

# LEAN SIX SIGMA MEASURE PHASE

*"That which cannot be measured cannot be proven"* - Anthony W. Richardson



# COURSE CONTENT

## Coverage:

- Introduction to QUALITY and LEAN SIX SIGMA
  - History of Quality (Deming, Juran, Ishikawa, Taguchi, etc.)
  - Evolution of Six Sigma
  - Evolution of LEAN
  - Lean Six Sigma – philosophy and objectives
  - Deliverables of a Lean Six Sigma Project
  - How Organizations deploy Six Sigma
  - Data driven decision making
  - The Problem Solving Strategy  $Y = f(x)$
  - Understanding LSS frame workWhy MSA ?
  - Terminologies - Resolution, Bias, Repeatability & Reproducibility
  - Analysis using software, Interpretation of results
  - SWIPE factors
  - Variation - Special & Common Cause Variation
  - Process Stability
  - Normal Distribution
  - Process Capability - Cp,Cpk & Pp,Ppk

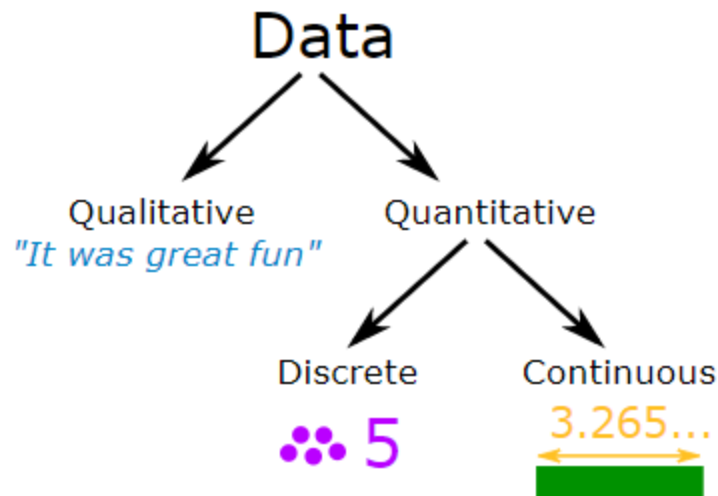
## Understanding “DATA”

Data is a collection of facts, such as numbers, words, measurements, observations or just descriptions of things

Data can be qualitative or quantitative.

Qualitative data is descriptive information (it describes something)

Quantitative data is numerical information (numbers)



# Understanding “STATISTICS”

A branch of mathematics dealing with the collection, analysis, interpretation, and presentation of masses of numerical data

## Descriptive statistics

Descriptive statistics uses the data to provide descriptions of the population, either through numerical calculations or graphs or tables. Four types of Descriptive statistics are Measures of Frequency, Measures of Central Tendency, Measures of Dispersion or Variation, Measures of Position

## Inferential statistics

With Inferential statistics, you take data from samples and make generalizations about a population, Three types of Inferential statistics are Hypothesis tests, confidence intervals, and regression analysis

## Predictive statistics

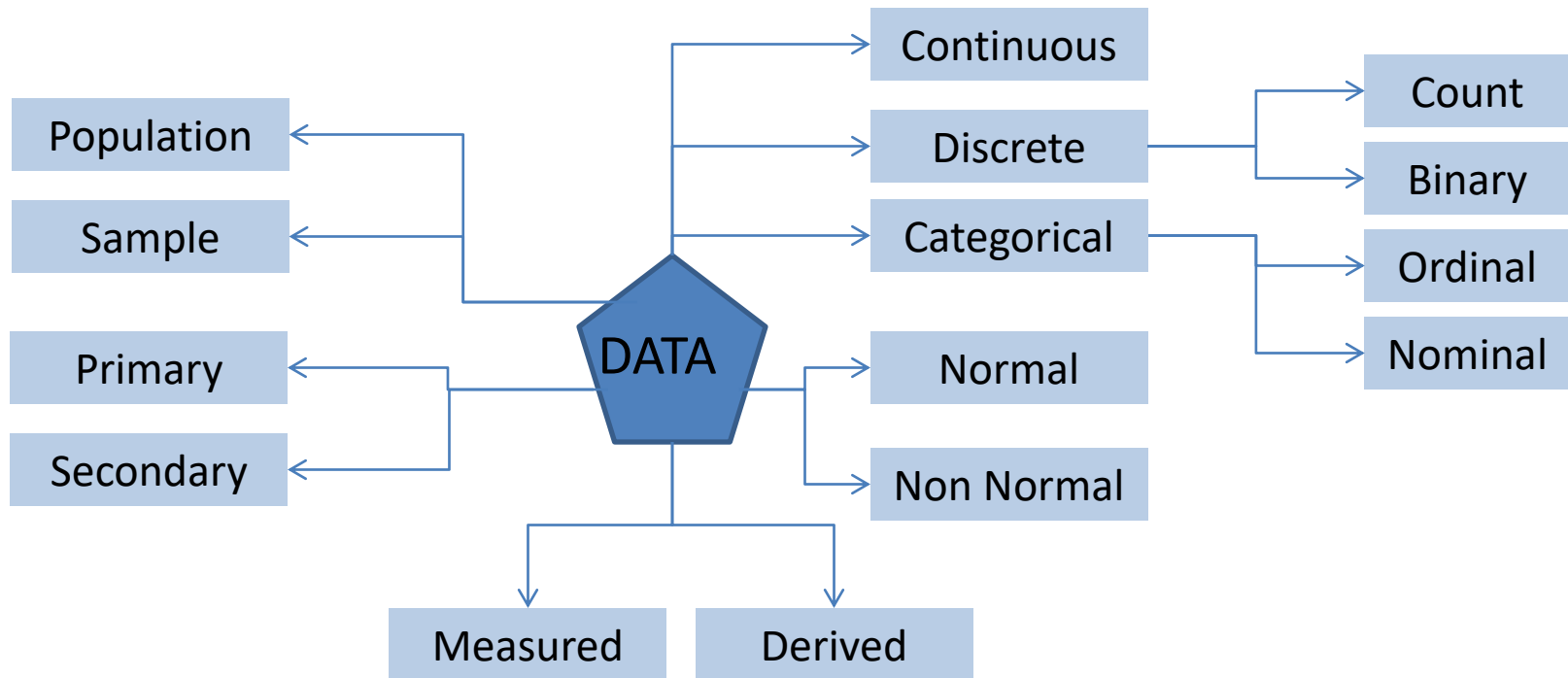
Predictive statistics encompasses a variety of statistical techniques from data mining, predictive modelling, and machine learning, that analyze current and historical facts to make predictions about future

## Prescriptive statistics

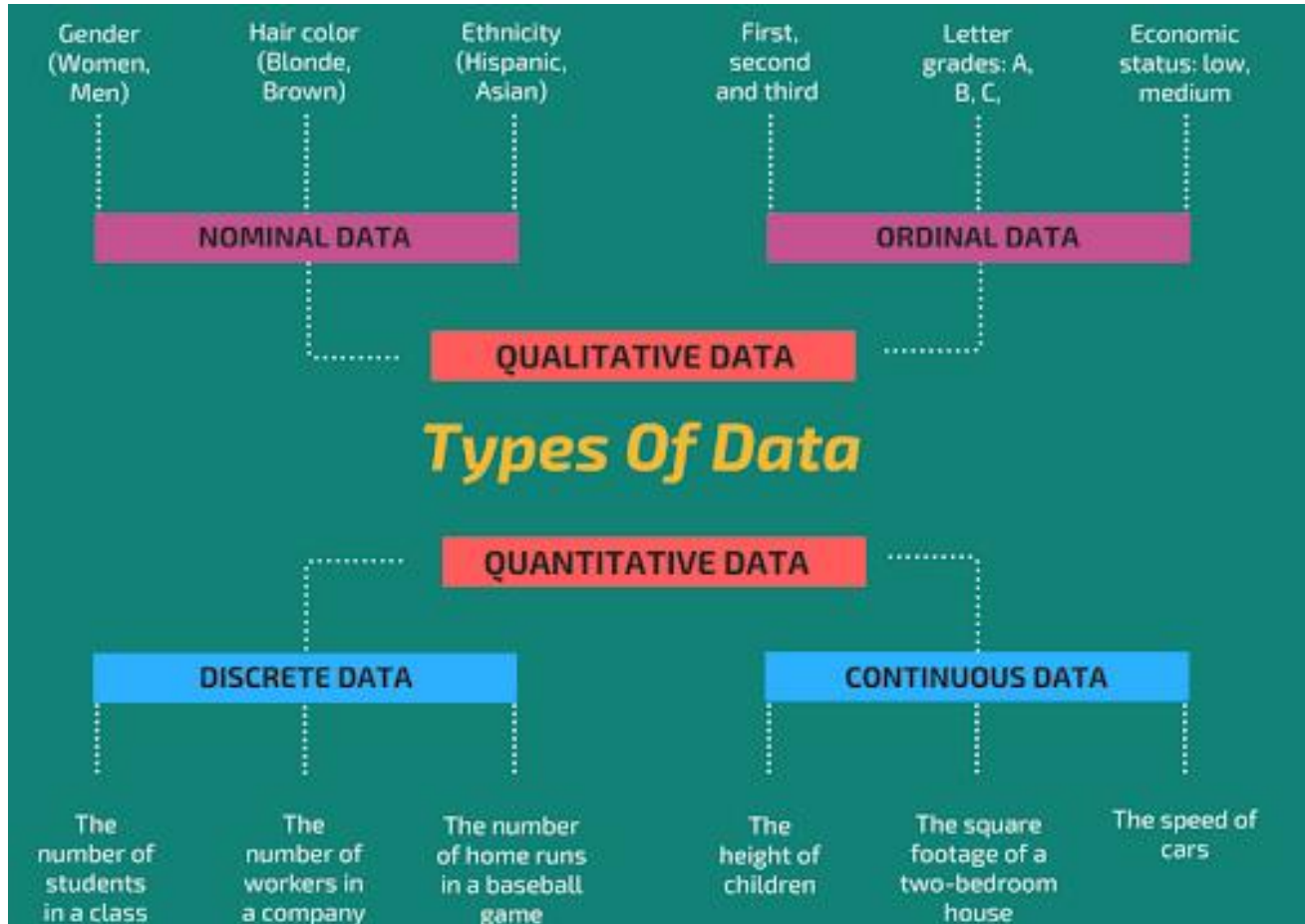
Prescriptive statistics is the area of statistics dedicated to finding the best course of action for a given situation

## Dimensions of DATA

Understanding the different dimensions of the data is very important before we start to work with the data



# Examples for types of data



## Dimensions of DATA

Since we understand the different dimensions of data, we need to answers to all the below questions before we start analysis

- ➡ What is the source of the data ?
- ➡ Are we going to work with population data or sample data?
- ➡ What type of data we are dealing with?
- ➡ Is the data collected and directly used for analysis (or) the data is fed in to a formula, metric is derived and analyzed ?
- ➡ What is the distribution of the data under study ?

## First step in measure phase

Do Brainstorming to establish the below equation

$$Y = f(x1, x2, x3, x4, x5)$$

## Dimensions of DATA

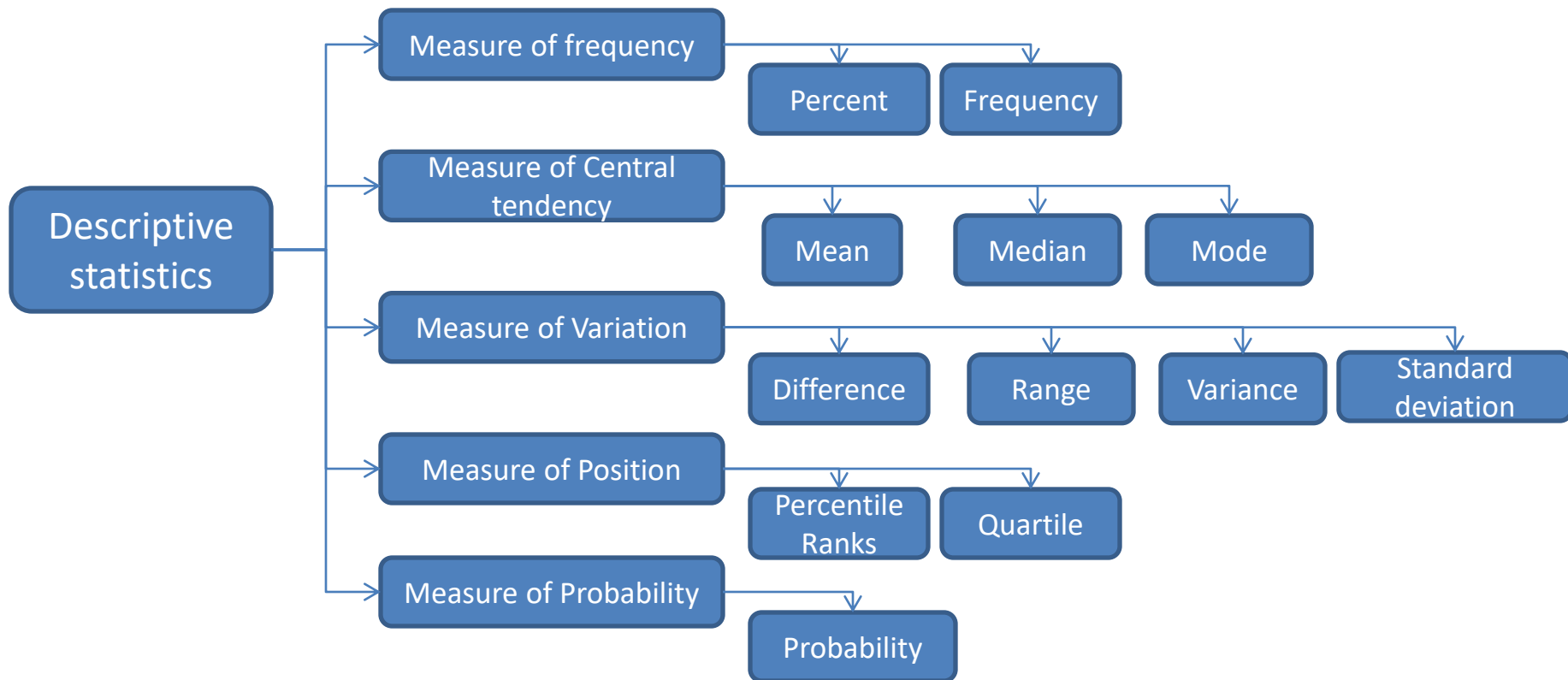
Below grid will help us to understand the data on all its dimensions

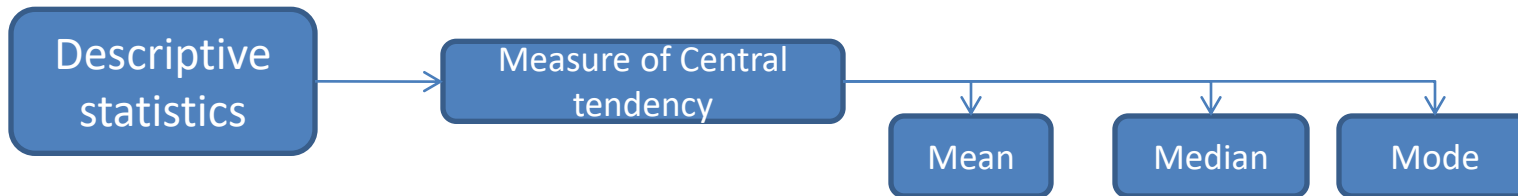
	$Y = f(x1, x2, x3, x4, x5)$					
Source						
Data type						
Measured / Derived						
Population/ Sample						
Distribution type						



### Descriptive statistics

Descriptive statistics uses the data to provide descriptions of the population, either through numerical calculations or graphs or tables. Four types of Descriptive statistics are Measures of Frequency, Measures of Central Tendency, Measures of Dispersion or Variation, Measures of Position





### Mean

Add all the numbers then divide by the amount of numbers

9, 3, 1, 8, 3, 6

$$9 + 3 + 1 + 8 + 3 + 6 = 30$$

$$30 \div 6 = 5$$

The mean is 5

### Median

Order the set of numbers, the median is the middle number

9, 3, 1, 8, 3, 6

1, 3, 3, 6, 8, 9

The median is 4.5

### Mode

The most common number

9, 3, 1, 8, 3, 6

The mode is 3



Excel Analysis

Note : Mode can be multi mode also

Descriptive statistics

Measure of Variation

Difference

Range

Variance

Standard deviation

### Difference (or) Delta

Difference between A and B

13, 15  
 $15 - 13 = 2$   
 Difference is 2

### Range

The difference between the highest number and lowest number

9, 3, 1, 8, 3, 6

$9 - 1 = 8$

The range is 8

## Standard Deviation

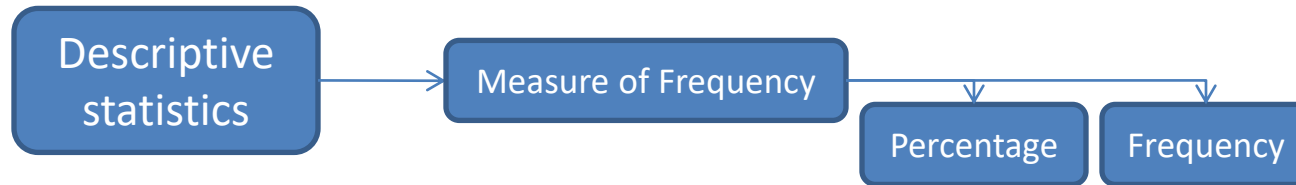
$$\sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

76	84	69	92	58
89	73	97	85	77

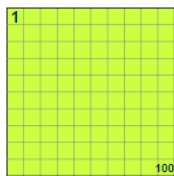
$$\bar{X} = \frac{\text{Sum}}{n}$$



Excel Analysis



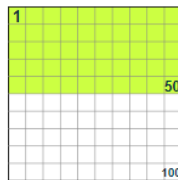
Percentage refers to a number or ratio that represents a fraction of 100. It is often denoted by the symbol "%" or simply as "percent"



100% means **all**.

Example:

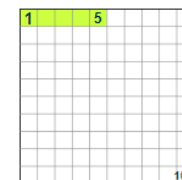
$$100\% \text{ of } 80 \text{ is } \frac{100}{100} \times 80 = 80$$



50% means **half**.

Example:

$$50\% \text{ of } 80 \text{ is } \frac{50}{100} \times 80 = 40$$



5% means  $\frac{5}{100}$ ths.

Example:

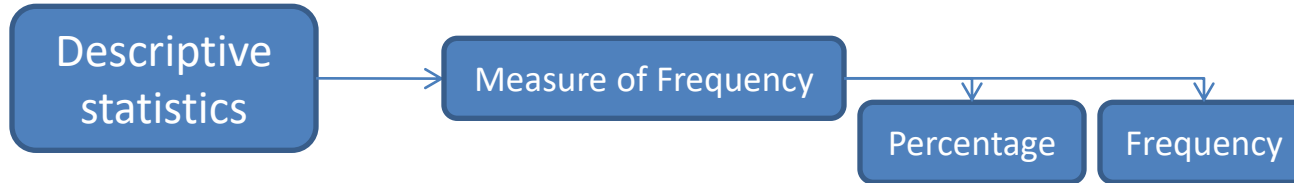
$$5\% \text{ of } 80 \text{ is } \frac{5}{100} \times 80 = 4$$

Percentage achieve the following

- What will have if 100 opportunities are given
  - But what will happen if my opportunities are already greater than 100?
- Compare two performance when the opportunities are not standard



Excel Analysis



In statistics the frequency (or absolute frequency) of an event is the number of times the observation occurred/recorded

These frequencies are often graphically represented in histograms.

### EXAMPLE:

A survey was done with a class of 31 students to see how many times they had played sport last month.

score	tally	frequency (f)
1		4
2	 	9
3	 	6
4	 	7
5		3
6		2

Most people only played twice.



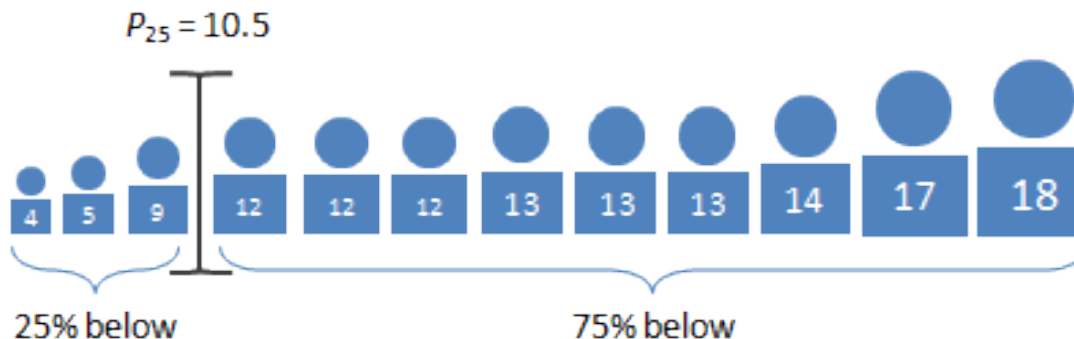
Minitab exercise



Excel Analysis



Percentile : The value below which a percentage of data falls, Percentiles are commonly used to report test scores



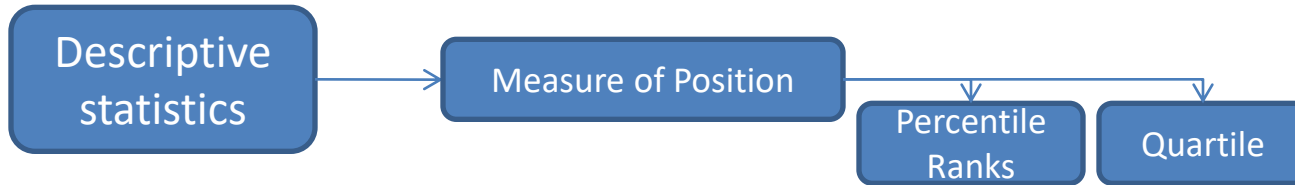
The key difference between percentage and percentile is the percentage is a mathematical value presented out of 100 and percentile is the per cent of values below a specific value. The percentage is a means of comparing quantities. A percentile is used to display position or rank.



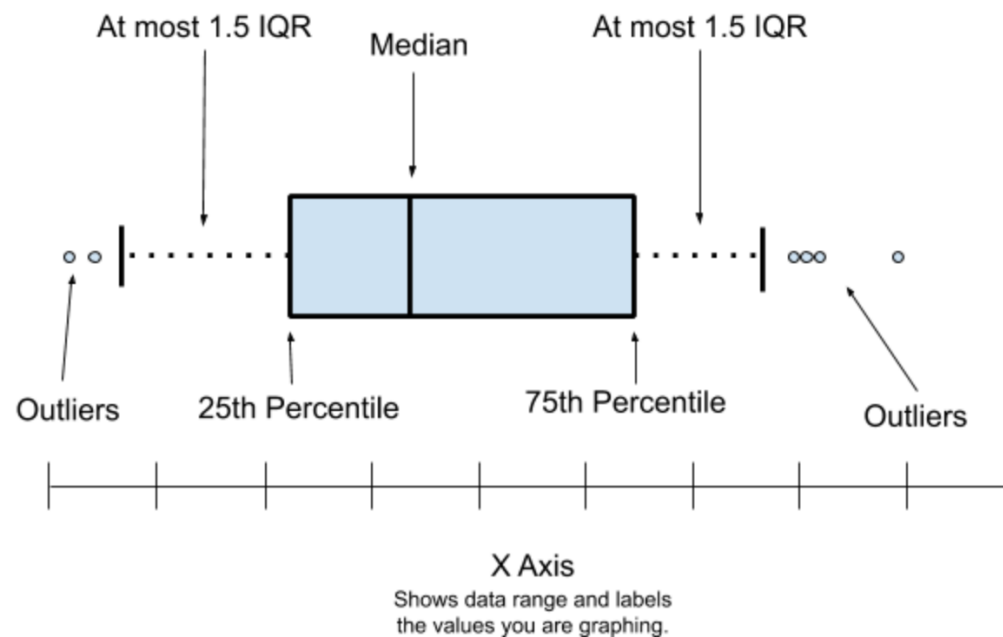
Minitab exercise



Excel Analysis



**Quartile** : Each of four equal groups into which a population / sample data can be divided according to the distribution of values of a particular variable.

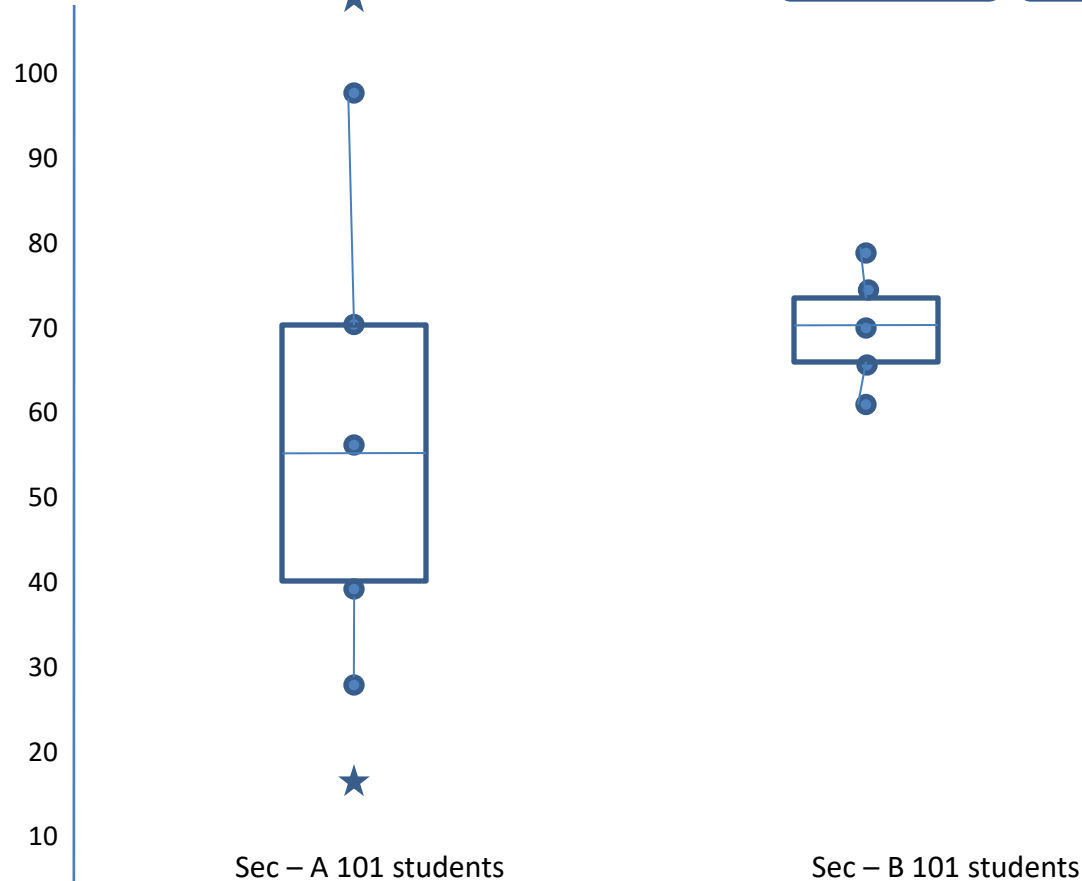


Descriptive statistics

Measure of Position

Percentile Ranks

Quartile



	101	101
Ranks	Sec A	Sec B
101	98	80
75	70	75
51	55	70
25	40	65
1	28	60

$$\text{IQR} = P75 - P25$$

$$\text{Outlier} = P75 + (1.5 * \text{IQR})$$

$$P25 - (1.5 * \text{IQR})$$

$$\text{Stability Ratio} = Q1/Q3$$

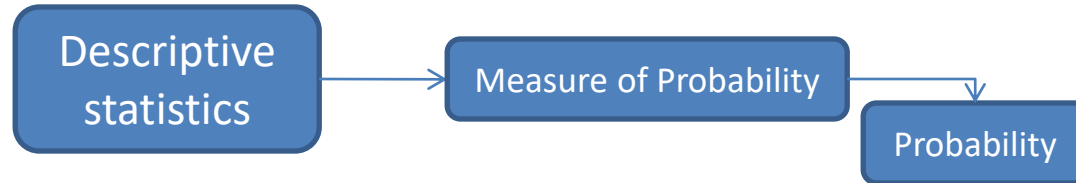


Minitab exercise



Excel Analysis





Probability is simply how likely something is to happen



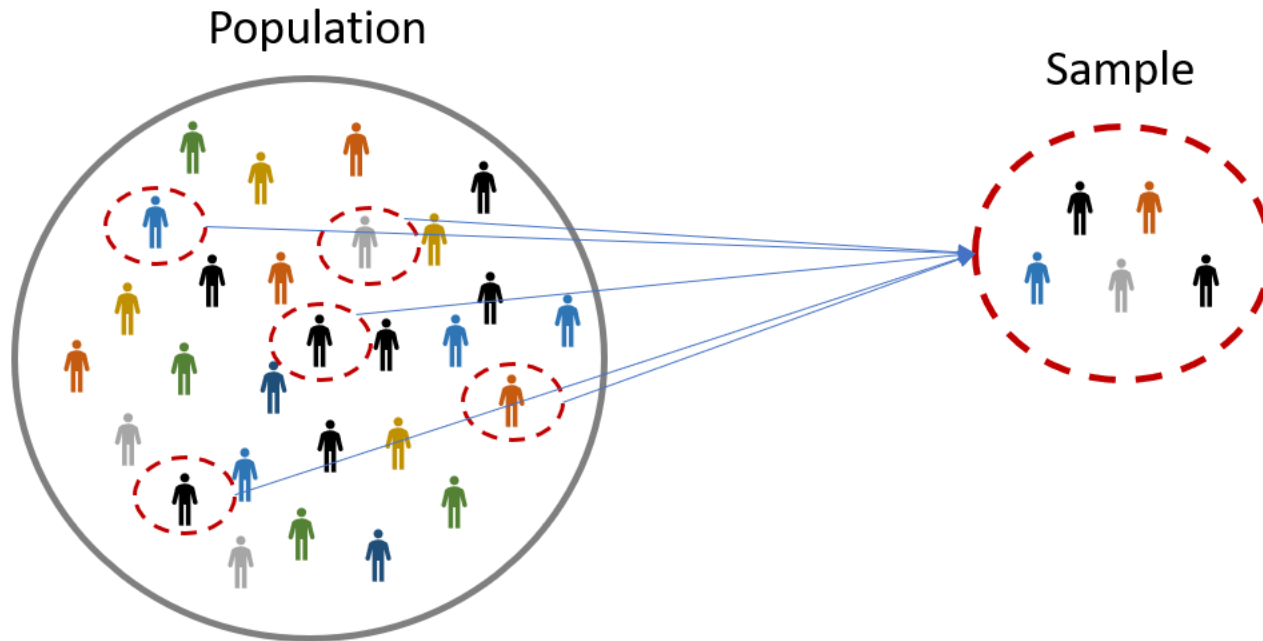
### What is Sample statistic and Population parameter

#### Population

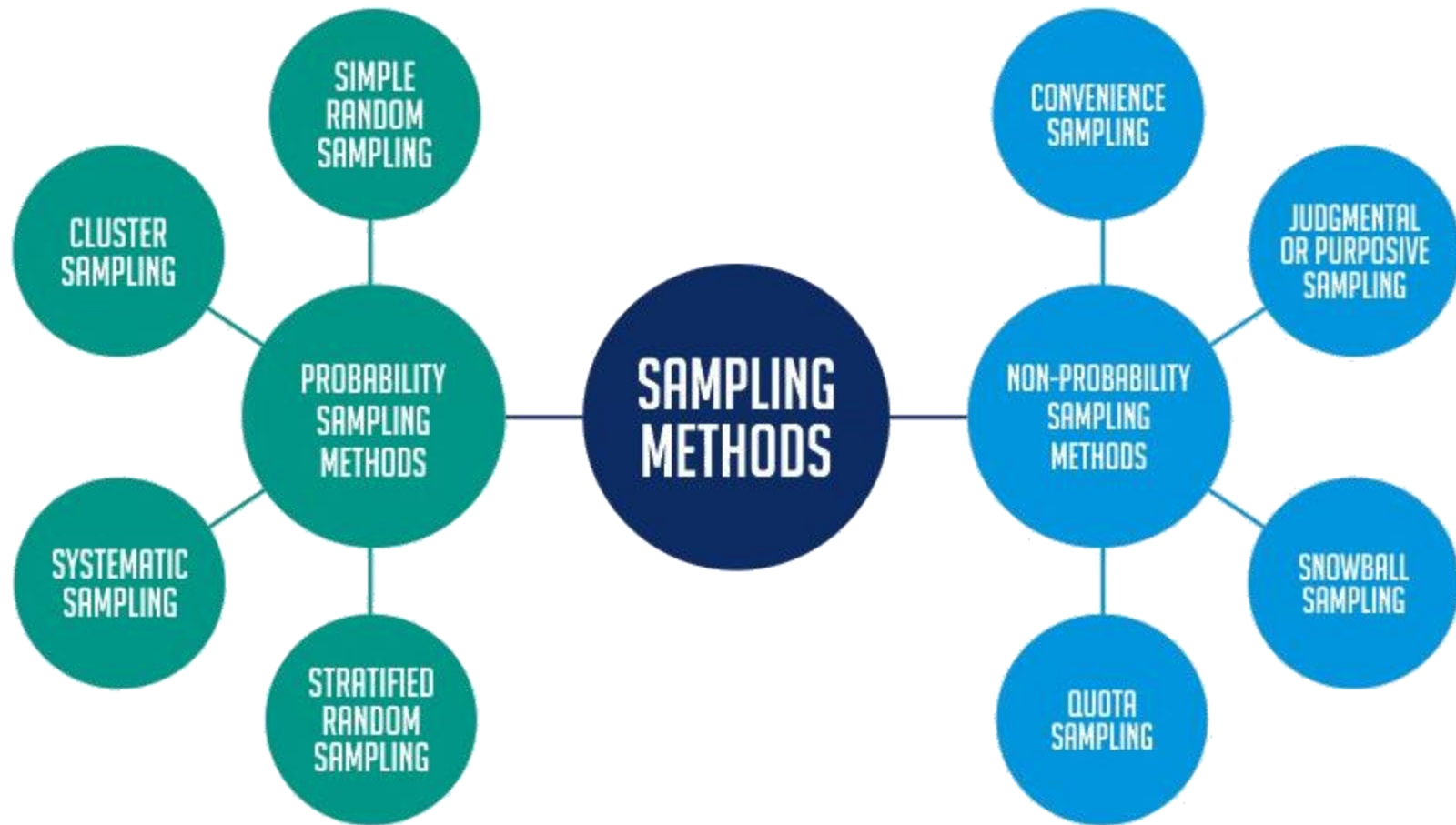
A population data set contains all members of a specified group (the entire list of possible data values)

#### Sample

A sample data set contains a part, or a subset, of a population. The size of a sample is always less than the size of the population from which it is taken



## Sampling techniques



### Probability sampling

A probability sampling method is any method of sampling that utilizes some form of random selection. In order to have a random selection method, you must set up some process or procedure that assures that the different units in your population have equal probabilities of being chosen

#### Simple Random sampling

Simple random sampling is the basic sampling technique where we select a group of subjects (a sample) for study from a larger group (a population)

#### Cluster sampling

Cluster sampling is a sampling plan used when mutually homogeneous yet internally heterogeneous groupings are evident in a statistical population. The total population is divided into these groups (known as clusters) and a simple random sample of the groups is selected

#### Systematic sampling

Systematic sampling is a type of probability sampling method in which sample members from a larger population are selected according to a random starting point but with a fixed, periodic interval. This interval, called the sampling interval, is calculated by dividing the population size by the desired sample size

#### Stratified sampling

Stratified sampling is a probability sampling technique wherein the researcher divides the entire population into different subgroups or strata, then randomly selects the final subjects proportionally from the different strata

### Non Probability sampling

Non-probability sampling is a sampling technique where the odds of any member being selected for a sample cannot be calculated. It's the opposite of probability sampling

#### convenience sampling

A convenience sample is a type of non-probability sampling method where the sample is taken from a group of population easy to reach

#### Judgment sampling

Judgment sampling is a common nonprobability method. The researcher selects the sample based on judgment. This is usually an extension of convenience sampling. For example, a researcher may decide to draw the entire sample from one "representative" city, even though the population includes all cities

#### Snowball sampling

Snowball sampling is where research participants recruit other participants for a test or study. It is used where potential participants are hard to find. It's called snowball sampling because (in theory) once you have the ball rolling, it picks up more "snow" along the way and becomes larger and larger

#### Quota sampling

In quota sampling, a population is first segmented into mutually exclusive sub-groups, just as in stratified sampling. Then judgment is used to select the subjects or units from each segment based on a specified proportion

Note : Widely used in areas like Psychology

## What is Sample statistic of Population parameter

A statistic and a parameter are very similar. They are both descriptions of groups, The difference between a statistic and a parameter is that statistics describe a sample. A parameter describes an entire population

Parameter name	Population parameter symbol	Sample statistic
Number of cases	N	n
Mean	$\mu$ (mu)	$\bar{x}$ (Sample mean)
Proportion	$\pi$ (Pi)	P (Sample proportion)
Variance	$\sigma^2$ (Sigma-square)	$s^2$ (Sample variance)
Standard deviation	$\sigma$ (Sigma)	s (sample standard deviation)
Correlation	$\rho$ (rho)	r (Sample correlation)
Regression Coefficient	$\beta$ (beta)	b (sample regression coefficient)

## Lets understand how to equate sample statistics and population parameter

Population Mean ( $\mu$ ) can fall any where between

“Sample Mean  $(\bar{X})$   $\pm$  Margin of error”

### Sample size

Sample size measures the number of individual samples measured or observations used in a survey or experiment

### Standard Error

The standard error of a statistic is the standard deviation of its sampling distribution

### Margin of Error

A margin of error tells you how many percentage points your results will differ from the real population value

### CI & CL

A 95% confidence interval is a range of values that you can be 95% certain contains the true mean of the population

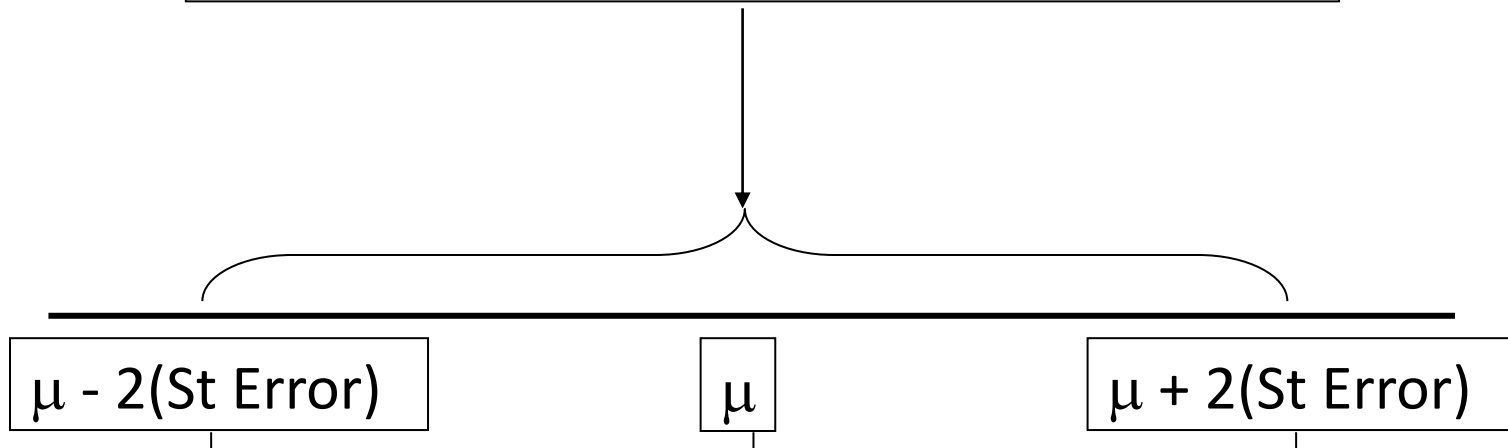
### Some Formulas

CONCEPT	FOR PROPORTIONS	FOR MEANS
When we take lots of random samples, we use the standard deviation of the sample statistics to describe the error that is due to random chance. This is called the standard error. The standard error depends on a population parameter and sample size.	Standard Error = $\sqrt{\frac{p(1-p)}{n}}$	Standard Error = $\sigma/\sqrt{n}$
If a normal model is a good fit for the sampling distribution, then 95% of sample statistics estimate the population parameter within 2 standard errors. This describes the margin of error.	Margin of Error = $2 \sqrt{\frac{p(1-p)}{n}}$	Margin of Error = $2\sigma/\sqrt{n}$
We create an interval to estimate the population parameter. We say we are 95% confident that the confidence interval contains the population parameter.	Confidence interval is $\hat{p} \pm \text{margin of error}$ $\hat{p} \pm 2 \sqrt{\frac{p(1-p)}{n}}$	Confidence interval is $\bar{X} \pm \text{margin of error}$ $\bar{X} \pm 2\sigma/\sqrt{n}$
These confidence intervals require a normal model. So we can only use these formulas when the normality criteria are met.	Both expected successes and failures at least 10	Variable is normally distributed in the population OR sample size is more than 30



## Sample statistic and Population parameter

Roughly 95% of the time, sample means,  $\bar{X}$  will fall in this interval.



### Key take away

- Step1 : Understand the difference between sample statistics and population parameter
- Step2 : Consider taking statistically significant sample
- Step3 : Calculate the correct CI with 95% CL
- Step4 : Visualize  $\mu$  and  $\bar{X}$  with +/- CI
- Step5 : Now make the appropriate decision

# Data collection

Check sheet one of the 7QC tools is a form used to collect data in real time at the location where the data is generated. The data it captures can be quantitative or qualitative

Let us Understand how to construct a data collection plan, data collection sheet or check sheet with a exercise



Excel Analysis

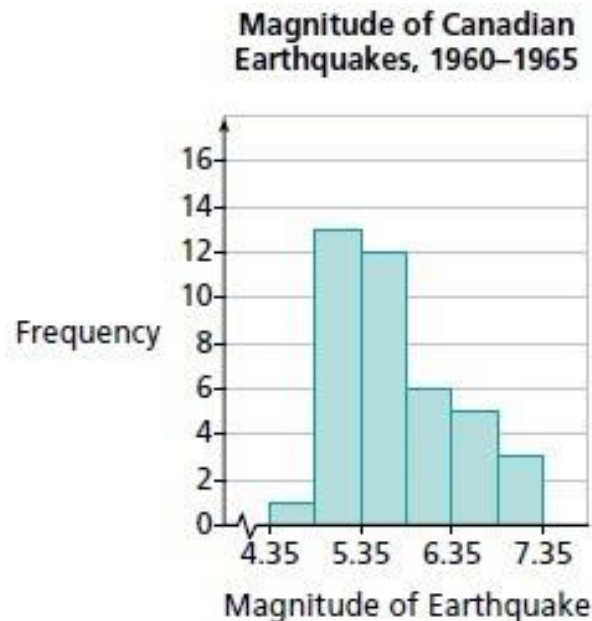
## What do mean by Data distribution

A data distribution is a function or a listing which shows all the possible values (or intervals) of the data. It also (and this is important) tells you how often each value occurs

Histogram is the graphical tool used to identify distributions

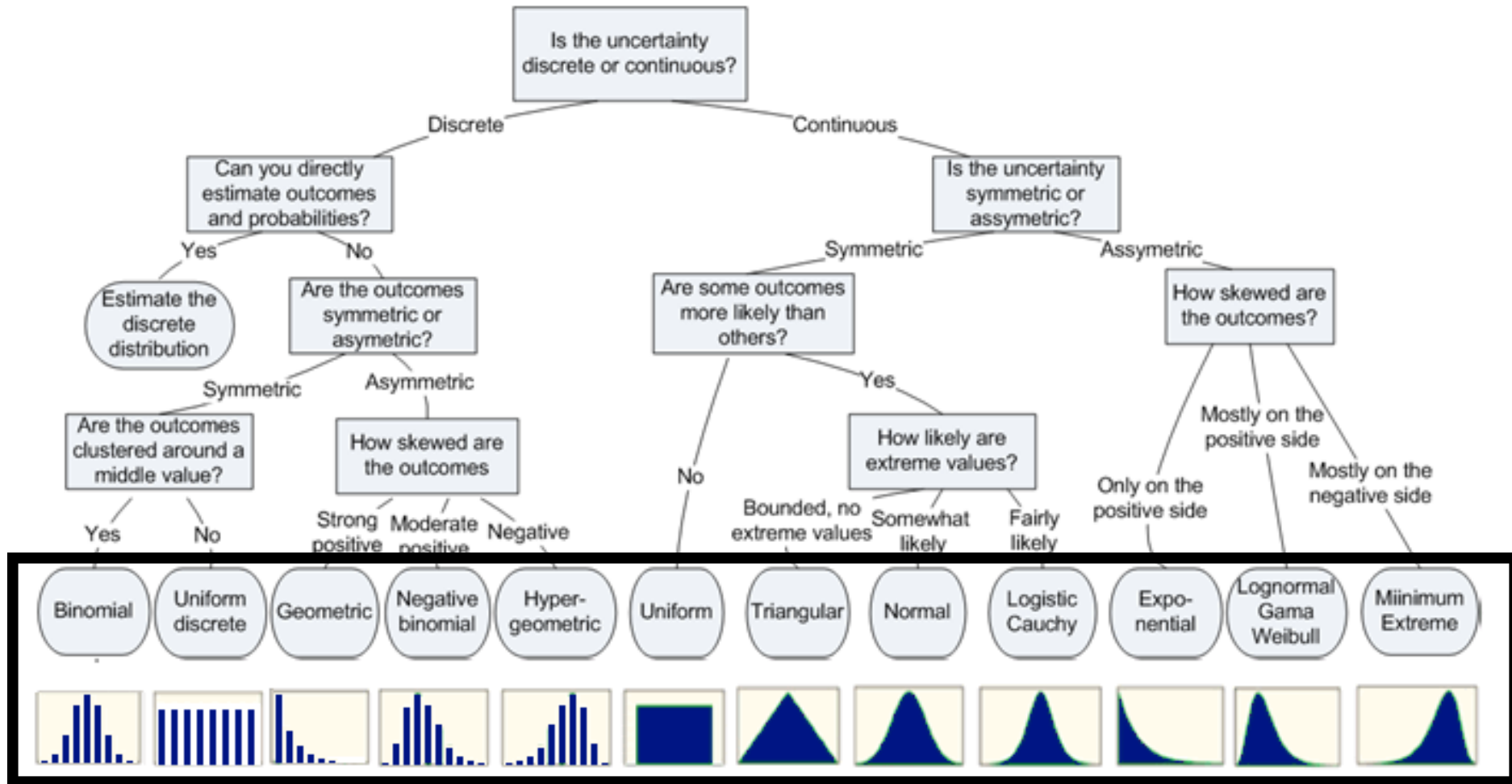
Frequency Distribution

Class	Tally	Frequency
4.35–4.85		1
4.85–5.35		13
5.35–5.85		12
5.85–6.35		6
6.35–6.85		5
6.85–7.35		3



Minitab exercise  
Refer : Histogram

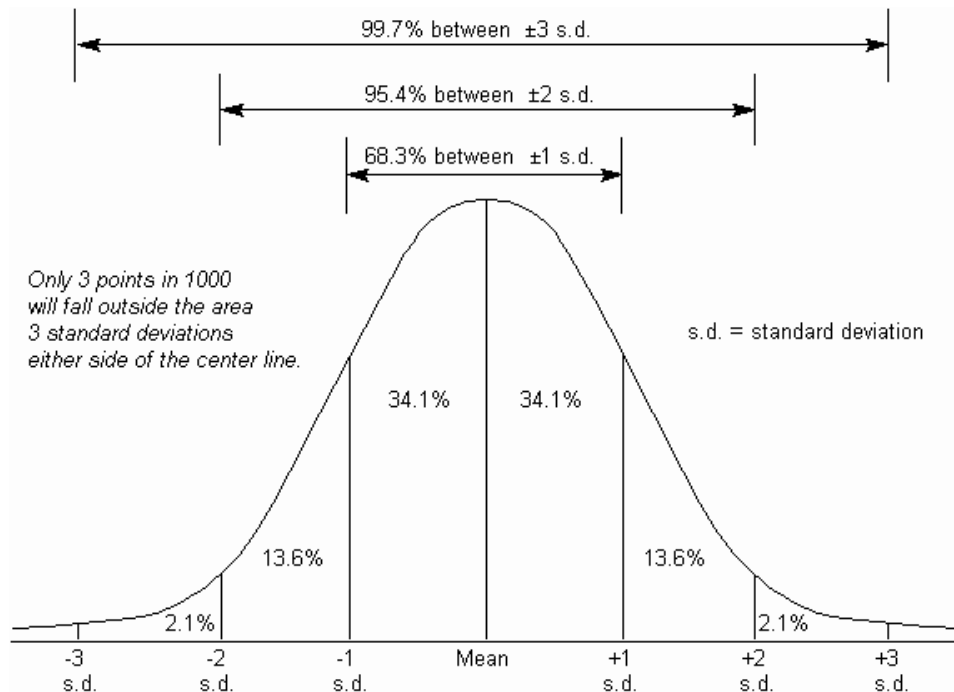
### Different types of distribution



## Different types of distribution

### Normal distribution

Normal distribution, also known as the Gaussian distribution, is a probability distribution that describes how the values of a variable are distributed. It is a symmetric distribution where most of the observations cluster around the central peak and the probabilities for values further away from the mean taper off equally in both directions.



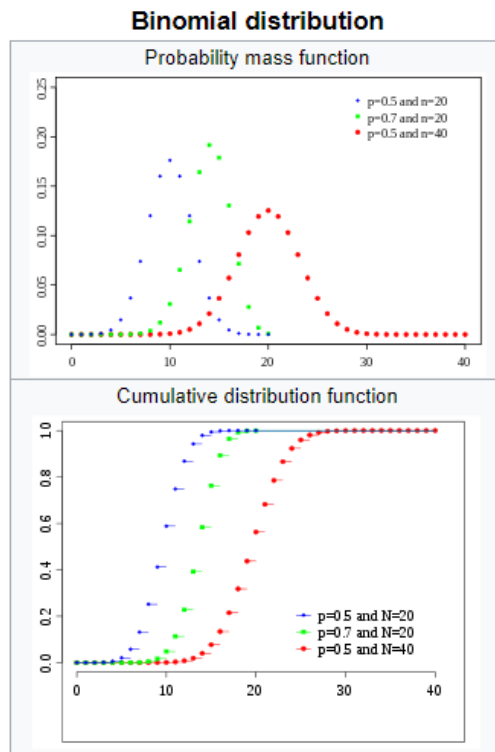
Normal distributions are symmetric, unimodal, and asymptotic, and the mean, median, and mode are all equal. A normal distribution is perfectly symmetrical around its center. That is, the right side of the center is a mirror image of the left side.

Continuous (Variable) data generally tend to take normal distribution

## Different types of distribution

### Binomial distribution

A Binomial distribution also called Bernoulli distribution can be thought of as simply the probability of a SUCCESS or FAILURE outcome in an experiment or survey that is repeated multiple times. The binomial is a type of distribution that has two possible outcomes



Probability Mass function (PFM) is the probability of getting exactly  $k$  successes in  $n$  independent Bernoulli trials is given by the probability mass function

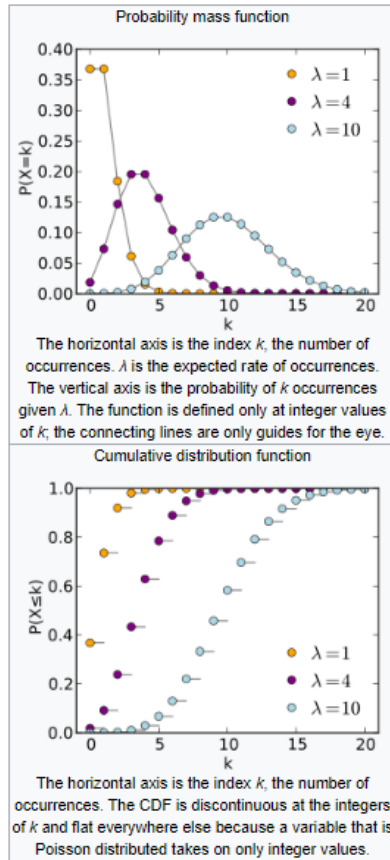
Example : Suppose a biased coin comes up heads with probability 0.3 when tossed. The probability of seeing exactly 4 heads in 6 tosses

Cumulative Distribution function (CDF) The cumulative distribution function (cdf) is the probability that the variable takes a value less than or equal to  $x$

Example : What is the probability of GDP of next quarter being less than 0

Binomial (Discrete Binary) generally tend to take normal distribution

## Different types of distribution



The Poisson distribution named after French mathematician Siméon Denis Poisson is used to model the number of events occurring within a given time interval

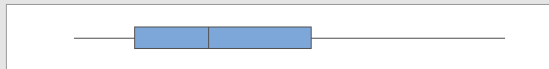
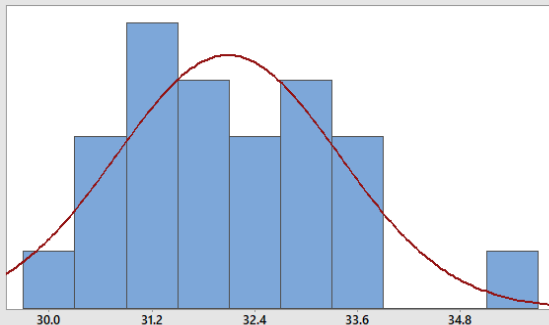
Example : This is a Poisson experiment in which we know the following:  $\mu = 2$ ; since 2 homes are sold per day, on average.  $x = 3$ ; since we want to find the likelihood that 3 homes will be sold tomorrow

Defects (Discrete Count) generally tend to take Poisson distribution

## Distribution identification

### Normality Test

Summary Report for AHT1



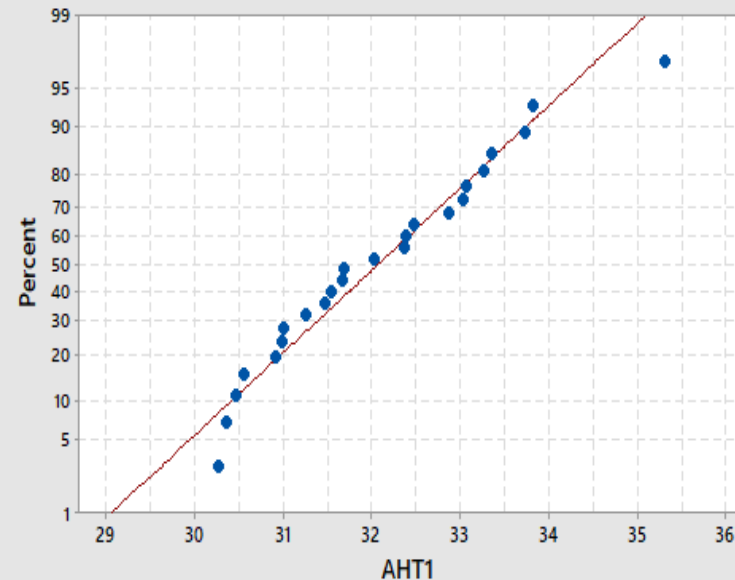
95% Confidence Intervals



Anderson-Darling Normality Test

A-Squared	0.31
P-Value	0.539
Mean	32.085
StDev	1.296
Variance	1.680
Skewness	0.566703
Kurtosis	-0.029489
N	24
Minimum	30.279
1st Quartile	31.001
Median	31.854
3rd Quartile	33.061
Maximum	35.322
95% Confidence Interval for Mean	31.538 32.632
95% Confidence Interval for Median	31.211 32.905
95% Confidence Interval for StDev	1.008 1.818

Probability Plot of AHT1  
Normal



Mean	32.09
StDev	1.296
N	24
AD	0.306
P-Value	0.539

P-Value > 0.05 Refers to Normal Distribution

P-Value < 0.05 Refers to Non Normal Distribution

Minitab exercise





## Why MSA ?

### Calibration Vs MSA



Calibration



MSA

## Terminologies - Resolution

The ability of the measurement system to detect variation to a meaningful extent compared to the process variation or specification.

General rule :

Less than 1/10 of total tolerance.

(Refer ISO10012:2003 for more details)



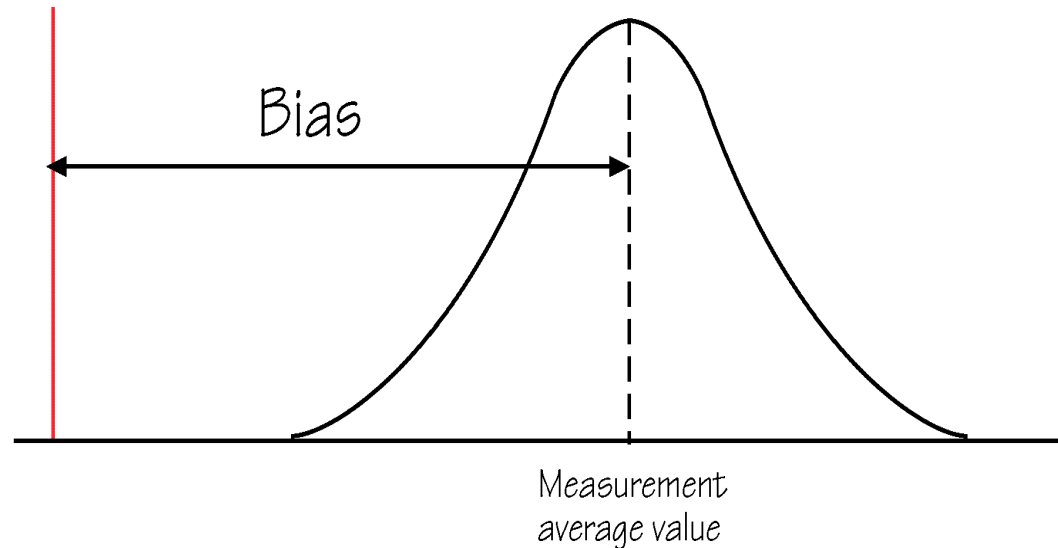
## Terminologies - Bias

Practical version of the term Accuracy.

Difference between the average measurement and a reference value.

- One part  
(Get reference from NABL Accredited Lab)
  - One operator
  - One equipment
- Repeat 10-15 times

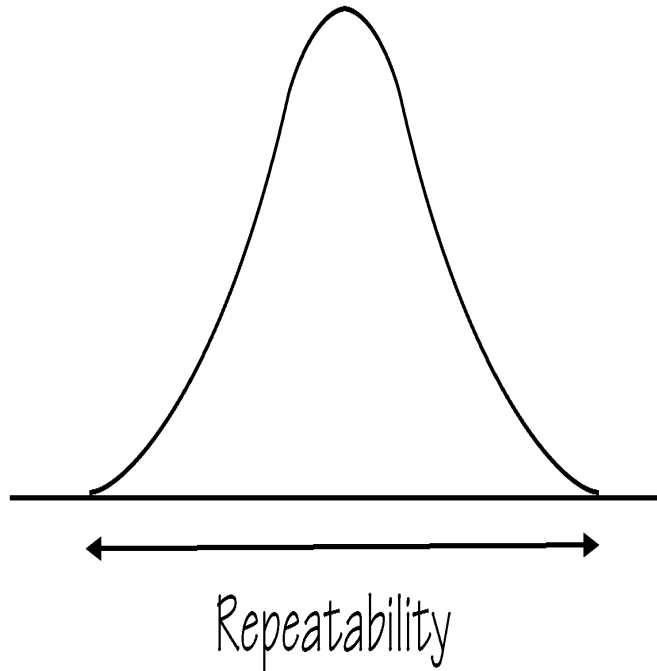
Reference value



### Terminologies - Repeatability (EV)

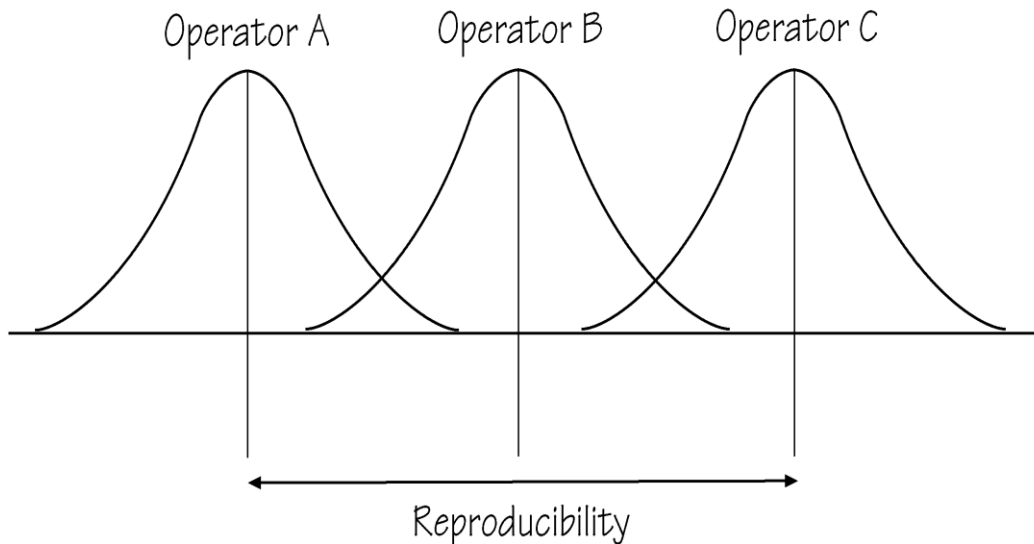
Inherent variation in the measurement system.

- One part
  - One operator
  - One equipment
- Measured 10-15 times



## Terminologies - Reproducibility (AV)

The ability of different inspectors to achieve the same result.



Same part, Same equipment, More than one appraiser.

## Setting up & Conducting GR&R study

### GR&R

Combined *repeatability* and *reproducibility*.

Defines the capability of measurement system.

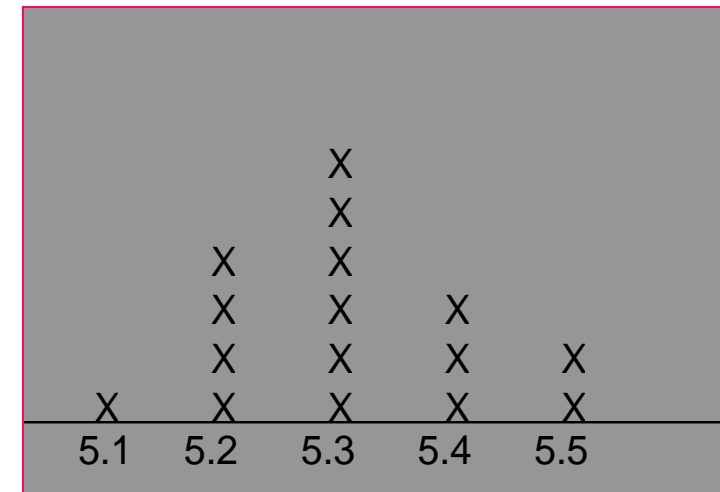
< 10% - Good, 10 – 30% - Conditionally Acceptable

> 30% - Not Acceptable

## Setting up & Conducting GR&R study

### No. of distinct data categories – ndc

Determines whether the measurement system is capable to discriminate different parts.



## Attribute Measurement Systems

- These measurement systems utilize accept /reject criteria are ratings to determine the acceptable level of quality.
- Kappa techniques are used to evaluate these measurement systems.

## What is Kappa (k) ?

For a given scenario Kappa values indicate the strength of agreement.

Values of kappa range from -1 to 1.

- Negative values reflect disagreement
- Positive values reflect agreement

## Steps to calculate Kappa - I

The first entry in each cell of the table represents the no. of observed agreements during the study.

		Rater 1		
		Count	Yes	No
Rater 2	Yes	40	50	90
	No	0	10	10
		40	60	100



## Steps to calculate Kappa - II

The second entry in each cell of the table represents no. of expected, agreement.

		Rater 1		
Rater 2	Count	Yes	No	
	Expected			
	Yes	40	50	90
	No	0	10	10
		36	54	
		4	6	
		40	60	100

The expected counts are:

In the Yes/Yes cell:

$$(40 \times 90) / 100 = 36.0.$$

In the No/Yes cell:

$$(40 \times 10) / 100 = 4.0.$$

In the Yes/No cell:

$$(60 \times 90) / 100 = 54.0.$$

In the No/No cell:

$$(60 \times 10) / 100 = 6.0.$$

### Steps to calculate Kappa - III

$$K = \frac{\text{No. Observed Agreement} - \text{No. Expected Agreement}}{\text{Total Observations} - \text{No. Expected Agreement}}$$

**No. of Observed agreement**  
**= 40+10 = 50**

**No. of Expected agreement**  
**= 36 + 6 = 42**

**Total Observation = 100**  
**Kappa = 0.138**

		Rater 1		
Rater 2	Count	Yes	No	
	Expected			
	Yes	40	50	90
	No	36	54	
		0	10	10
		4	6	
		40	60	100

### What is the acceptable value?

Kappa	Strength of Agreement
< 0.00	Poor or None
0.00 – 0.20	Slight
0.21 – 0.40	Fair
0.41 – 0.60	Moderate
0.61 – 0.80	Substantial
0.81 – 1.00	Almost perfect

### Special Cause & Common Cause

Day of the week	Travel Time (Minutes)
Monday	36
Tuesday	34
Wednesday	38
Thursday	91 ( Special Cause)
Friday	39
Saturday	37
Sunday	17 (Special Cause)

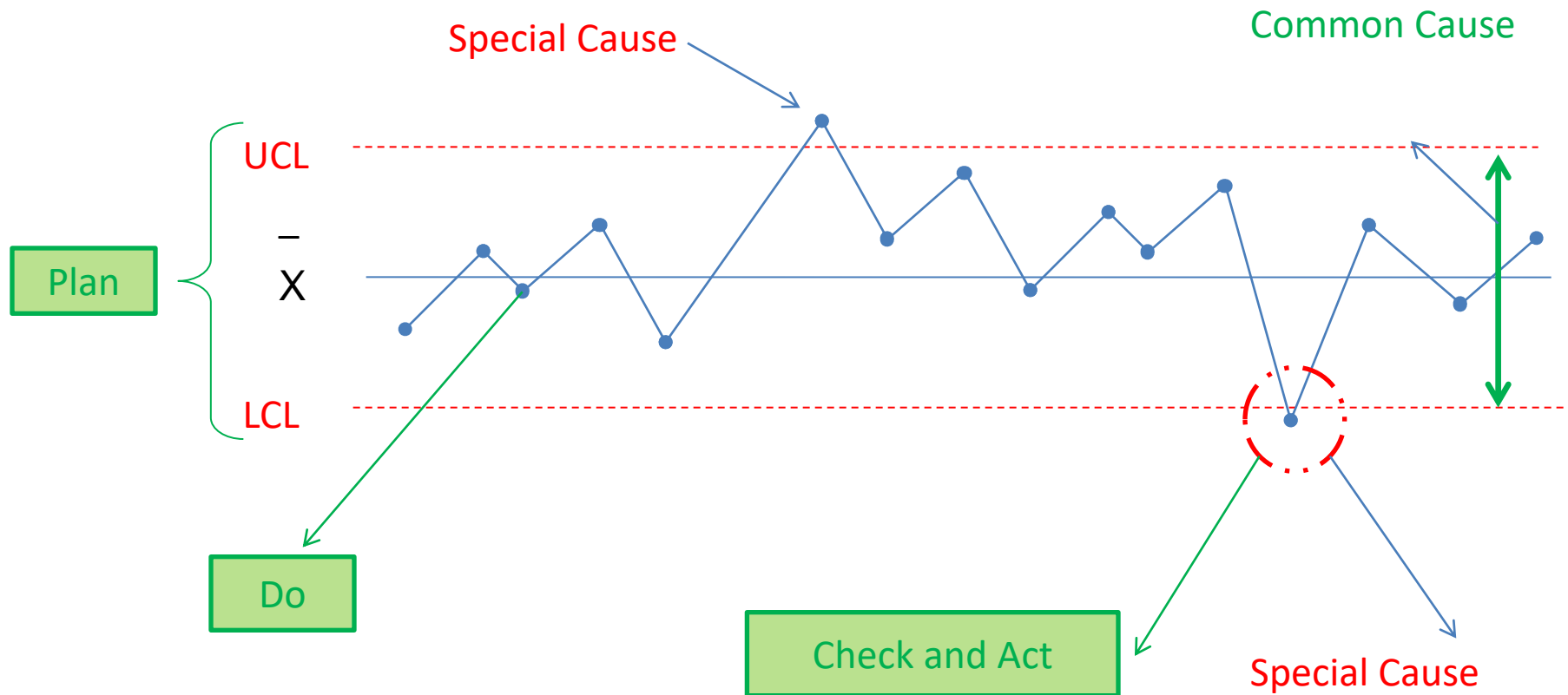


**Walter Shewhart 16<sup>th</sup>  
May 1924**

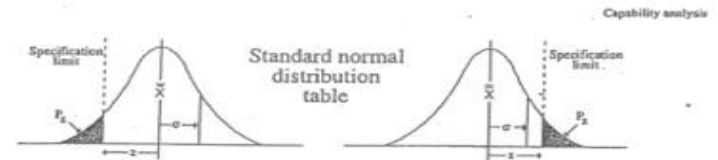
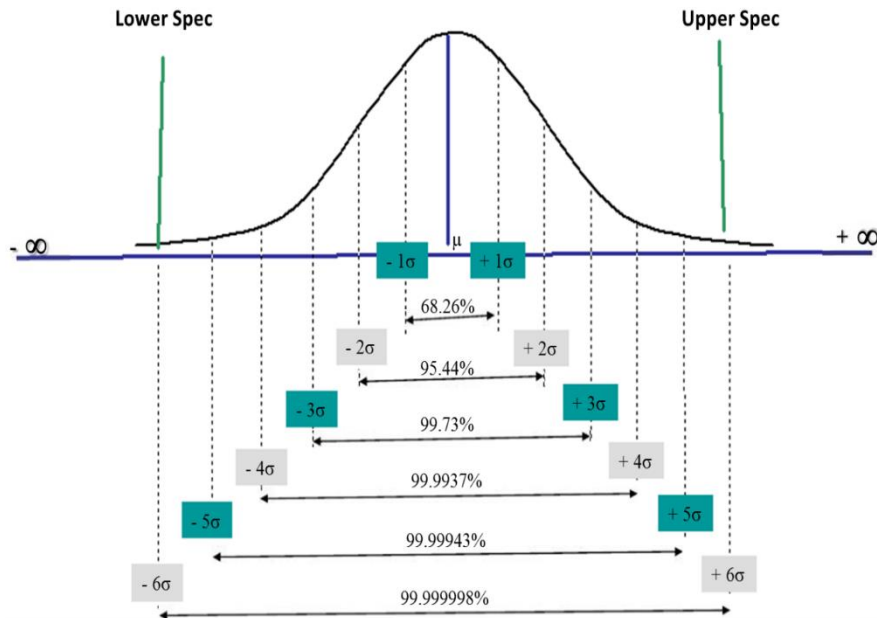
**What should you do when you observe a special cause ?**

**If the time taken to reach office the following Monday is 45 minutes, Is it a special cause or common cause ?**

# Concept of Stability



### Normal Distribution



z	x.x0	x.x1	x.x2	x.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
4.0	.00003									
3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

## Concept of Capability

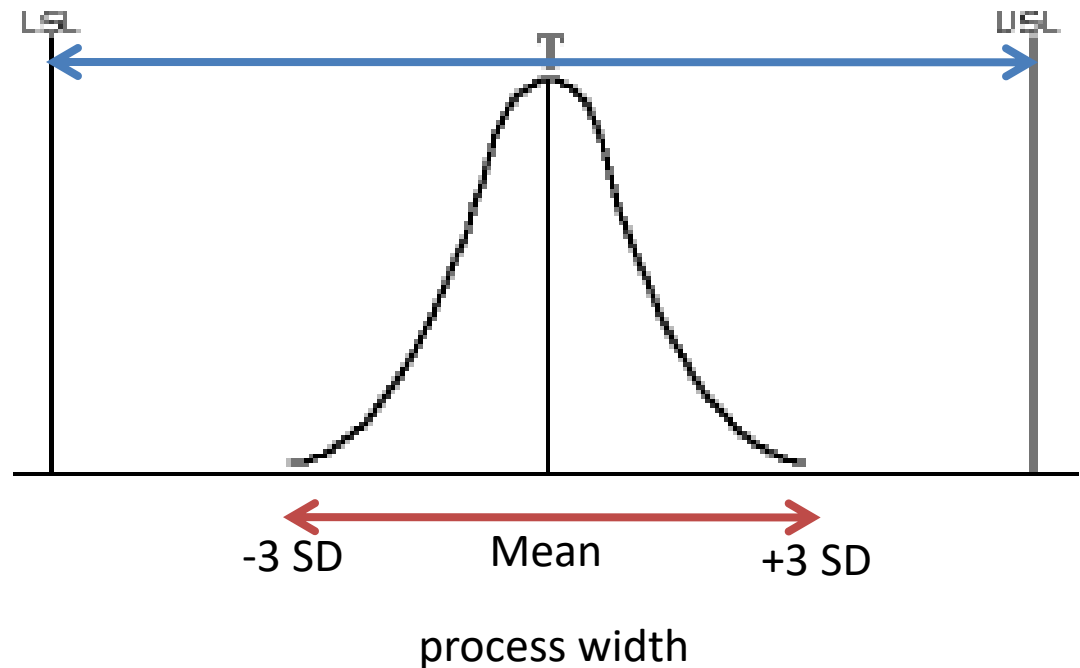
**Process output meets customer specifications**

### Specification Width

(USL-LSL)

### Process Width

( $\pm 3$  standard deviations from the mean)



### Data for process capability

Date	Time	1	2	3	4	5
2-Sept.2012	8am	35	37	41	36	40
2-Sept.2012	9am	40	39	42	38	43
2-Sept.2012	10am	47	41	39	40	41
2-Sept.2012	11am	40	36	37	39	36
2-Sept.2012	12noon	43	44	42	40	39
2-Sept.2012	1pm	38	39	41	40	37
2-Sept.2012	2pm	42	36	40	41	38
2-Sept.2012	3pm	43	44	43	39	41
2-Sept.2012	4pm	36	38	35	39	37
2-Sept.2012	5pm	36	41	40	42	39
3-Sept.2012	8am	42	41	40	38	37
3-Sept.2012	9am	37	38	40	40	37
3-Sept.2012	10am	44	41	40	39	40
3-Sept.2012	11am	39	38	35	36	40
3-Sept.2012	12noon	42	43	45	40	39
3-Sept.2012	1pm	35	38	37	39	41
3-Sept.2012	2pm	35	36	39	38	37
3-Sept.2012	3pm	43	42	43	43	39
3-Sept.2012	4pm	43	39	41	42	44
3-Sept.2012	5pm	39	37	36	35	36
4-Sept.2012	8am	43	40	41	40	39
4-Sept.2012	9am	35	39	36	41	38
4-Sept.2012	10am	41	38	37	37	40
4-Sept.2012	11am	34	39	40	36	37
4-Sept.2012	12noon	43	39	41	42	40

Grand Average = 39.39

Average Range = 4.96

n= 5

USL = 42 , LSL = 36

Sample size	A2	d2	D3	D4	E2
5	0.577	2.326	--	2.114	1.290

Control chart weighing factors



$$\hat{\sigma} = \frac{\bar{R}}{d_2} = 2.13$$

$$Z_{upper} = \frac{USL - \bar{X}}{\hat{\sigma}} = 1.22 \rightarrow 0.1112$$

$$Z_{lower} = \frac{\bar{X} - LSL}{\hat{\sigma}} = 1.59 \rightarrow 0.0559$$

Total out of specification = 11.12% + 5.59% = 16.71%

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = 0.47$$

$$C_{pk} = \frac{Z_{\min}}{3} = 0.41$$

New product or service.

- Little data.
- Trial run, with data collection.
- Complete control chart and histogram.
- Complete analysis as before but use *actual* standard deviation instead of estimated standard deviation.

*Cp becomes Pp*

$$Pp = \frac{USL - LSL}{6\sigma}$$

*Interpret as Cp*

*Cpk becomes Ppk*

$$Ppk = \frac{Z_{\min}}{3}$$

*Interpret as Cpk*

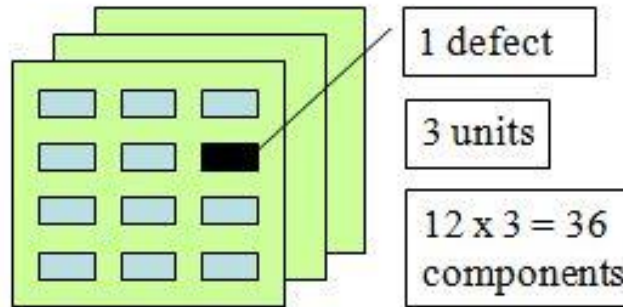
$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

## DPMO

### Defects: DPU and DPMO

#### Defects Per Unit

$$\text{DPU} = \frac{\text{Defects detected}}{\text{Units processed}} = 1/3$$



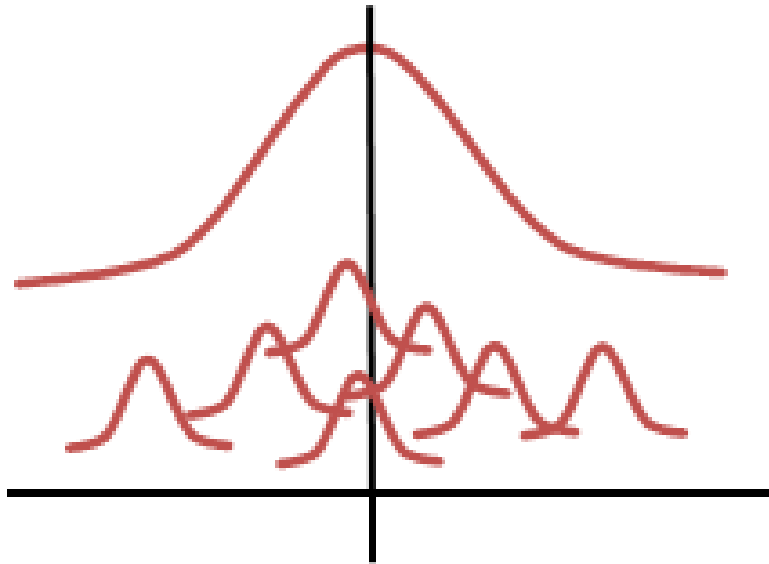
#### Defects Per Million Opportunities

$$\text{DPMO} = \frac{\text{Defective components X 1 000 000}}{\text{Components used}} = 27\,777$$

DPMO

# Sigma Level Conversion Table

Yield	DPMO	Sigma	Yield	DPMO	Sigma	Yield	DPMO	Sigma
6.6%	934,000	0	69.2%	308,000	2	99.4%	6,210	4
8.0%	920,000	0.1	72.6%	274,000	2.1	99.5%	4,660	4.1
10.0%	900,000	0.2	75.8%	242,000	2.2	99.7%	3,460	4.2
12.0%	880,000	0.3	78.8%	212,000	2.3	99.75%	2,550	4.3
14.0%	860,000	0.4	81.6%	184,000	2.4	99.81%	1,860	4.4
16.0%	840,000	0.5	84.2%	158,000	2.5	99.87%	1,350	4.5
19.0%	810,000	0.6	86.5%	135,000	2.6	99.90%	960	4.6
22.0%	780,000	0.7	88.5%	115,000	2.7	99.93%	680	4.7
25.0%	750,000	0.8	90.3%	96,800	2.8	99.95%	480	4.8
28.0%	720,000	0.9	91.9%	80,800	2.9	99.97%	330	4.9
31.0%	690,000	1	93.3%	66,800	3	99.977%	230	5
35.0%	650,000	1.1	94.5%	54,800	3.1	99.985%	150	5.1
39.0%	610,000	1.2	95.5%	44,600	3.2	99.990%	100	5.2
43.0%	570,000	1.3	96.4%	35,900	3.3	99.993%	70	5.3
46.0%	540,000	1.4	97.1%	28,700	3.4	99.996%	40	5.4
50.0%	500,000	1.5	97.7%	22,700	3.5	99.997%	30	5.5
54.0%	460,000	1.6	98.2%	17,800	3.6	99.9980%	20	5.6
58.0%	420,000	1.7	98.6%	13,900	3.7	99.9990%	10	5.7
61.8%	382,000	1.8	98.9%	10,700	3.8	99.9992%	8	5.8
65.6%	344,000	1.9	99.2%	8,190	3.9	99.9995%	5	5.9
						99.99966%	3.4	6



**Long Term** (most representative of the population and may be the entire population)

**Short Term** samples

Long term sigma = Short term sigma – 1.5

**Without Data,  
you are just another person  
with an Opinion.**

**Data is absolutely useless.  
It's what you do with it that matters.**

