Capacitated Vehicle Routing Problem (CVRP) with Max-Min Ant System

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1 Introduction

The Capacitated Vehicle Routing Problem (CVRP) is a well-known optimization problem in logistics, where the objective is to find the optimal set of routes for a fleet of vehicles that must serve a number of customers, each with a specific demand, while respecting vehicle capacity limits. The goal is to minimize the overall travel cost, typically represented by distance or time. CVRP is crucial in applications involving transportation, delivery services, and supply chain management, as it directly impacts operational efficiency and cost reduction.

2 Proposed Solution

To solve the CVRP, we employed a Max-Min Ant System (MMAS) approach, which is an extension of the Ant Colony Optimization (ACO) algorithm. MMAS is well-suited for combinatorial optimization problems like CVRP due to its use of pheromone trails and heuristic information to guide solution construction.

In MMAS, artificial ants build solutions by probabilistically choosing the next customer to visit based on a combination of pheromone levels and heuristic desirability (inverse of distance). The pheromone values are updated iteratively, favoring paths that contribute to lower-cost solutions. Key parameters in the MMAS include:

- **Pheromone Influence (α)**: The weight given to pheromone trails in selecting the next node.
- **Heuristic Influence (β)**: The weight given to the inverse of distance between nodes.
- **Pheromone Evaporation Rate (ρ)**: Controls the rate at which pheromone trails evaporate, encouraging exploration of new solutions.

The MMAS also employs upper and lower limits on pheromone levels, which are updated using the following formula:

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \Delta\tau_{ij} \tag{1}$$

where τ_{ij} represents the pheromone level on the edge between nodes i and j, ρ is the evaporation rate, and $\Delta\tau_{ij}$ is the pheromone deposited by the ants.

The algorithm iteratively improves upon solutions, with ants collectively working to find routes that minimize the overall distance while respecting vehicle capacity constraints.

3 Dataset Description

The MMAS algorithm was tested on three CVRP instances from the TSPLIB library: `eil7.vrp`, `eil13.vrp`, and `eil31.vrp`. These datasets represent different sizes and complexities, with varying numbers of nodes and demands. The vehicle capacity for each instance was chosen to ensure feasibility while providing a challenge for the algorithm.

4 System Specifications

The experiments were conducted on a machine with the following specifications:

- Processor: Apple M1 (MacBook Air)
- RAM: 8GB
- Operating System: macOS Monterey
- Software: GCC Compiler for C, TSPLIB dataset

5 Results

The MMAS algorithm was tested with three different configurations for the number of ants: 100, 200, and 300. The results for each dataset are presented in separate tables below.

5.1 Results for 100 Ants

Dataset	Best Cost
`eil7.vrp`	81
`eil13.vrp`	278
`eil31.vrp`	498

Table 1: Summary of results for 100 ants on CVRP datasets

5.2 Results for 200 Ants

Dataset	Best Cost
`eil7.vrp`	81
`eil13.vrp`	278
`eil31.vrp`	448

Table 2: Summary of results for 200 ants on CVRP datasets

5.3 Results for 300 Ants

Dataset	Best Cost
`eil7.vrp`	81
`eil13.vrp`	278
`eil31.vrp`	505

Table 3: Summary of results for 300 ants on CVRP datasets

6 Analysis

From the results in Tables 1, 2, and 3, we can observe the following trends:

- For the smaller instance `eil7.vrp`, the best cost remained the same (81) regardless of the number of ants used. This suggests that the problem size is small enough that increasing the number of ants does not yield significant improvement.
- For the larger instances, such as `eil31.vrp`, increasing the number of ants led to better solutions in some cases (e.g., cost decreased from 498 to 448 when increasing ants from 100 to 200), but the cost increased slightly when further increasing ants to 300.
- This indicates that while increasing the number of ants can improve solution quality by providing more exploration, there may also be diminishing returns or even negative effects if too many ants are used, possibly due to over-reinforcement of certain paths.

Further tuning of parameters like α, β, and ρ may yield improved results, especially for larger problem instances.

7 Conclusion

The Max-Min Ant System (MMAS) proved effective in solving the CVRP instances provided, with the number of ants playing a significant role in the quality of the solutions. Future work could focus on parameter optimization and hybrid approaches to further enhance the performance of the algorithm for larger problem instances.

References

[1] Reinelt, G. (1991). TSPLIB - A Traveling Salesman Problem Library. ORSA Journal on Computing.

[2] Dorigo, M., Stutzle, T. (2004). Ant Colony Optimization. MIT Press.