Equations used in model

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1 Introduction

This documents the equations used in the lid-driven model. Please refer to "Numerical Simulation in Fluid Dynamics: A Practical Introduction", by Michael Griebel et. al., for more details

2 Equations used in the model

All the variables here are dimensionless variables
i,j : subscripts, indexing the x,y positions
n : superscripts, indexing the time step

2.1 comp_delt.f90

Adaptive time step

$$\delta t = \tau \ min \left(\frac{Re}{2} \left(\frac{1}{\delta x^2} + \frac{1}{\delta y^2} \right)^{-1}, \frac{\delta x}{|u_{max}|}, \frac{\delta y}{|v_{max}|} \right) \tag{1}$$

2.2 comp_fg.f90

Calculate a term in momentum equation: the part that is not related with pressure.

$$\begin{aligned} F_{i,j} = u_{i,j} + \delta t \left(\frac{1}{Re} \left(\left[\frac{\partial^2 u}{\partial x^2} \right]_{i,j} + \left[\frac{\partial^2 u}{\partial y^2} \right]_{i,j} \right) - \left[\frac{\partial \left(u^2 \right)}{\partial x} \right]_{i,j} - \left[\frac{\partial \left(uv \right)}{\partial y} \right]_{i,j} + g_x \right) \\ i = 1, ..., i_{max} - 1, \\ j = 1, ..., j_{max}, \end{aligned} \tag{2}$$

$$\begin{aligned} G_{i,j} = v_{i,j} + \delta t \left(\frac{1}{Re} \left(\left[\frac{\partial^2 v}{\partial x^2} \right]_{i,j} + \left[\frac{\partial^2 v}{\partial y^2} \right]_{i,j} \right) - \left[\frac{\partial \left(uv \right)}{\partial x} \right]_{i,j} - \left[\frac{\partial \left(v^2 \right)}{\partial y} \right]_{i,j} + g_y \right) \\ i = 1, ..., i_{max}, \\ j = 1, ..., j_{max} - 1, \end{aligned} \tag{3}$$

2.3 comp_rhs.f90

rhs term. used in Poisson Equation

$$rhs_{i,j} = \frac{1}{\delta t}\left(\frac{F_{i,j}^{(n)} - F_{i-1,j}^{(n)}}{\delta x} + \frac{G_{i,j}^{(n)} - G_{i,j-1}^{(n)}}{\delta y}\right) \\ i = 1, ..., i_{max}, \\ j = 1, ..., j_{max}, \tag{4}$$

2.4 poisson.f90

For incompressible fluid, pressure follows a Poisson Equation. $\nabla^2 P = rhs$ (Use momentum equations and mass equations to get here). Use successive overrelaxation (SOR) to solve this Poisson Equation.
Please refer to `http://en.wikipedia.org/wiki/Successive_over-relaxation` for SOR

Boundary Be careful with boundary values for F and G. We didn't define boundaries values of F and G in comp_fg.f90, for example, $F_{0,j}$. However, the problem is easy to solve. With continuous pressure condition, momentum equation and Poission equation, the boundary values will disappear. Since boundary F,G values are only used here, so we can actually set arbitrary values as we will.

2.5 adapt_uv.f90

Update u,v for the next time step

$$u_{i,j}^{n+1} = F_{i,j}^{n} - \frac{\delta t}{\delta x}\left(p_{i+1,j}^{n+1} - p_{i,j}^{n+1}\right) \\ i = 1, ..., i_{max} - 1 \\ j = 1, ..., j_{max} \tag{5}$$

$$v_{i,j}^{n+1} = G_{i,j}^{n} - \frac{\delta t}{\delta x}\left(p_{i,j+1}^{n+1} - p_{i,j}^{n+1}\right) \\ i = 1, ..., i_{max} \\ j = 1, ..., j_{max} - 1 \tag{6}$$

3 Conclusion

This document is subject to future updates.