a1

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STAT 441: Classification
Assignment 1

In [310]:
```
from scipy.io import loadmat
from scipy.linalg import sqrtm
```

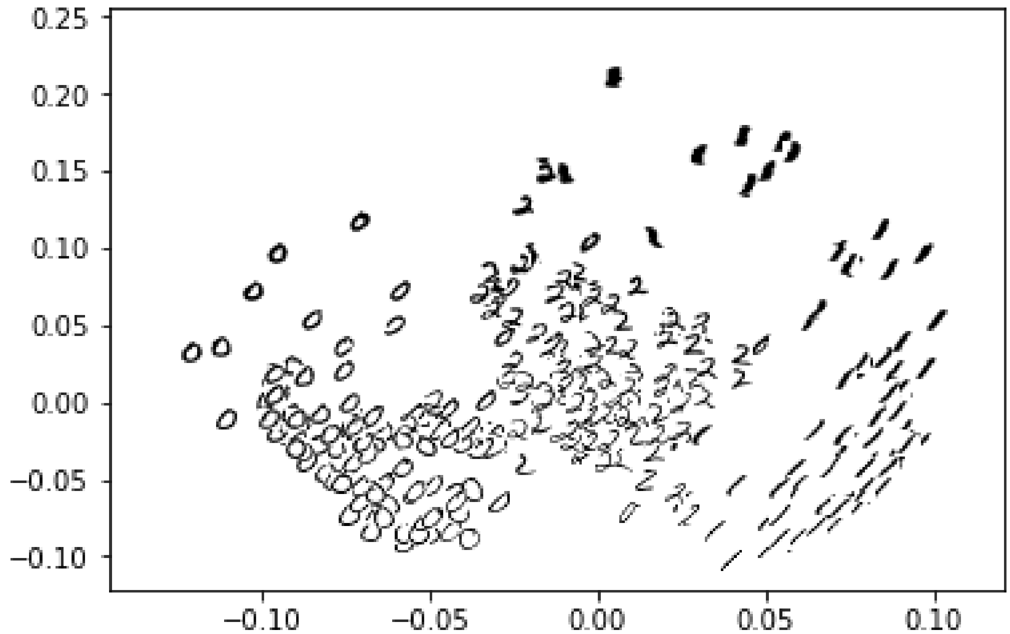
In [157]:
```
import matplotlib.pyplot as plt
from matplotlib.offsetbox import AnnotationBbox, OffsetImage
%matplotlib inline
import numpy as np
from numpy.linalg import norm
```

In [158]:
```
def plotimages(images, Y, scale=0.9, proportion=0.2):
    '''
    Input:
        images: images, must be in a 3-dimensional matrix (x by y by n)
            e.g. if X is 64 by 400 and size of each image is 8 by 8,
            images=np.reshape(X,(8,8,400));
        Y: coordinates of where to plot the image (Y(1,:) by Y(2,:)
        scale: scale of each image wrt to figure size (scale <= 1, e.g. 0.8)
        proportion: proportion of the data to be ploted (proportion <= 1)
            e.g. if there are 400 data points proportion = 1, plots
            all 400 data points and proportion = 0.5 plot only 200 data points
            (i.e. 1th, 3th, 5th, ...)
    Ali Ghodsi 2006
    '''
    Y /= norm(Y, axis = 1, ord=2).reshape((Y.shape[0],1))
    inc = np.floor(1/proportion)

    image_width = images.shape[0]
    image_height = images.shape[1]
    n_images = images.shape[2]

    plt.gray()
    fig = plt.gcf()
    fig.clf()
    ax = plt.subplot(111)
```

```
        ax.set_xlim((Y[0,:].min()*1.2,Y[0,:].max()*1.2))
        ax.set_ylim((Y[1,:].min()*1.2,Y[1,:].max()*1.2))

        for counter in np.arange(0,n_images,inc):
            counter = int(counter)
            xy = (Y[0,counter],Y[1,counter])
            current_image = 1-np.reshape(images[:,:,counter],
                                         (image_width,image_height))
            imagebox = OffsetImage(current_image, zoom=scale)
            ab = AnnotationBbox(imagebox, xy, xybox=(1., -1.), xycoords='data',
                                boxcoords="offset points", frameon=False)
            ax.add_artist(ab)
            plt.draw()
        #plt.show()
```

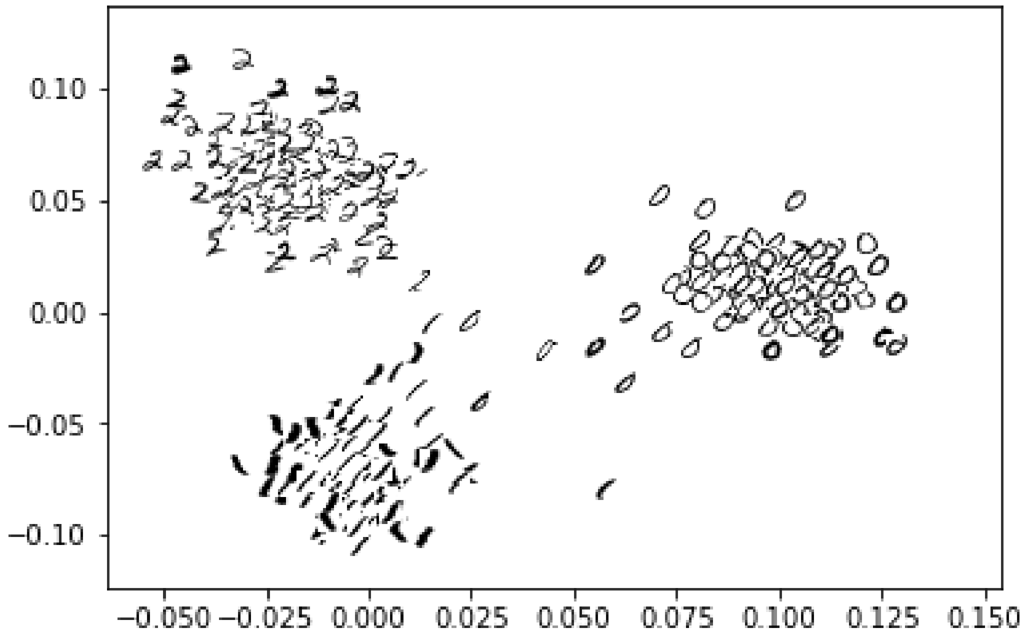
0.0.1 Part (a)

```
In [43]: X = loadmat('0_1_2.mat')['X']
         (d, n) = X.shape

In [44]: mu = np.mean(X, axis=1).reshape(d, 1)
         X_mean = np.dot(mu, np.ones((1, n)))
         X_tilde = X - X_mean

In [45]: U, S, V = np.linalg.svd(X_tilde)
         pcps = np.transpose(U[:, 0:2])
         Y_pca = np.dot(pcps, X_tilde)

In [46]: images = np.reshape(X,(8,8,n))
         plotimages(images, Y_pca, 1, 1)
         plt.show()
```



0.0.2 Part (b)

```
In [47]: S_w = np.zeros((d, d))

         for i in range(1,4):
             k = i*100
             j = k - 100
             X_i = X[:, j:k]
             mu_i = np.mean(X_i, axis=1).reshape(d, 1)
             mean_i = np.dot(mu_i, np.ones((1, 100)))
             X_i = X_i - mean_i
             S_w = S_w + np.dot(X_i, np.transpose(X_i))

In [48]: S_t = np.dot(X_tilde, np.transpose(X_tilde))/n
         S_b =  S_t - S_w
         M = np.dot(np.linalg.inv(S_w), S_b)
         E, W = np.linalg.eig(M)

In [49]: discrims = np.transpose(W[:, 0:2])
         Y_fda = np.dot(discrims, X)

In [50]: # using images = np.reshape(X,(8,8,n)) run above
         plotimages(images, Y_fda, 1, 1)
         plt.show()
```



0.0.3 Part (c)

```
In [277]: def summary_values(data, k):
              '''
              data: the dataset
              k: the number of classes in the dataset
              Assumes that there are m*k data points in the dataset,
                  where m is a positive integer, with the next i*k
                  data points being from class i
              '''
              means = []
              covars = []
              (dim, npoints) = data.shape
              ppc = npoints/k # the number of points per class
              for i in range(1,k+1):
                  # get the class mean
                  b = i*ppc
                  a = b - ppc
                  data_i = data[:, a:b]
                  mu_i = np.mean(data_i, axis=1).reshape(dim, 1)
                  means.append(mu_i)

                  # calculate the within class covariance
                  mean_i = np.dot(mu_i, np.ones((1, ppc)))
                  data_i = data_i - mean_i
```

```
        sigma_i = np.dot(data_i, np.transpose(data_i))/(ppc - dim)
        covars.append(sigma_i)
    return (means, covars)

class_means, class_covars = summary_values(Y_pca, 3)
```

In [283]:
```
# Linear boundaries
sigma = sum(class_covars)
xcoeffs = [np.dot(np.linalg.inv(sigma), mu) for mu in class_means]
consts = []
for i in range(len(slopes)):
    intercept = 0 - np.log(3) - (np.dot(
        np.transpose(class_means[i]), xcoeffs[i])/2)
    consts.append(intercept[0][0])

slopes = [xcoeffs[1]-xcoeffs[0],
          xcoeffs[1]-xcoeffs[2],
          xcoeffs[2]-xcoeffs[0]]
intercepts = [consts[1]-consts[0],
              consts[1]-consts[2],
              consts[2]-consts[0]]
```

In [284]:
```
# Quadratic boundaries
dets = [np.linalg.det(sig) for sig in class_covars]
mean_sig = []

for i in range(3):
    c = np.dot(np.linalg.inv(class_covars[i]), class_means[i])
    mean_sig.append(c)

# constant terms
a0 = [0]*3
a0[0] = (np.log(dets[1]) - np.log(dets[0]) +
          np.dot(np.transpose(class_means[1]), mean_sig[1]) -
          np.dot(np.transpose(class_means[0]), mean_sig[0]))
a0[1] = (np.log(dets[1]) - np.log(dets[2]) +
          np.dot(np.transpose(class_means[1]), mean_sig[1]) -
          np.dot(np.transpose(class_means[2]), mean_sig[2]))
a0[2] = (np.log(dets[2]) - np.log(dets[0]) +
          np.dot(np.transpose(class_means[2]), mean_sig[2]) -
          np.dot(np.transpose(class_means[0]), mean_sig[0]))
a0 = [-0.5*elem[0][0] for elem in a0]

# x coefficients
a1 = [0]*3
a1[0] = mean_sig[1] - mean_sig[0]
a1[1] = mean_sig[1] - mean_sig[2]
a1[2] = mean_sig[2] - mean_sig[0]
```

```
# x^2 coefficients
a2 = [0]*3
a2[0] = -0.5*(class_covars[1] - class_covars[0])
a2[1] = -0.5*(class_covars[1] - class_covars[2])
a2[2] = -0.5*(class_covars[2] - class_covars[0])
```

In [367]:
```
print("The formulas for the linear decision boundaries are:")
for i in range(3):
    formula = "   0 = y^T*[{},{}]^T %+f"
    print(formula.format(slopes[i][0][0], slopes[i][1][0]) %
          intercepts[i])
```

```
The formulas for the linear decision boundaries are:
   0 = y^T*[62.3646802321,9.46174639799]^T +0.079522
   0 = y^T*[29.6954023751,1.86377382065]^T -0.946712
   0 = y^T*[32.669277857,7.59797257734]^T +1.026235
```

The formulas for the quadratic boundaries are:

$$0 = \mathbf{y}^T \begin{bmatrix} -5.46437767e-05 & 4.18409303e-04 \\ 4.18409303e-04 & -1.94088829e-03 \end{bmatrix} \mathbf{y} + \mathbf{y}^T \begin{bmatrix} 154.87505264 \\ 24.61722339 \end{bmatrix} - 0.47397694$$

$$0 = \mathbf{y}^T \begin{bmatrix} -0.00027197 & 0.00040752 \\ 0.00040752 & -0.00221845 \end{bmatrix} \mathbf{y} + \mathbf{y}^T \begin{bmatrix} 75.19771341 \\ 0.95073167 \end{bmatrix} - 3.40368554$$

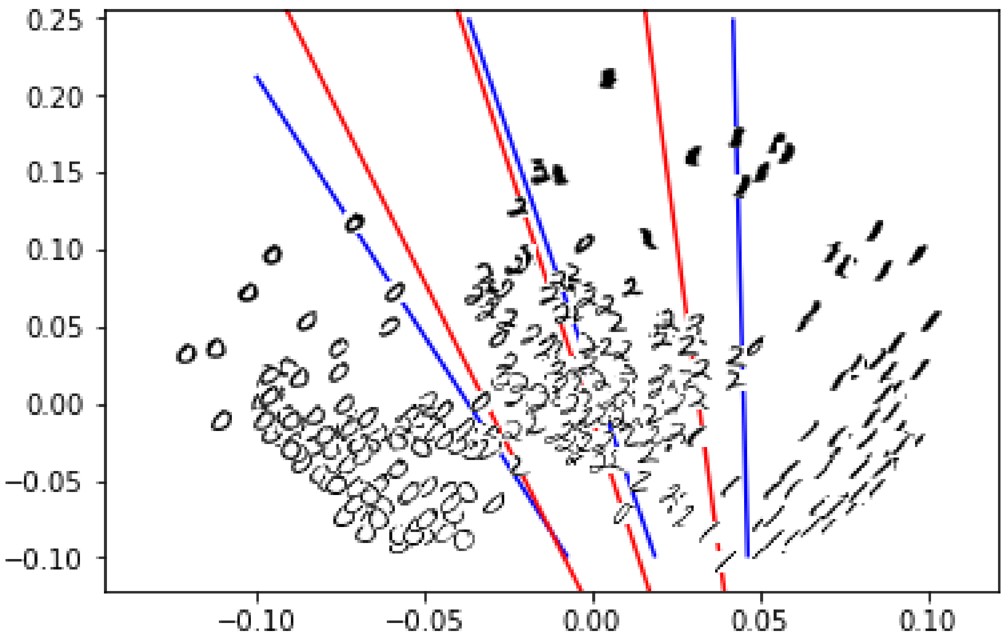
$$0 = \mathbf{y}^T \begin{bmatrix} 2.17323021e-04 & 1.08933396e-05 \\ 1.08933396e-05 & 2.77564444e-04 \end{bmatrix} \mathbf{y} + \mathbf{y}^T \begin{bmatrix} 79.67733923 \\ 23.66649172 \end{bmatrix} + 2.929708601$$

0.0.4 Part (d)

In [296]:
```
# using images = np.reshape(X,(8,8,n)) from above
plotimages(images, Y_pca, 1, 1)

x_min, x_max = -0.10, 0.10

# Linear Coefficients
xm1 = slopes[0][0][0]
ym1 = slopes[0][1][0]
xm2 = slopes[1][0][0]
ym2 = slopes[1][1][0]
xm3 = slopes[2][0][0]
ym3 = slopes[2][1][0]

c1 = intercepts[0]
c2 = intercepts[1]
```

```
c3 = intercepts[2]

lx = np.linspace(x_min, x_max)
lin_y1 = ((-1*lx*xm1) - c1)/ym1
lin_y2 = ((-1*lx*xm2) - c2)/ym2
lin_y3 = ((-1*lx*xm3) - c3)/ym3

plt.plot(lx, lin_y1, color = 'red')
plt.plot(lx, lin_y2, color = 'red')
plt.plot(lx, lin_y3, color = 'red')

# Quadratic Coefficients
ax1 = np.sum(a2[0][0])
ay1 = np.sum(a2[0][1])
b1 = np.sum(a2[0])
cx1 = a1[0][0][0]
cy1 = a1[0][1][0]
d1 = a0[0]

ax2 = np.sum(a2[1][0])
ay2 = np.sum(a2[1][1])
b2 = np.sum(a2[1])
cx2 = a1[1][0][0]
cy2 = a1[1][1][0]
d2 = a0[1]

ax3 = np.sum(a2[2][0])
ay3 = np.sum(a2[2][1])
b3 = np.sum(a2[2])
cx3 = a1[2][0][0]
cy3 = a1[2][1][0]
d3 = a0[2]

y_min, y_max = -0.10, 0.25
y_range = np.linspace(y_min, y_max)

x, y = np.meshgrid(x_range, y_range)

plt.contour(x, y,
            (ax1*x*x)+(ay1*y*y)+(b1*x*y)+(cx1*x)+(cy1*y)+d1,
            [0], colors = 'blue')
plt.contour(x, y,
            (ax2*x*x)+(ay2*y*y)+(b2*x*y)+(cx2*x)+(cy2*y)+d2,
            [0], colors = 'blue')
plt.contour(x, y,
            (ax3*x*x)+(ay3*y*y)+(b3*x*y)+(cx3*x)+(cy3*y)+d3,
            [0], colors = 'blue')
```

```
plt.show()
```



0.0.5 Part (e)

```
In [364]: def euclidean_lda(data, means):
              (d, n) = data.shape
              U, S, V = np.linalg.svd(data)
              S = np.diag(S)
              T = np.dot(np.linalg.inv(sqrtm(S)), np.transpose(U))
              X_hat = np.dot(T, data)
              X_mus = [np.dot(T, mu) for mu in means]

              Y = [0]*n
              for i in range(n):
                  dist0 = np.dot(np.transpose(X_hat[:, i] - X_mus[0]),
                                 X_hat[:,i] - X_mus[0])[0][0]
                  dist1 = np.dot(np.transpose(X_hat[:,i] - X_mus[1]),
                                 X_hat[:,i] - X_mus[1])[0][0]
                  dist2 = np.dot(np.transpose(X_hat[:,i] - X_mus[2]),
                                 X_hat[:,i] - X_mus[2])[0][0]

                  if dist0 <= dist1 and dist0 <= dist2:
                      Y[i] = 0
                  elif dist1 <= dist0 and dist0 <= dist1:
                      Y[i] = 1
                  else:
```

```
                Y[i] = 2
        return Y
```

```
In [365]: # Error rates
          X_class_means = []
          for i in range(1,4):
              b = i*100
              a = b - 100
              data_i = X[:, a:b]
              mu_i = np.mean(data_i, axis=1).reshape(d, 1)
              X_class_means.append(mu_i)

          Y_lda = euclidean_lda(X, X_class_means)
```

```
In [369]: Y_wrong = 0
          for i in range(n):
              if i < 100:
                  if Y_lda[i] != 0:
                      Y_wrong = Y_wrong + 1
              elif i < 200:
                  if Y_lda[i] != 1:
                      Y_wrong = Y_wrong + 1
              else:
                  if Y_lda[i] != 2:
                      Y_wrong = Y_wrong + 1

          L_n = Y_wrong/float(n)
          print("The empirical error rate for the LDA is: {0:.000f}%."
                .format(L_n))
```

```
The empirical error rate for the LDA is: 1%.
```

0.0.6 Part (f)

Yes, we can. We estimate some vector $\mathbf{w}$ such that $\mathbf{y} = \mathbf{w}^T\mathbf{x}$, where $\mathbf{w}$ is a d-by-1 column vector. In this case, we let $\mathbf{w}$ be the principals calculated via PCA.

We take the quadratic function that we are unable to estimate, which is $\mathbf{y} = \mathbf{x}^T\mathbf{v}\mathbf{x} + \mathbf{w}^T\mathbf{x}$. Then, extend $\mathbf{w}$ and the data $\mathbf{x}$ and by the square of values to $\hat{\mathbf{x}}$, as shown below:

$$\begin{aligned}\hat{\mathbf{w}} &= [w_1, w_2, ..., w_d, v_1, ..., v_d] \\ \hat{\mathbf{x}} &= [x_1, x_2, ..., x_d, x_1^2, ..., x_d^2]\end{aligned}$$

This projects the data into two more dimensions, making it quadratic. We can then apply LDA to estimate the new function $\hat{\mathbf{y}}^* = \hat{\mathbf{w}}^T\hat{\mathbf{x}}$, since it is linear in $\hat{\mathbf{x}}$.