

# **ADVANCED DATA STRUCTURES and** **ALGORITHMS**

## **Red-Black Trees**

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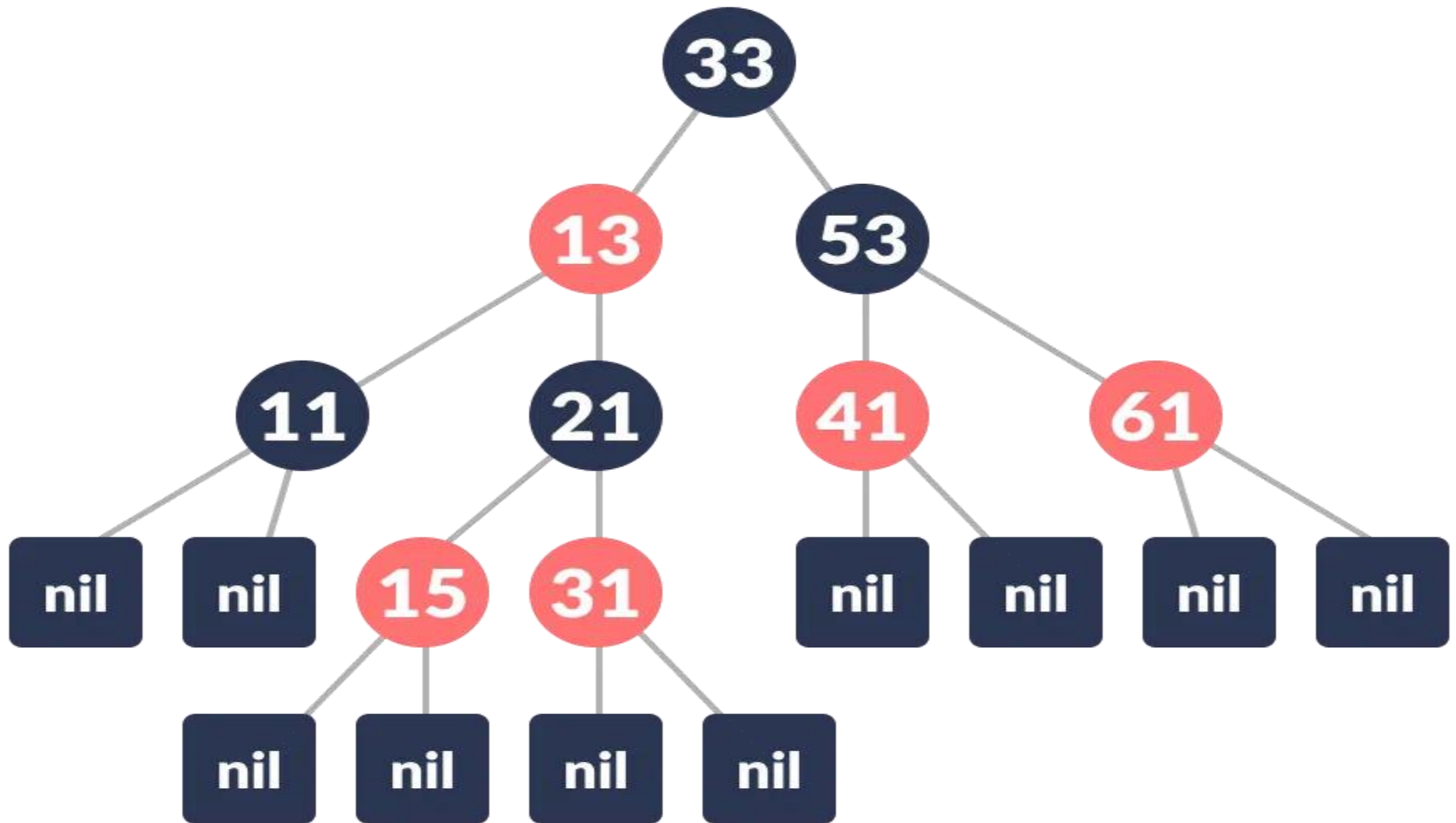
**Department of Information Technology**

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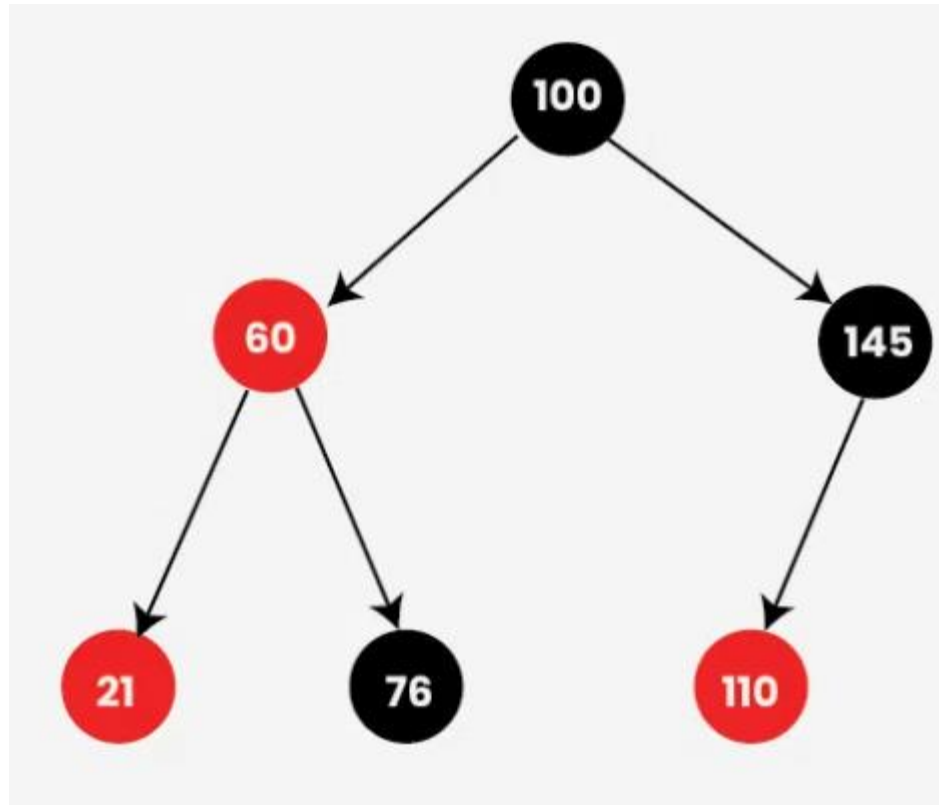
# Introduction to Red-Black Trees

- **Definition:** A self-balancing binary search tree with additional properties.
- **Properties:**
  - **Node-color Property:** Every node should be colored either red or black.
  - **Root Property:** The root is always black.
  - **Red Property:** Red nodes cannot have red children.
  - **Black Property:** Every path from a root node to every leaf node has the same number of black nodes.
  - **Leaf Property:** All NULL nodes are black.

# Example for Red-Black Tree



# Is it Red-Black Tree?



# Operations on Red-Black Trees

- The common operations
  - Traversing
  - Searching
  - Insertion
  - Deletion

# Operations on Red-Black Trees

- 2 ways to perform the insertion/deletion operations in Red-Black Trees.
- **Bottom-Up operations:**
  - First perform the operation by searching in top down manner
  - Restore the properties by recursively travel back the tree from bottom.
- **Top-Down Operations:**
  - Rotations and recoloring are done while traversing down the tree to the insertion or deletion point.

# Bottom-Up Insertion Overview

## Steps Involved:

- Insert the node like in a BST.
- Color the new node red.
- Fix any violations of the red-black properties.

# Insertion: Fixing the violations Cases

**Case 1: The new node is the root node.**

- Recolor it to black.

**Case 2: The parent node is black.**

- The tree remains valid.

**Case 3: Parent and parent sibling are both red.**

- Recolor the parent and parent sibling to black
- Grandparent to red.

**Case 4: Parent is red, parent sibling is black**

- Apply suitable Rotation
- After rotation assign parent to black and both the child's to red.



# Example-1 of Insertion

- **Create a Red-Black Tree for the following Data:**

**8, 18, 5, 15, 17, 25, 40, and 80**

# Example-1 of Insertion

create a Red-Black tree for the numbers  
8, 18, 5, 15, 17, 25, 40 and 80.

insert 8  
⇒



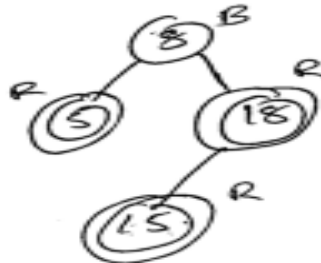
insert 18  
⇒



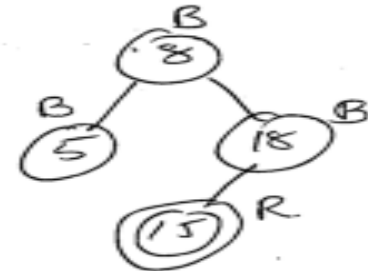
insert 5  
⇒



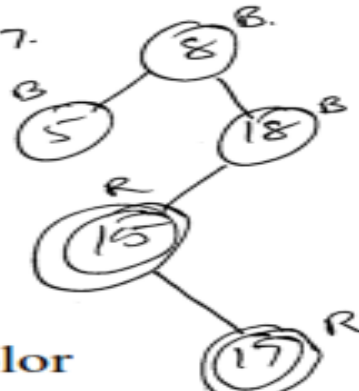
insert 15.  
⇒



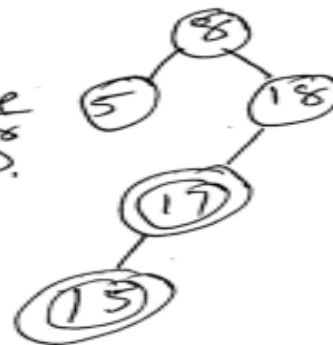
Recolor  
⇒



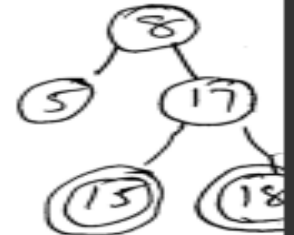
insert 17.  
⇒



LR & Recolor  
⇒

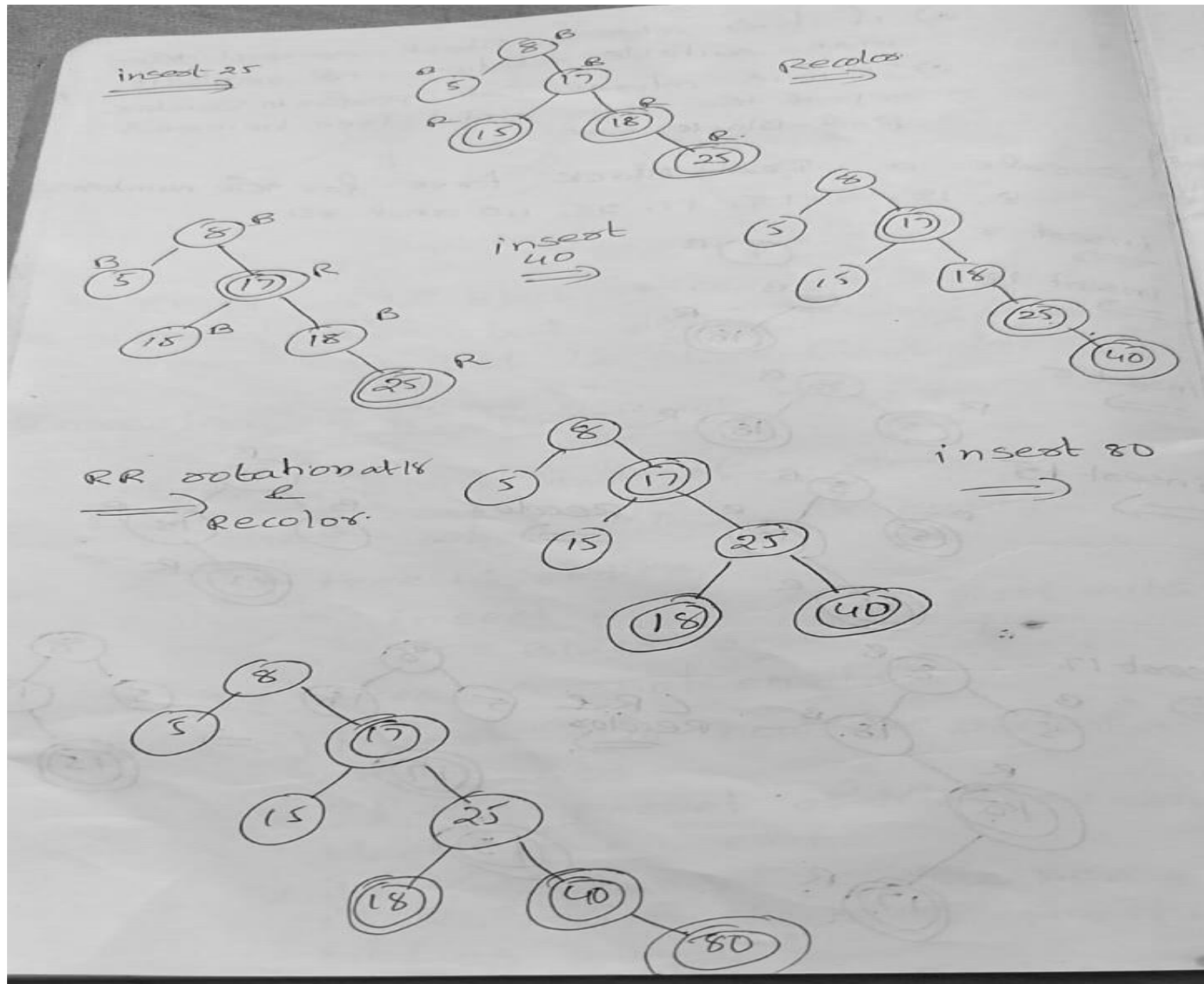


⇒



Sibling color

# Example-1 of Insertion



## Example-2 of Insertion

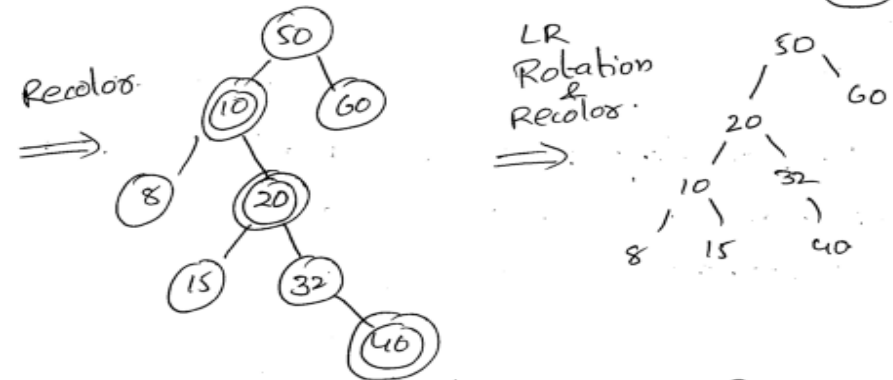
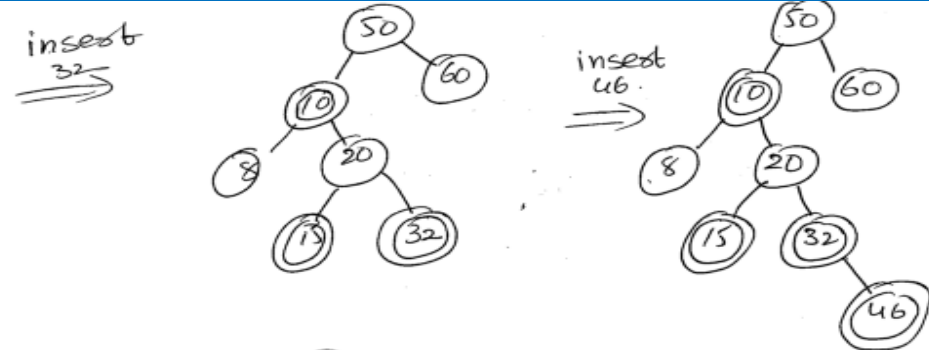
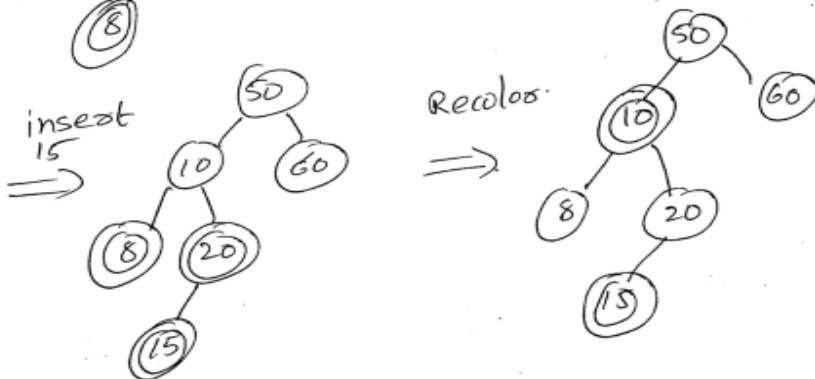
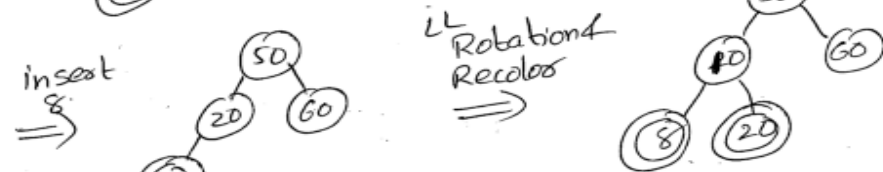
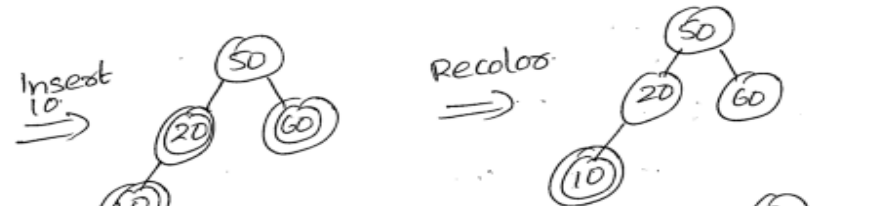
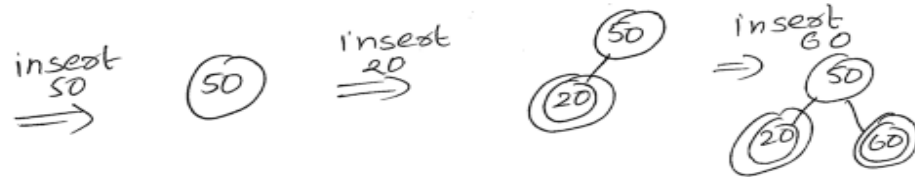
ex: create a Red-Black tree for the numbers

50, 20, 60, 10, 8, 15, 32, 46, 11, 48

# Example-2 of Insertion

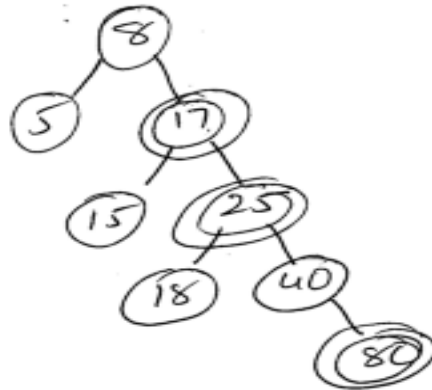
ex2: create a Red-Black tree for the numbers

50, 20, 60, 10, 8, 15, 32, 46, 11, 48



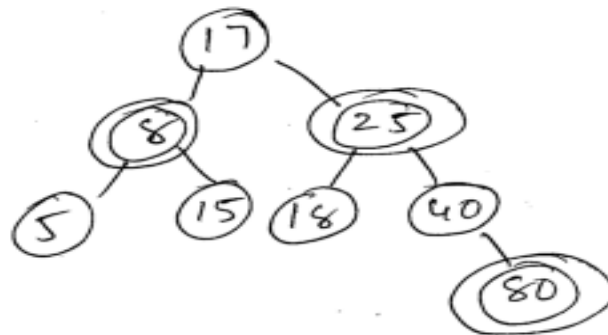
# Example-2 of Insertion

Recolor



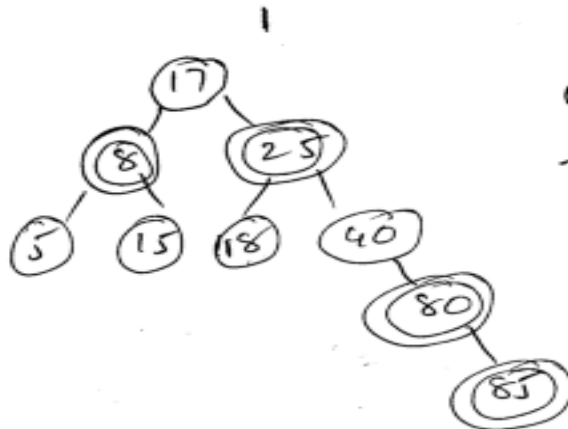
RR Rotation at 8  
& Recolor

⇒



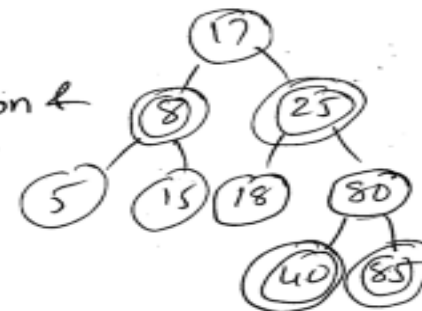
insert 85

⇒



RR rotation &  
Recolor

⇒



# Bottom-Up Deletion Overview

- **Steps Involved:**
  - Perform a standard BST deletion.
  - **If the deleted node was red**, the tree remains valid.
  - **If the deleted node was black**, fix any violations by recoloring and rotations.

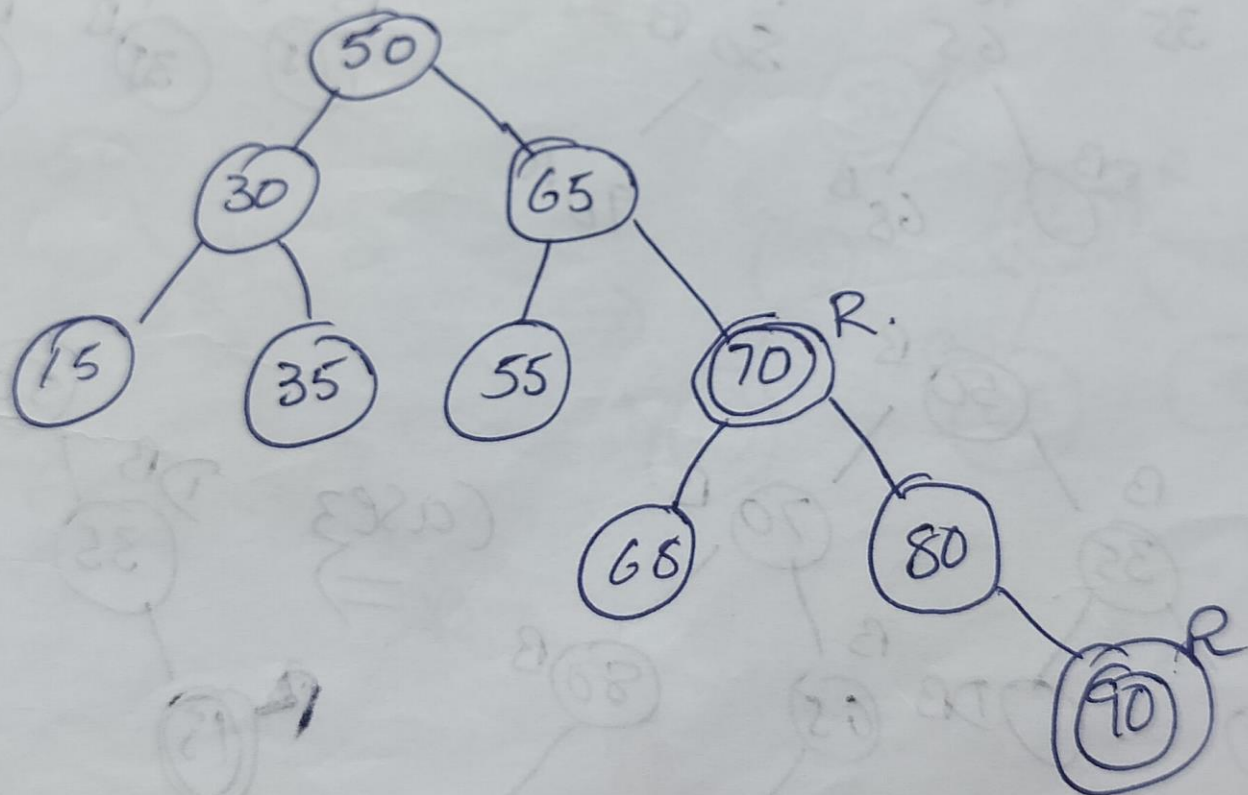
# Deletion Cases in Detail

- if the node to be deleted is black mark it as double black (DB).
- **Case-1: If root is "DB"** → just remove DB consider it as black.
- **Case-2: if "DB" sibling is black & both its children are also black.**
  - i. add extra black to its Parent (  $P$  )  
if  $P$  is red → becomes black.      if  $P$  is black → "DB".
  - ii. make the sibling Red.
  - iii. if "DB" still exists repeat the cases.
- **Case 3: if "DB" sibling is Red**
  - i. swap the color of  $P$  and sibling.
  - ii. Rotate the  $P$  towards "DB".
  - iii. reapply the cases by considering "DB".
- **Case 4: if "DB" sibling is black and the sibling child who is near to "DB" is red and far from "DB" is black.**
  - i. swap the sibling color with its red child color.
  - ii. rotate the sibling opposite to "DB".
  - iii. Apply case 5.
- **Case 5: if "DB" sibling is black and the sibling far child is red and near child is black.**
  - i. swap the parent and sibling color of DB.
  - ii. rotate parent towards "DB".
  - iii. give the extra black of  $DB$  to red child of sibling.

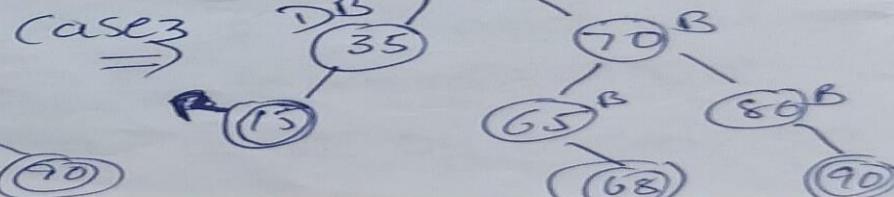
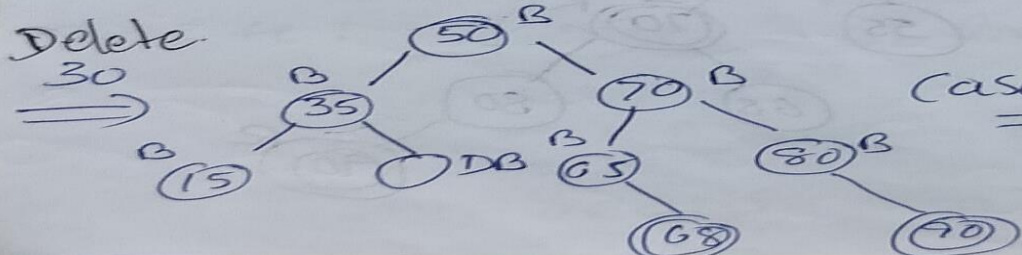
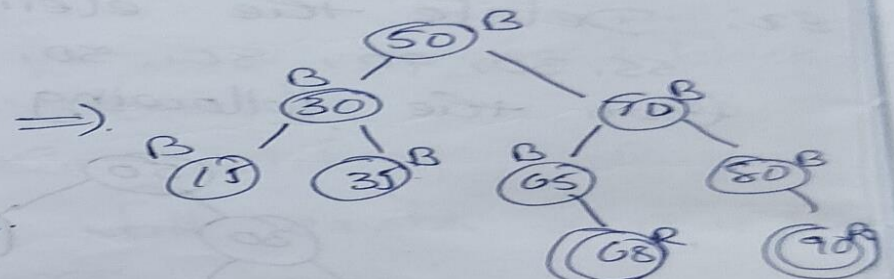
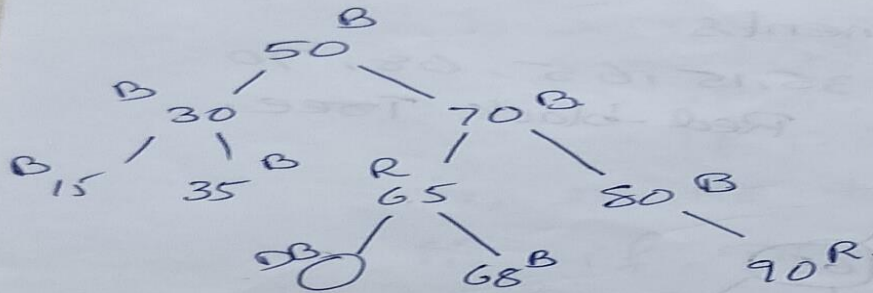
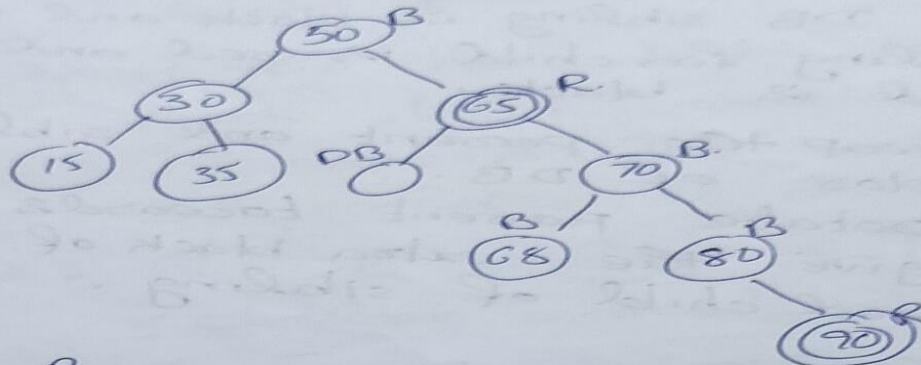
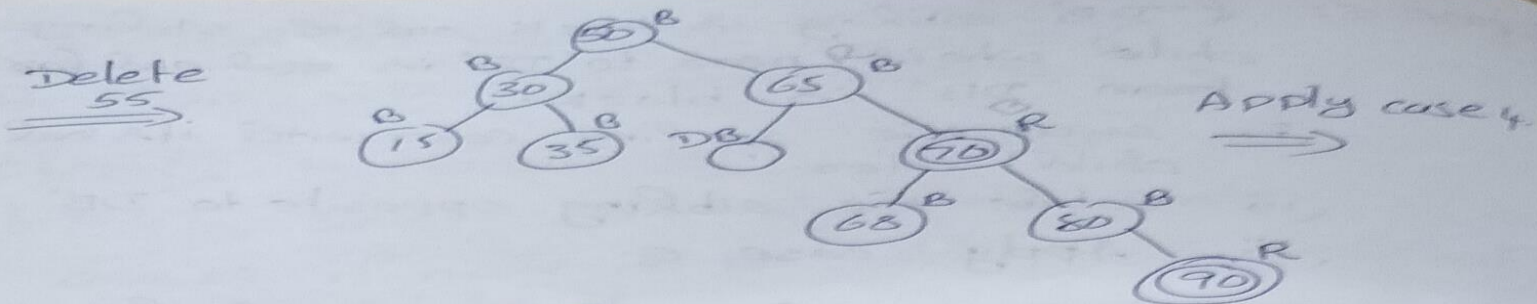


# Example of Deletion

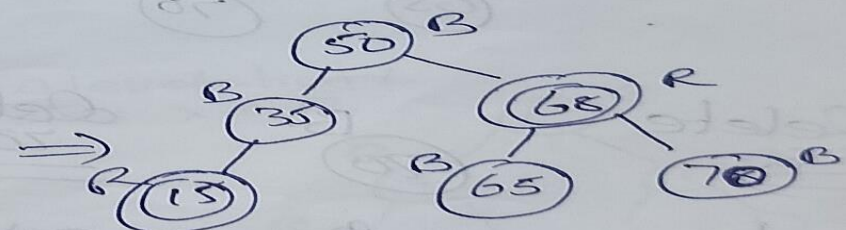
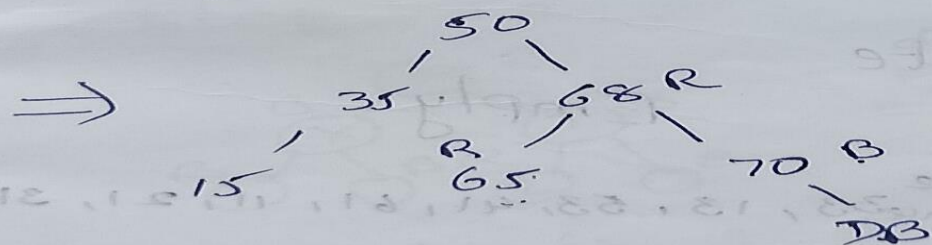
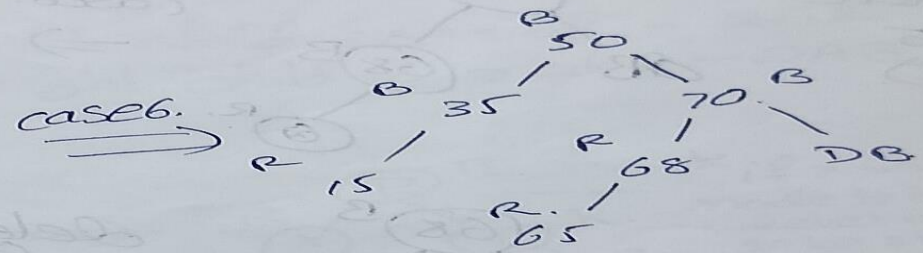
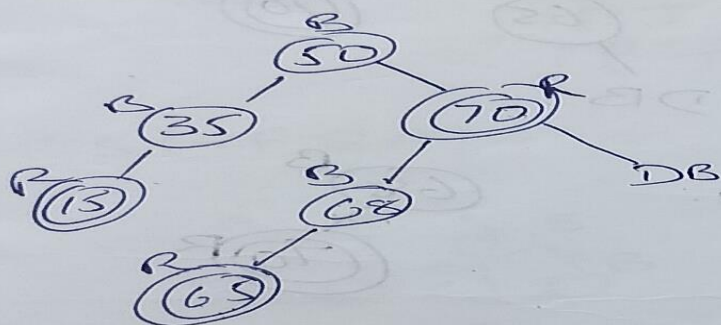
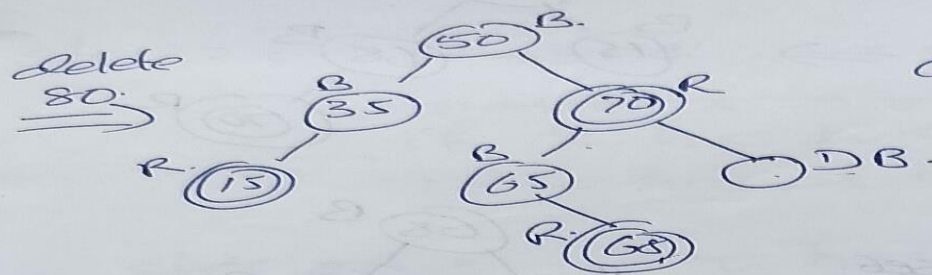
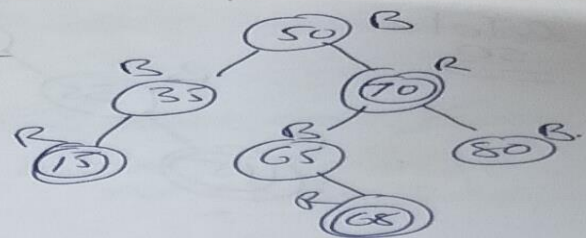
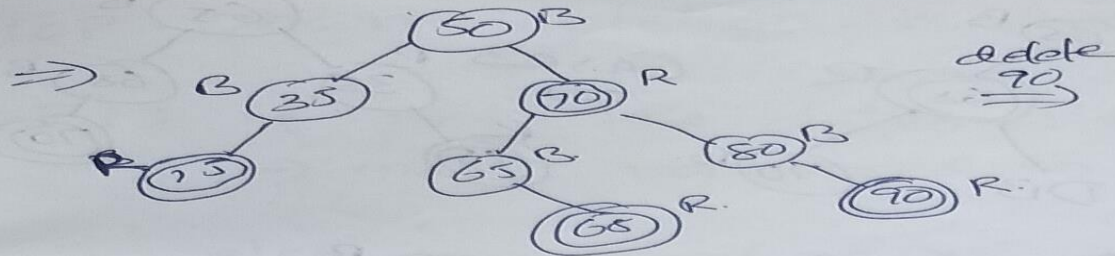
Ex: Delete the elements  
55, 30, 90, 80, 50, 35, 15, 65, 68, 70  
from the following Red-black Tree.



# Example of Deletion



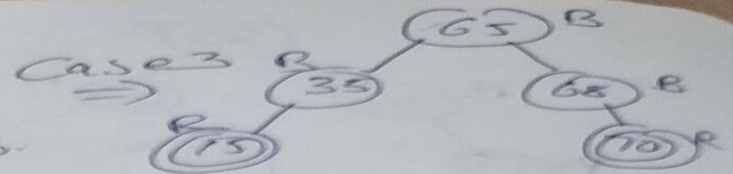
# Example of Deletion





# Example of Deletion

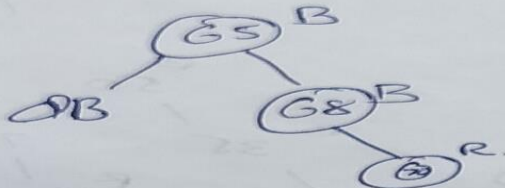
Delete 50



Delete 35



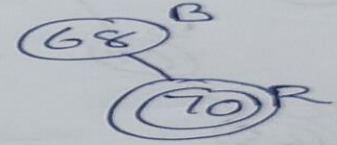
Delete 15



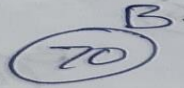
Case 6



Delete 65



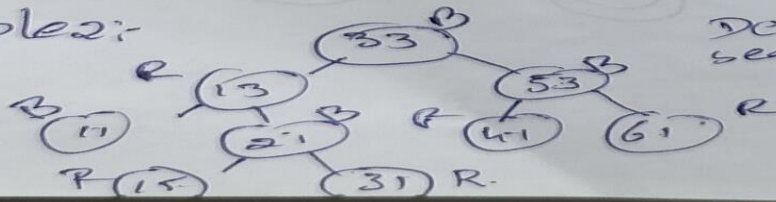
Delete 68



Delete 70

Empty

Example 2:



Delete sequence

33, 13, 53, 41, 61, 11, 21, 31, 15

# Advantages and Disadvantages

- **Advantages:**

- Efficient insertions, deletions, and lookups.
- More straightforward balancing rules compared to AVL trees.

- **Disadvantages:**

- More cases than AVL trees in some operations.
- Complexity in understanding the balancing rules.

# Applications

- **Use Cases in Real-World Systems:**
  - Databases
  - File systems
  - Memory management
  - Associative containers in programming languages  
(e.g., C++, Java)

# Summary

- **Red-Black Trees ensure  $O(\log n)$  time complexity for operations.**
- **Insertion and deletion involve fixing the tree using rotations and recoloring.**
- **The tree maintains balance through its properties, making it efficient for large datasets.**

All birds find shelter during  
a rain. But eagle avoids rain  
by flying above the clouds.  
Problems are common,  
but **attitude makes  
the difference.**

— *Abdul Kalam*

