Introduction to Basic Fixed Income Securities
Question 1

Lottery payments
A major lottery advertises that it pays the winner $10 million. However, this prize money is paid at the rate of $500,000 each year (with the first payment being immediate) for a total of 20 payments. What is the present value of this prize at 10% interest compounded annually?

Correct
The PV is given by

$(1- (\frac{1}{1.1\text{^}20}))$

$$PV = A\sum_{i=0}^{T-1} \frac{1}{(1+r)^i} = \left(\frac{A(1+r)}{r}\right)\left(1-\frac{1}{1+r^T}\right) = \left(\frac{500{,}000(1+0.1)}{0.1}\right)\left(1-\frac{1}{1.01^{20}}\right) = 4.6825M$$

Question 2

Sunk Costs (Exercise 2.6 in Luenberger)
A young couple has made a deposit of the first month's rent (equal to $1,000) on a 6-month apartment lease. The deposit is refundable at the end of six months if they stay until the end of the lease. The next day they find a different apartment that they like just as well, but its monthly rent is only $900. And they would again have to put a deposit of $900 refundable at the end of 6 months. They plan to be in the apartment only 6 months. Should they switch to the new apartment? Assume an (admittedly unrealistic!) interest rate of 12% per month compounded monthly.

Stay
Switch

Correct
Compare the two alternatives: Stay in the original apartment, for an NPV, C_1, given by

$$C_1 = -\sum_{i=0}^{5} \frac{1000}{1.12^i} + \frac{1000}{1.12^6} \approx -4,222.$$

-4111

Take the new apartment where we assume a security deposit is again required. The NPV then is

$$C_2 = -900 - \sum_{i=0}^{5} \frac{900}{1.12^i} + \frac{900}{1.12^6} \approx -4700.$$

-5600

The couple should take the $1000 apartment.

Question 3

Relation between spot and discount rates
Suppose the spot rates for 1 and 2 years are $s_1 = 6.3\%$ and $s_2 = 6.9\%$ with annual compounding. Recall that in this course interest rates are always quoted on an annual basis unless otherwise specified. What is the discount rate d(0,2)?

Correct
From the relation in the slides we get

$$d(0,2) = \frac{1}{(1+s_2)^2} = 0.8751$$

Question 4

Relation between spot and forward rates
Suppose the spot rates for 1 and 2 years are s_1 = 6.3% and s_2 = 6.9% with annual compounding. Recall that in this course interest rates are always quoted on an annual basis unless otherwise specified. What is the forward rate, $f_{1,2}$ assuming annual compounding?

Correct
From the relation in the slides we get

$(1+s_2)^2 = (1+s_1)(1+f_{1,2}) \Rightarrow f_{1,2} = \frac{(1+s_2)^2}{(1+s_1)} - 1 = 0.075 = 7.5\%$

Question 5

Forward contract on a stock
The current price of a stock is \$400 per share and it pays no dividends. Assuming a constant interest rate of 8% per year compounded quarterly, what is the stock's theoretical forward price for delivery in 9 months?

Correct
The discount factor is $d(0,9) = \frac{1}{(1+r/4)^3} = \frac{1}{1.02^3}$. Therefore, the forward price is

$F = S_0/d(0,9) = 400(1.02)^3 = 424.4832$

Question 6

Bounds using different lending and borrowing rate
Suppose the borrowing rate r_B =10% compounded annually. However, the lending rate (or equivalently, the interest rate on deposits) is only 8% compounded annually. Compute the difference between the upper and lower bounds on the price of an perpetuity that pays \(A = 10,000\\)\$ per year.

Correct
The upper bound on the price of the annuity is $\frac{A}{r_L} = \frac{10000}{0.08} = 125,000.$

The lower bound on the price of the

annuity is $\frac{A}{r_B} = \frac{10000}{0.1} = 100,000.$

The difference

is $25,000.$

Question 7

Value of a Forward contract at an intermediate time

Suppose we hold a forward contract on a stock with expiration 6 months from now. We entered into this contract 6 months ago so that when we entered into the contract, the expiration was T = 1 year. The stock price 6 months ago was S0 = 100, the current stock price is 125 and the current interest rate is r = 10 compounded semi-annually. (This is the same rate that prevailed 6 months ago.) What is the current value of our forward contract?

Correct

The forward price F_0 at the time we entered into the forward contract

is given by

$$F_0 = S_0/d(0,T) = S_0(1+r/2)^2 = 110.25.$$

The forward

price F_t at

the current time t for a

forward contract with expiration 6 months is given by

$$F_t = S_t/d(t,T) = S_t(1+r/2) = 131.25.$$

Therefore, the value is $f_t = (F_t - F_0)d(t,T) = (131.25 - 110.25)/(1+r/2) = 20.$

Introduction to Derivative Securities

Question 1Term structure of interest rates and swap valuation

Suppose the current term structure of interest rates, assuming annual compounding, is as follows:

s_1	s_2	s_3	s_4	s_5	s_6
7.0%	7.3%	7.7%	8.1%	8.4%	8.8%

What is the discount rate $d(0,4)$? (Recall that interest rates are always quoted on an annual basis unless

stated otherwise.)

Please submit your answer rounded to three decimal places. So for example, if your answer is 0.4567 then you should submit an answer of 0.457.

0.732

Correct

The discount rate is given by the formula

$d(0,t) = \frac{1}{(1+s_t)^t}$

Using this formula we get 0.7323.

Question 2 Swap Rates

Suppose a 6-year swap with a notional principal of $10 million is being configured. What is the fixed rate of interest that will make the value of the swap equal to zero. (You should use the term structure of interest rates from Question 1.)

Correct

The discount rates are:

$d(0,1)$	$d(0,2)$	$d(0,3)$	$d(0,4)$	$d(0,5)$	$d(0,6)$
0.9346	0.8686	0.8005	0.7323	0.6681	0.6029

The value of the fixed interest rate, typically called the swap rate, is

given by the formula

$r = \frac{1-d(0,6)}{\sum_{t=1}^{6} d(0,t)} = 0.0862 = 8.62\%$

Question 3 Hedging using futures

Suppose a farmer is expecting that her crop of oranges will be ready for harvest and sale as 150,000 pounds of orange juice in 3 months time. Suppose each orange juice futures contract is for 15,000 pounds of orange juice, and the current futures price is F_0 = 118.65 cents-per-pound. Assuming that the farmer has enough cash liquidity to fund any margin calls, what is the risk-free price that she can guarantee herself.

Correct

She can go short 10 futures contracts each for $15,000$ pounds. This way

she can ensure that the payoff from the futures contract would remove

all the uncertainty in the spot price of orange juice, and she can lock

the price to 118.65 cents-per-pound.

Question 4 Minimum variance hedge

Suppose a farmer is expecting that her crop of grapefruit will be ready for harvest and sale as 150,000 pounds of grapefruit juice in 3 months time. She would like to use futures to hedge her risk but unfortunately there are no futures contracts on grapefruit juice. Instead she will use orange juice futures. Suppose each orange juice futures contract is for 15,000 pounds of orange juice and the current futures price is F_0 = 118.65 cents-per-pound. The volatility, i.e. the standard deviation, of the prices of orange juice and grape fruit juice is 20% and 25%, respectively, and the correlation coefficient is 0.7. What is the approximate number of contracts she should go short to minimize the variance of her payoff?

Correct

Suppose the farmer goes short h orange juice contracts. Then the

payoff from the contract and the sale of the grape fruit juice would

be $C = 150,000 S_T^G - 15,000 h(S_T^O - F_t^O)$ where S_T^G and

S_T^O denote the price of grapefruit juice and orange juice at

expiration, and F_t^O denotes the current price of orange juice

futures.

Using the formula in the module slides, the minimum-variance hedge

is given by

$$h^* = \frac{150{,}000}{15{,}000} \cdot \frac{\mathbf{cov}(S_t^G, S_t^O)}{\mathbf{var}(S_t^G)} = 10 \cdot \frac{\rho\sigma_G}{\sigma_O} = 10 \cdot \frac{(0.7)(0.25)}{0.20} = 8.75$$

Since the solution h^* has to be an integer, the number is

either 8 or 9.

Question 5 Call Options

Consider a 1-period binomial model with R=1.02, S_0 = 100, 1.05=u=1/d. Compute the value of a European call option on the stock with strike K=102. The stock does not pay dividends.

Correct

The arbitrage-free value is $C_0 = \frac{1}{R} E_0^Q [C_1] = \frac{1}{R} [qC_u + (1-q)C_d]$ where $q = \frac{R-d}{u-d}$, $1-q = \frac{u-R}{u-d}$, $C_u = 3$ and

$C_d = 0.$

Therefore $C_0 = qC_u/R = 2.0373.$

Question 6 Call Options II

When you construct the replicating portfolio for the option in the previous question how many dollars do you need to invest in the cash account?

Correct

We need to solve two equations in two unknowns as

described in the lecture slides:

$uS_0 x + Ry = C_u$

$dS_0 x + Ry = C_d$

which yields a solution of $x = .3073$ and $y = -28.694$. The answer is therefore $y = -28.694$.