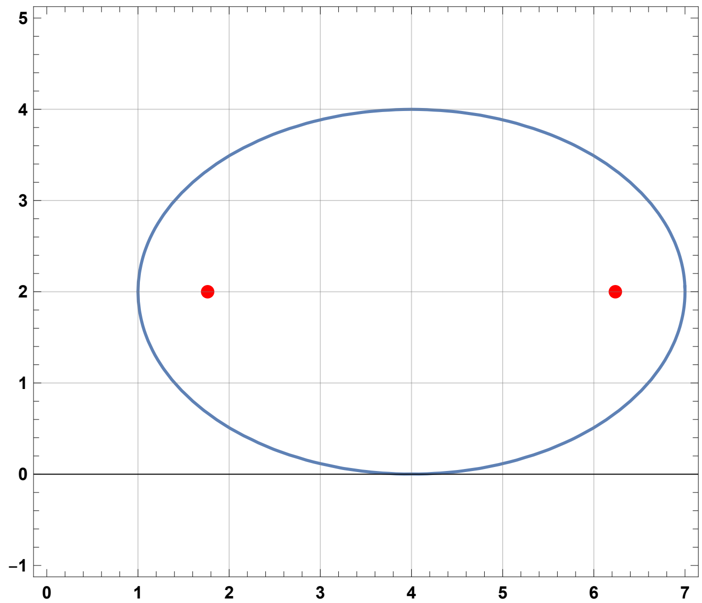
$4(x - 4)^2 + 9(y - 2)^2 = 36$

$\frac{(x-4)^2}{9} + \frac{(y-2)^2}{4} = 1$

$h = 4$, $k = 2$, $a = 3$, $b = 2$, $c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$

Vertices = (1, 2), (7, 2), Foci = $\left(4 \pm \sqrt{5}, 2\right)$

In[•]:=
```
ellipse = ContourPlot[{4 x^2 + 9 y^2 - 32 x - 36 y + 64 == 0}, {x, 0, 7},
   {y, -1, 5}, GridLines → Automatic, Axes → True, AspectRatio → Automatic];
focus1 = Graphics[{PointSize[Large], Red, Point[{4 + √5, 2}]}];
focus2 = Graphics[{PointSize[Large], Red, Point[{4 - √5, 2}]}];
Show[ellipse, focus1, focus2]
```

Out[•]=


$4x^2 + 9y^2 + 24x + 18y + 9 = 0$

$4(x^2 + 6x) + 9(y^2 + 2y) = -9$
$4(x^2 + 6x + 9) + 9(y^2 + 2y + 1) = 36$
$4(x+3)^2 + 9(y+1)^2 = 36$
$\frac{(x+3)^2}{9} + \frac{(y+1)^2}{4} = 1$
$h = -3,\ k = -1,\ a = \pm 3,\ b = \pm 2,\ c = \sqrt{5}$
Vertices = (−6, −1), (0, −1), Foci = $\left(-3 \pm \sqrt{5}, -1\right)$

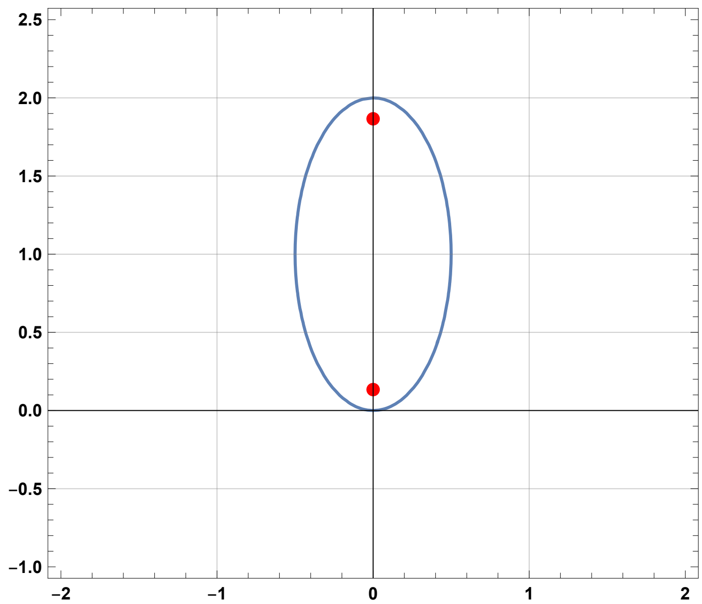
$4x^2 + y^2 = 2y$

$4x^2 + y^2 - 2y = 0$
$4(x-0)^2 + (y^2 - 2y + 1) = 1$
$4(x-0)^2 + (y-1)^2 = 1$
$\frac{(x-0)^2}{\frac{1}{4}} + (y-1)^2 = 1$
$h = 0,\ k = 1,\ a = \pm 1,\ b = \pm \frac{1}{2}$
$c^2 = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$
$c = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$

Vertices = (0, 2), (0, 0), Foci = $\left(0,\ 1 \pm \frac{\sqrt{3}}{2}\right)$

In[ ]:=
```
ellipse = ContourPlot[{4 x^2 + y^2 == 2 y}, {x, -2, 2}, {y, -1, 2.5},
   GridLines → Automatic, Axes → True, AspectRatio → Automatic];
focus1 = Graphics[{PointSize[Large], Red, Point[{0, 1 + Sqrt[3]/2}]}];
focus2 = Graphics[{PointSize[Large], Red, Point[{0, 1 - Sqrt[3]/2}]}];
Show[ellipse, focus1, focus2]
```

Out[ ]=



Find an equation for the ellipse that has its center at the origin and satisfies the given conditions.

Vertices V(±8, 0), Foci F(±5, 0)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a = 8,\ c = 5,\ c^2 = a^2 - b^2,\ b^2 = a^2 - c^2,\ b^2 = 64 - 25 = 39$

$\frac{x^2}{64} + \frac{y^2}{39} = 1$

Vertices V(0, ±7), Foci F(0, ±2)

$a = \pm 7,\ c = \pm 2$

$c^2 = a^2 - b^2,\ b^2 = 49 - 4 = 45$

$\frac{x^2}{45} + \frac{y^2}{49} = 1$

Vertices V(0, ±5), Minor axis of length 3

$a = \pm 5, \; b = \pm 1.5$

$\frac{x^2}{(1.5)^2} + \frac{y^2}{25} = 1$