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Example Test

Read carefully the points below

- This test is intended to assess your active knowledge and your command of some basic mathematical skills at this moment.
- Use of a calculator or table of formulas is not permitted.
- The test consists of 22 multiple-choice questions. For each question exactly one of the given possibilities is correct.
- The test will last one hour.

1. One of the following statements is not true. Which one?

   a. $\dfrac{5}{2^{-3}} = 40$ b. $64^{\frac{2}{3}} = 16$ c. $\dfrac{1}{\frac{1}{2}+\frac{1}{3}} = 5$ d. $\sqrt{\frac{1}{11}} = \frac{1}{11}\sqrt{11}$

2. $\sqrt[3]{a}\,\sqrt[5]{a}$ is equal to

   a. $\sqrt[15]{a}$ b. $\sqrt[8]{a}$ c. $\sqrt[8]{a^2}$ d. $\sqrt[15]{a^8}$

3. Which number is the largest one?

   a. $\sqrt{2}$ b. $\sqrt[3]{4}$ c. $\sqrt[4]{8}$ d. $\sqrt[5]{16}$

4. The expression $\dfrac{a}{2-a} + \dfrac{a}{2+a}$ is equal to

   a. $\dfrac{4\,a}{4-a^2}$ b. $\dfrac{2\,a^2}{a^2-4}$ c. $\dfrac{2\,a^2}{4-a^2}$ d. $\dfrac{4\,a}{a^2-4}$

please turn over

5. The expression $\left(\sqrt{11}-\sqrt{7}\right)^2-\left(\sqrt{11}+\sqrt{7}\right)^2$ is equal to

a. 0 b. $36-4\sqrt{77}$ c. -14 d. $-4\sqrt{77}$

6. How many different zeros has the polynomial $f(x)=x^3-8\,x^2+16\,x$?

a. 3 b. 2 c. 1 d. 0

7. The expression $\dfrac{\ln(\sqrt{\mathrm{e}\sqrt{\mathrm{e}}})}{\ln(\sqrt{\mathrm{e}})}$ is equal to

a. $\sqrt{\mathrm{e}}$ b. $\dfrac{1}{2}$ c. $\dfrac{3}{2}$ d. $\dfrac{1}{4}$

8. If $3\ln(y)=x^3+\ln(8)$, then y is equal to

a. $2\,\mathrm{e}^{x}$ b. $8\,\mathrm{e}^{\frac{1}{3}x^3}$ c. $\dfrac{8}{3}\,\mathrm{e}^{\frac{1}{3}x^3}$ d. $2\,\mathrm{e}^{\frac{1}{3}x^3}$

9. If $f(x)=x^2$ and $g(x)=1+x$, then $f(g(x))$ is equal to

a. $1+x^2$ b. $(1+x)^2$ c. $x^2\,(1+x)$ d. $x^2+(1+x)$

10. Someone is asked to solve the next two equations:

(1) $\ln(x^2)=4$,

(2) $(\ln(x))^2=4$

He solves these equations as follows:

(1) $\ln(x^2)=4\rightarrow 2\ln(x)=4\rightarrow \ln(x)=2\rightarrow x=\mathrm{e}^2$

(2) $(\ln(x))^2=4\rightarrow \ln(x)=2\rightarrow x=e^2$

Which statement is true?

a. Only solution (1) is complete

b. Only solution (2) is complete

c. Both solutions are complete

d. Both solutions are incomplete.

11. The expression $\ln(\mathrm{e}^5-\mathrm{e}^3)$ is equal to

a. 2 b. $\dfrac{5}{3}$ c. $3+\ln(\mathrm{e}^2-1)$ d. $3-\ln(\mathrm{e}^2-1)$

12. The function f is given by $f(x) = \dfrac{\sqrt{1-x^2}}{{}^{10}\log(x)}$.

The domain of f consists of all x such that

a. $0 < x$

b. $0 < x < 1$

c. $-1 \leq x \leq 1$

d. $-1 \leq x \leq 1$ ánd $x \neq 0$

13. The expression $7^{\,{}^{49}\log(3)}$ is equal to

a. ${}^{7}\log(9)$

b. $\sqrt{3}$

c. ${}^{7}\log(\sqrt{3})$

d. 9

14. Solve the equation $2\,x + 1 = \sqrt{x^2+5}$.
The equation has

a. just one solution x_1. It is true that $x_1 > 1$.

b. no solutions

c. just one solution x_1. It is true that $0 < x_1 < 1$.

d. two solutions

15. If $h(x) = f(g(x))$, then $h'(x)$ is equal to

a. $f'(g(x)) \cdot x$

b. $f'(g(x)) \cdot g'(x)$

c. $f'(g(x)) + f(g'(x))$

d. $f'(g(x)) \cdot g(x) + f(g'(x)) \cdot g'(x)$

16. If $y = \sqrt[3]{x^3+8}$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ is equal to

a. $\dfrac{3\,x^2}{2\,\sqrt[3]{x^3+8}}$

b. $\dfrac{x^2}{\sqrt[3]{(x^3+8)^2}}$

c. 1

d. $\dfrac{1}{3\,\sqrt[3]{(x^3+8)^2}}$

17. For $k > 0$ the integral $\displaystyle\int_k^{3k} \frac{1}{x}\,\mathrm{d}x$ can be simplified as

a. $\ln(3)$

b. $\ln(2\,k)$

c. ${}^{k}\log(3\,k)$

d. $\dfrac{8}{9\,k^2}$

18. The integral $\displaystyle\int_1^2 \left(\frac{1}{x}\right)^3 \mathrm{d}x$ is equal to

a. $\dfrac{1}{4}\ln^4(2)$

b. $\ln(8)$

c. $\dfrac{15}{16}$

d. $\dfrac{3}{8}$

please turn over

19. The function f is given by $f(x) = \sin(a\,x) + \cos(a\,x)$ for some $a \neq 0$.
The maximum value of $f(x)$ is

a. 1
b. 2
c. $\sqrt{2}$
d. a value dependent on a.

20. The function f, given by $f(x) = \cos^2(\frac{1}{2}\,x) - \sin^2(\frac{1}{2}\,x)$, has

a. period $2\,\pi$
b. period $\dfrac{1}{2}\,\pi$
c. period π
d. a horizontal line as its graph

21. The derivative of $f(x) = (\cos(x) + \sin(x))^2$ is

a. 0
b. $2\,\cos^2(x) - 2\,\sin^2(x)$
c. $2\,\sin^2(x) - 2\,\cos^2(x)$
d. $-2\sin(x)\,\cos(x)$

22. An antiderivative of $f(x) = \cos(x)\sin(x)$ is

a. $\dfrac{1}{2}\,\cos^2(x)$
b. $\dfrac{1}{2}\,\sin^2(x)$
c. $-\sin^2(x) + \cos^2(x)$
d. $-\dfrac{1}{4}\,\cos^2(x)\,\sin^2(x)$

end of the test

Notes to the Test

Below you will find the correct answers to the example test. So you can discover what you can do well and where there are gaps in your mathematical knowledge. Below are notes on each problem and references to practice material. You can use these to brush up on the theory behind the troublesome problems and work some exercises. Of some formulas we say that it is useful to know them by heart. The obvious advice is, of course, to do something about that, soon.

Exponents/powers (problems 1, 2, 3 and 7)

In this Handout, following these notes, there is a chapter called *Review of Algebra*. Under the heading of *Exponents* you will find a summary of the definitions and rules for working with exponents. If you had any difficulties with these problems then read that section and do some problems from the range 83–100 in the Review (answers at the end).

Fractions and brackets (problems 1, 4, 5 and 9)

These problems are about adding, subtracting and simplifying fractions. This also involves manipulating brackets and factoring numbers. See under *Fractions* and *Factoring* in the Review for a summary. Problems 17–28 and 49–54 are recommended.

Quadratic and higher-order equations (problems 6, 12 and 14)

Quadratic equations occur very frequently; you should be able to solve them without difficulty — see problems 61–68 of the *Review*. In the review there is also an explanation of the quadratic formula. You are probably used to solve inequalities with the aid of a Graphic Calculator. It is useful to be able to solve simple cases without that device, for example using a quick sketch or a table of signs. In Appendix A of Stewart's book you will find, under *Inequalities*, more about this subject and in problems 13–38 plenty to practice with. In the test we did not deal with the *absolute value*. If you do not know what that is (anymore) then read in Appendix A the section of that name. For cubic equations there is a general 'cubic formula' (known as Cardan's formula) but we do not assume that you know it. We do expect that you are able to note and perform simple steps like "bringing x outside the brackets".

Roots (problems 2, 3, 5,7, 12 and 14)

Roots (square, cube, etc) are collectively called radicals (radix is the Latin word for root), you will find their theory in the review in the section of that name. Suitable problems are 95–100.

Exponentials and logarithms (problems 7, 8, 10, 11 and 13)

You can find the definitions and properties of exponential and logarithmic functions in sections 1.5 and 1.6 (*Logarithmic functions* and *Natural logarithms*) of Stewart's book. Suitable problems are 35–41 and 49–52 from 1.6.

Differentiation (problems 15, 16 and 21)

We expect you to know a few standard derivatives by heart: those of x^n (also for negative and rational n), e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, ... We also expect that you know the rules for differentiating sums, differences, products and quotients. Finally there is the *chain rule*, you must know that one too, otherwise you will keep making unnecessary mistakes. The theory can be found in sections 2.7, 2.8 and 2.9 of Stewart's book; the various rules are in 3.1, 3.2 and 3.4. In section 3.5 you will find a range of problems, 7–34, in which the rules are combined. Section 2.8 will explain in more detail how to set up an equation of a tangent line at a point on a graph; see example 2 and in problems 7–10 of that section.

Integration (problems 17, 18 and 22)

The Analysis course will cover advanced integration techniques extensively. These problems deal with simple functions that can be integrated using the basic rules. Here too we expect that you know how to integrate a number of standard functions — x^n, e^x, $\sin x$, $\cos x$ — and simple combinations thereof in your head. In section 5.4 of Stewart's book you will find problems with elementary integrals, for example 5–9, 17–22 and 25–28.

Trigonometry (problems 19, 20, 21 and 22)

Trigonometry uses a lot of formulas, called *trigonometric identities*, wherein one expression is transformed into another. The tables of formulae in the front (and back) of Stewart's book (they are detachable) contain many of them and we do not expect you to know them all by heart. A few of them occur so often that it is impracticable to have to look them up each time you need them. This is especially true of the rule for $\sin(-x)$, $\cos(-x)$, $\tan(-x)$, $\sin(\frac{\pi}{2} - x)$ and $\cos(\frac{\pi}{2} - x)$. It is easy to reconstruct these formulas, and also those that say what happens when you add π to (or subtract it from) the argument of the sine or cosine, by sketching the graphs of the functions or of the unit circle. The double-angle formulas $\sin 2x$ and $\cos 2x$ are less easy to derive but they are used a lot. Our advice is to learn them by heart — if you have not done so yet. The definitions and many properties of the trigonometric functions and their graphs can be found in Appendix D of Stewart's book. It is very useful to verify simple identities using graphs or the unit circle. We strongly recommend that you have the standard values of sine and cosine for 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$ at your fingertips; they are needed virtually everywhere.

Additional study material

An important source of explanations and exercises is the book *Basisboek Wiskunde* by Jan van de Craats and Rob Bosch (ISBN 90-430-1156-8, €29,95). Though this book is in Dutch we recommend it as an opportunity to get acquainted with Dutch mathematical terminology (for starters: 'Wiskunde' is Dutch for 'Mathematics'). Parts of the book can be found at `http://staff.science.uva.nl~craats`. After the notes we give a table with for each problem suitable problem sets from the book. *Foundation Maths* by Anthony Croft and Robert Davison (ISBN 0-131-97921-3) is recommended for students who prefer to studie in English. LR students are provided with this book.

Finally there is also practice material in the *Toetsbank Analyse*, which can be found on the TU *Blackboard* site, under 'courses'; its number is 0000.

References to *Basisboek Wiskunde*

This table shows, per problem, where the relevant material can be found in *Basisboek Wiskunde* by J. van de Craats en R. Bosch.

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14	124, 128
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17	150, 198
18	192, 198
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20	140
21	138, 140, 170
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Answers Test

1	c	12	b
2	d	13	b
3	d	14	c
4	a	15	b
5	d	16	b
6	b	17	a
7	c	18	d
8	d	19	c
9	b	20	a
10	d	21	b
11	c	22	b