Advanced Data Structures and Algorithms

Introduction to Algorithms

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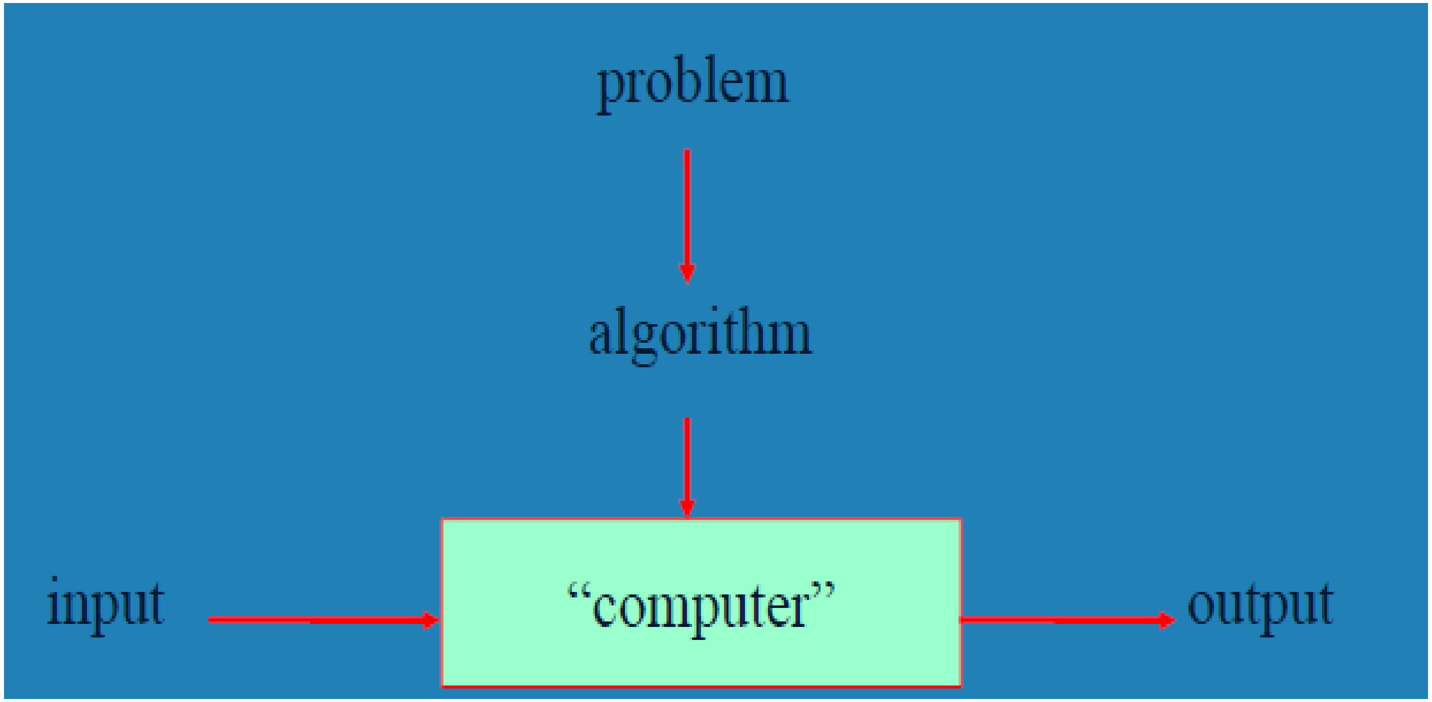
Topics

- **Algorithm Specification**
- **Pseudocode Convention**
- **Performance Analysis**
- **Asymptotic Notations**

What is an Algorithm

- **Definition**
  - An ***Algorithm*** is a finite set of instructions that, if followed, accomplishes a particular task.



Characteristics of an Algorithm

All algorithms must satisfy the following criteria:

(1)***Input***: There are zero or more quantities that are externally supplied.

(2)***Output***: At least one quantity is produced.

(3)***Definiteness***: Each instruction is clear and unambiguous.

(4)***Finiteness***: If we trace out the instructions of an algorithm, then for all cases, the algorithm terminates after a finite number of steps.

(5)***Effectiveness***: Every instruction must be basic enough to be carried out, in principle, by a person using only pencil and paper. It is not enough that each operation be definite and also must be feasible.

Study of Algorithms

- The study of algorithms includes many important and active areas of research.

- **Four distinct areas of study:**

  - **1. How to Devise Algorithm**

  - **2. How to Validate Algorithm**

  - **3. How to Analyze Algorithms**

  - **4. How to Test a Program**

Devise and Validate an Algorithm

- **Devise an Algorithm:**
  - Study various design techniques that have proven to be useful in that they have often yielded good algorithms.
  - Various design Strategies:
    - **Divide and Conquer**
    - **Greedy Method**
    - **Dynamic Programming**
    - **Backtracking**
    - **Branch & Bound**

- **Validate an Algorithm:**
  - Once an algorithm is devised, it is **necessary to show that it computes the correct answer for all possible legal inputs**.
  - Once the validity of the method has been shown, a program can be written and a second phase begins.

Analyze and Test the Algorithms

- **3. Analyze the Algorithm**
  - Analysis of algorithms or performance analysis refers to the task of **determining how much computing time and storage an algorithm requires.**

- **4. Test a Program**
  - Testing a program consists of two phases: **debugging and profiling** (or performance measurement).
  - Debugging is the process of executing programs on sample data sets to determine whether faulty results occur and, if so, to correct them.
  - A proof of correctness is much more valuable than a thousand tests(if that proof is correct),since it guarantees that the program will work correctly for all possible inputs.
  - Profiling or performance measurement is the process of executing a correct program on datasets and measuring the time and space it takes to compute the result.

Topics

- **Algorithm Specification**
- **Pseudocode Convention**
- **Performance Analysis**
- **Asymptotic Notations**

Describing Algorithms

- **Natural language**
  - English
    - Instructions must be definite and effectiveness
- **Graphic representation**
  - Flowchart
    - work well only if the algorithm is small and simple
- **Pseudocode**
  - Readable
  - Instructions must be definite and effectiveness

Pseudocode

- **Pseudocode:**
  - Implementation of an algorithm in the form of annotations and informative text written in plain English.
  - It has no syntax like any of the programming language and thus can’t be compiled or interpreted by the computer.
- **Advantages of Pseudocode**
  - Improves the readability of any approach.
  - Acts as a bridge between the program and the process. Also works as a rough documentation, so the logic of one developer can be understood easily when a pseudo code is written out.
  - Explains what exactly each line of a program should do, hence making the code construction phase easier for the programmer.
- **Disadvantages of Pseudocode**
  - Pseudocode does not provide a visual representation of the logic of programming.
  - There are no proper format for writing the for pseudocode.

Pseudocode Conventions

- **1.** Comments begin with **//** and continue until the end of line.
- **2.** Block of statements are indicated with matching braces:{ **and** }.
- **3.** Statements are delimited by **";"**.
- **4.** The data types of variables are not explicitly declared. Compound data types can be formed with records.

```
node = record
        {
            data type_1 data;
               .
               .
               .
            data type_n data;
            node *link;
        }
```

Pseudocode Conventions

- **5.** Assignment of values to variables is done using the assignment statement
  variable:=expression or value;

- **6.** There are two Boolean values true and false. In order to produce these values, the logical operators and, or, and not and the relational operators <, and > are provided.

- **7.** Elements of multi dimensional arrays are accessed using '[' and ']'.
  - For example, if A is a two dimensional array, the (i,j)th element of the array is denoted as - **A[i,j].**

- **8.** The while, repeat-until and for loops takes the following form:

```
while (condition)do
 {
statement 1   ...
statement n
}
```

```
Repeat
 {
Statement 1 ....
Statement n
} until(condition)
```

```
for variable:= value l to value 2 step value do
{
     (statement 1) ....
     (statement n)
}
```

Pseudocode Conventions

- **9.** A conditional statement has the following forms:
  - **if** (condition) then statement;
  - **if** (condition) then statement 1 **else** statement 2;
  - **case**

    {

    : condition 1: statement 1 .....

    : condition n: statement n

    : else: statement n + 1

    }

- **10.** Input and output are done using the instructions **read and write**.

- **11.** There is only one type of procedure: Algorithm.
  - An algorithm consists of a **heading and a body**.
  - The heading takes the form **Algorithm Name(parameter list)**

Tasks on Algorithms

- Write an algorithm to merge the given two sorted arrays as one sorted array.
- Write an algorithm for printing nth Fibonacci number.
- Write an algorithm to find the maximum product of two integers in the given array.

Tasks on Algorithms

```
Algorithm merge_sorted_arrays(arr1, arr2)
  {
     i := 0;          //Pointer for arr1
     j := 0 ;         // Pointer for arr2
     k := 0;          // Pointer for mergedArray
     while (i < n1 and j < n2) do // copy the
                                  //elements with
                                  //comparison
     {
         if (arr1[i] < arr2[j]) then
         {
             mergedArray[k] = arr1[i] ;
              i + := 1 ; k+ :=1;
          }
          else
          {
               mergedArray[k] = arr2[j];
                j += 1; k += 1;
           }
       }
```

```
If(i<n1) then
{
   for x := i to n1-1
    {
        mergedArray[k]:=  arr1[x];
        x +:= 1; k +:= 1;
     }
 }
 else
  {
     for x= j to n2-1
      {
          mergedArray[k]:=  arr2[x];
           x +:= 1 k +:= 1
        }
   }
}   // body close
```

Topics

- **Algorithm Specification**
- **Pseudocode Convention**
- **Performance Analysis**
- **Asymptotic Notations**

Performance Evaluation

- Evaluate a program
  - ***MWGWRERE***

    Meet specifications, **W**ork correctly,

    Good user-interface, **W**ell-documentation,

    Readable, **E**ffectively use functions,

    Running time acceptable,

    Efficiently use space

- How to achieve them?
  - Good programming style, experience, and practice

Performance Evaluation

- **Performance Evaluation**
  - Performance **Analysis**
  - Performance **Measurement**

- **Performance Analysis** - prior
  - estimate ***time*** – ***Time Complexity***
  - estimate ***space*** – ***Space Complexity***
  - machine independent

- **Performance Measurement** -posterior
  - The actual ***time*** and ***space*** requirements
  - machine dependent

Space Complexity

- Definition
  - The ***space complexity*** of a program is the amount of memory that it needs to run to completion
- The space needed is the sum of
  - ***Fixed*** space and ***Variable*** space

- Fixed space
  - Includes the instructions, variables, and constants
  - Independent of the number and size of Input and Output
- Variable space
  - Depends on an instance '*I*' of the problem
  - Includes dynamic allocation, functions' recursion
- Total space of any program
  - $S(P) = c + \boldsymbol{S_p(Instance)}$

Examples of Evaluating Space Complexity

```
float abc(float a, float b, float c)
{
  return a+b+b*c+(a+b-c)/(a+b)+4.00;
}
```

```
float sum(float list[], int n)
{
 float fTmpSum= 0;
 int i;
 for (i= 0; i< n; i++)
   fTmpSum+= list[i];
 return fTmpSum;
}
```

$S_{sum(n)}=0$

```
float rsum(float list[], int n)
{
  if (n) return rsum(list, n-1)+ list[n-1];
  return 0;
}
```

$S_{rsum}(n) = 3*n$

parameter: float(list[])	1
parameter: integer(n)	1
return address	1

Time Complexity

- **Definition**

  The ***time complexity, T(p),*** taken by a program P is the sum of the compile time and the run time

  T(P)= compile time + **run** (or **execution**) **time**

  $= \boldsymbol{c} + \boldsymbol{t}_p(n)$

  *Compile time does not depend on the instance characteristics

- **How to evaluate?**
  - Use the system clock (machine dependent)
  - Number of ***steps*** performed (machine-independent)

- **Definition of a program step**
  - A ***program step*** is a syntactically or semantically meaningful instruction whose execution time is independent of the instance characteristics.

Examples of Determining Steps

- *the first method: count increment by a step*
- *EX: Algorithm for calculating sum of numbers in an array*

```
Algorithm sum(list[], n)
{
        tempsum:= 0;
        count++;                        /* assignment of zero */
        for i= 0 to n
        {
              count++;                  /* for the for loop */
              tempsum+:= list[i];
              count++;                  /* for assignment */
        }
        count++;                        /* last execution of for */
        count++;                        /* for return */
        return tempsum;
}
```

2n+ 3

Examples of Determining Steps(Cont.)

```
void add_matrices(int a[ ], int b[ ], int c[ ], int R, int C)
{
    int i, j;
    for i=0 to R
    {
        count++;                 /* for the for i loop */
        for j=0 to C
        {
            count++;             /* for the for j loop */
            count++;             /* for the addition  */
            c[i, j]:= a[i, j] + b[i,j];
        }
        count++;                 /* last execution of for j */
    }
    count++;                     /* last execution of for i */
}
```

y

x

= (R* x)+1

= (R*(2+y))+1

= (R*(2+(2*C)))+1

= (2*R*C+2*R)+1

Examples of Determining Steps(Cont.)

```
float rsum(float list[], int n)
{
  count ++;          /* for if condition  */
  if (n!=1) then
 {
   count++;     /* for return and rsum invocation */
   return rsum(list, n-1)+ list[n-1];
  }
  count++;          /* return */
  return list[0];
}
```

$$
\begin{aligned}
Trsum(1) &= 2 \\
Trsum(n) &= 2 + Trsum(n-1) \\
&= 2 + (2 + Trsum(n-2)) \\
&= 2*2 + Trsum(n-2) \\
&= 2*2+(2+ Trsum(n-3)) \\
&= 3*2+ Trsum(n-3) \\
&= \ldots\ldots\ldots\ldots \\
&= (n-1)*2 + Trsum(n-(n-1)) \\
&= 2*n-2+2 \\
&= 2*n
\end{aligned}
$$

Examples of Determining Steps(Cont.)

- **The second method: build a table to count the number of steps**

  s/e: steps per execution

  frequency: total numbers of times each statements is executed

Statement	***s/e***	***Frequency***	***Total Steps***
float sum(float list[], int n)			
{			
float sum=0;			
int i;			
for i=0 to n			
sum:= sum + list[i];			
return sum;			
}			

Examples of Determining Steps(Cont.)

- **The second method: build a table to count the number of steps**

  s/e: steps per execution

  frequency: total numbers of times each statements is executed

Statement	*s/e*	*Frequency*	*Total Steps*
float sum(float list[], int n)	*0*		
{	*0*		
float sum=0;	*1*		
int i;	*0*		
for i=0 to n	*1*		
sum:= sum + list[i];	*1*		
return sum;	*1*		
}	*0*		

Examples of Determining Steps(Cont.)

- **The second method: build a table to count the number of steps**

s/e: steps per execution

frequency: total numbers of times each statements is executed

Statement	***s/e***	***Frequency***	***Total Steps***
float sum(float list[], int n)	***0***	***0***	
{	***0***	***0***	
float sum=0;	***1***	***1***	
int i;	***0***	***0***	
for (i=0 to n	***1***	***n+1***	
sum:= sum + list[i];	***1***	***n***	
return sum;	***1***	***1***	
}	***0***	***0***	

Examples of Determining Steps(Cont.)

- **The second method: build a table to count the number of steps**

s/e: steps per execution

frequency: total numbers of times each statements is executed

Statement	*s/e*	*Frequency*	*Total Steps*
float sum(float list[], int n)	*0*	*0*	*0*
{	*0*	*0*	*0*
float sum=0;	*1*	*1*	*1*
int i;	*0*	*0*	*0*
for i=0 to n	*1*	*n+1*	*n+1*
sum:= sum + list[i];	*1*	*n*	*n*
return sum;	*1*	*1*	*1*
}	*0*	*0*	*0*

Total *2*n +3*

Examples of Determining Steps(Cont.)

- **The second method: build a table to count**

  s/e: steps per execution

  frequency: total numbers of times each statements is executed

Statement	***s/e***	***Frequency***	***Total Steps***
void add(int a[][], . . .			
{			
int i, j;			
for i=0 to R			
for j=0 to C			
c[i , j]: = a[i , j] + b[i , j];			
}			
Total			

Examples of Determining Steps(Cont.)

- **The second method: build a table to count**

  s/e: steps per execution

  frequency: total numbers of times each statements is executed

Statement	*s/e*	*Frequency*	*Total Steps*
void add(int a[][], . . .	*0*	*0*	*0*
{	*0*	*0*	*0*
int i, j;	*0*	*0*	*0*
for i=0 to R	*1*	*R+ 1*	*R+ 1*
for j=0 to C	*1*	*R*(C+1)*	*R*C+ R*
c[i , j]= a[i , j] + b[i , j];	*1*	*R*C*	*R*C*
}	*0*	*0*	*0*
Total			*2*R*C+2*R+1*

Examples of Determining Steps(Cont.)

- **The second method: build a table to count**

  s/e: steps per execution

  frequency: total numbers of times each statements is executed

Statement	***s/e***	***Frequency***	***Total Steps***
float rsum(float list[], int n)			
{			
if (n!=1)			
return rsum(list,n-1)+list[n-1]			
return list[0];			
}			
Total			

Examples of Determining Steps(Cont.)

- **The second method: build a table to count**

s/e: steps per execution

frequency: total numbers of times each statements is executed

Statement	*s/e*	*Frequency*	*Total Steps*
float rsum(float list[], int n)	*0*	*0*	*0*
{	*0*	*0*	*0*
if (n!=1)	*1*	*n+1*	*n+1*
return rsum(list,n-1)+list[n-1]	*1*	*n*	*n*
return list[0];	*1*	*1*	*1*
}	*0*	*0*	*0*
Total			***2*n + 2***

Tasks on Performance Analysis

- Write an algorithm for matrix multiplication and calculate its time complexity.
- Write an algorithm to print ‘n’ numbers in the Fibonacci series and estimate its time complexity.
- Write an algorithm to find the largest element in an array and estimate the time complexity.
- Write an algorithm to evaluate a polynomial using Horner’s rule and estimate the time complexity.
- Write an algorithm to check whether the given number is Armstrong Number or not and estimate the time complexity.
- Estimate the time complexity of factorial of a number using recursion.

Topics

- **Algorithm Specification**
- **Pseudocode Convention**
- **Performance Analysis**
- **Asymptotic Notations**

Algorithm Analysis

- To analyze the given algorithm, we need to know with which inputs the algorithm takes less time and with which inputs the algorithm takes a long time.

- There are three types of analysis:

- **Worst case Analysis**
  - Defines the input for which the algorithm takes a long time (slowest time to complete).
- **Best case Analysis**
  - Defines the input for which the algorithm takes the least time (fastest time to complete).
- **Average case Analysis**
  - Assumes that the input is random.
  - Run the algorithm many times, using many different inputs.
  - compute the total running time (by adding the individual times), and divide by the number of times the algorithm has executed.

Asymptotic Notations

- Asymptotic notations are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value (say ‘n’).

- The simplest example is a function f (n) = n^2+3n,
  - the term 3n becomes insignificant compared to n^2 when n is very large.
  - The function "f (n) is said to be asymptotically equivalent to n^2 as n $\to \infty$", and here is written symbolically as f (n) ~ n^2.

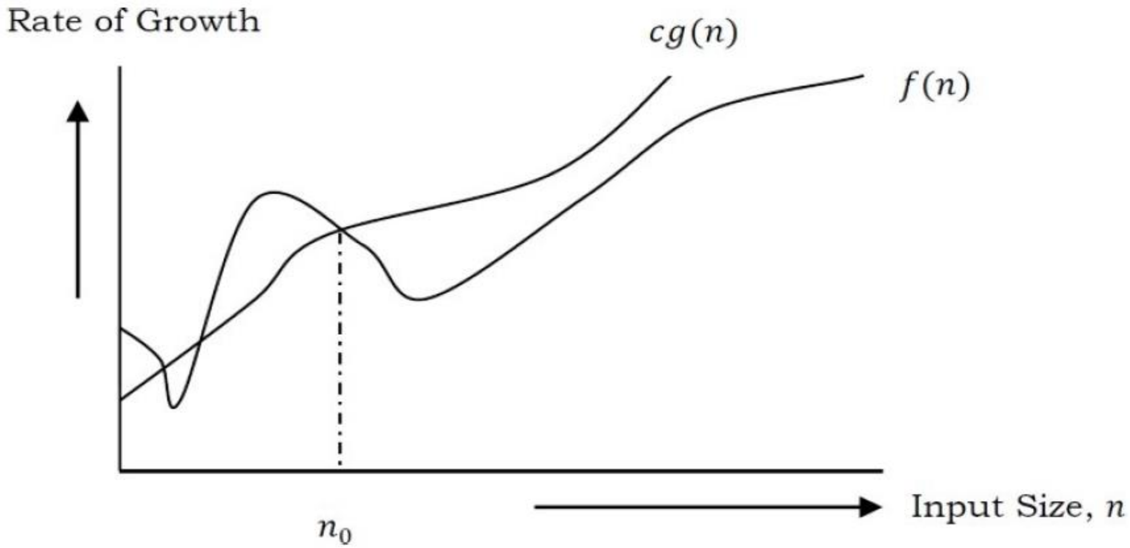
Asymptotic Notations

- The commonly used asymptotic notations to represent the time complexity of an algorithm:
  - **O (Big-Oh) Notation**
  - **Ω (Omega) Notation**
  - **θ (Theta) Notation**
  - o (Little-oh) Notation
  - ω (Little-omega) Notation

O (Big-Oh) Notation-Upper Bounding function

- It represents the upper bound running time complexity of an algorithm.
- It is the measure of the longest amount of time.
- **Definition:** The function **f (n) = O (g (n))** [read as "f of n is big-oh of g of n"] if and only if there exists positive constant **c** and **n0** such that

$$f(n) \leqslant c*(g(n)) \text{ for } \forall n >= n0$$



Examples on O (Big-Oh) Notation

- **Example-1:** Find upper bound for $f(n) = 3n + 8$
  - **Solution:** $3n + 8 \leq 4n$, for all $n \geq 8$
  - $\therefore 3n + 8 = O(n)$ with $c = 4$ and $n0 = 8$

- **Example-2:** Find upper bound for $f(n) = n^2 + 1$
  - **Solution:** $n2 + 1 \leq 2n2$, for all $n \geq 1$
  - $\therefore n2 + 1 = O(n2)$ with $c = 2$ and $n0 = 1$

- **Example-3:** Find upper bound for $f(n) = n^4 + 100n^2 + 50$
- **Example-4:** Find upper bound for $f(n) = 2n^3 - 2n^2$
- **Example-5:** Find upper bound for $f(n) = n$
- **Example-6:** Find upper bound for $f(n) = 410$

O (Big-Oh) Notation

Theorem 1.2 If $f(n) = a_m n^m + \cdots + a_1 n + a_0$, then $f(n) = O(n^m)$.

$$f(n) \qquad \sum_{i=0}^{m} |a_i| n^i$$

Proof:

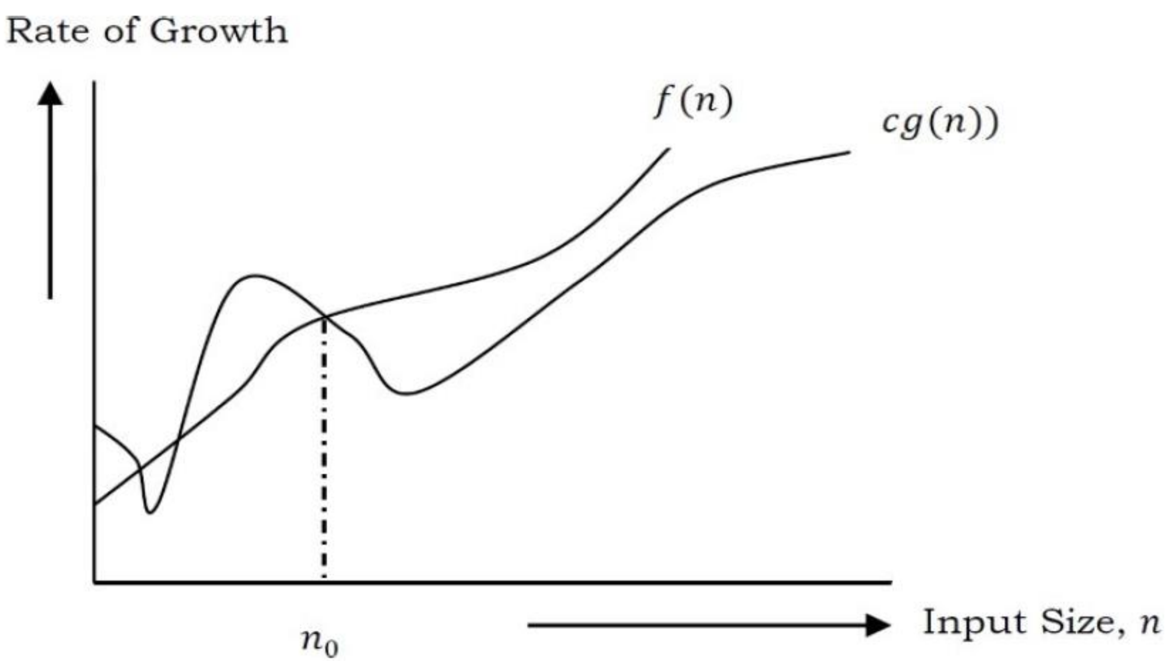
$$\begin{aligned} f(n) &\leq \sum_{i=0}^{m} |a_i| n^i \\ &\leq n^m \sum_{i=0}^{m} |a_i| n^{i-m} \\ &\leq n^m \sum_{i=0}^{m} |a_i| \qquad \text{for } n \geq 1 \end{aligned}$$

So, $f(n) = O(n^m)$ (assuming that m is fixed).

Ω (Omega) Notation-Lower Bounding function

- The notation Ω(n) is the formal way to express the lower bound of an algorithm's running time.
- **Definition:** The function f (n) = Ω (g (n)) [read as "f of n is omega of g of n"] if and only if there exists positive constant c and n_0 such that

$$F(n) \geq c^* g(n) \text{ for all } n, n \geq n_0$$



Examples on Ω (Omega) Notation

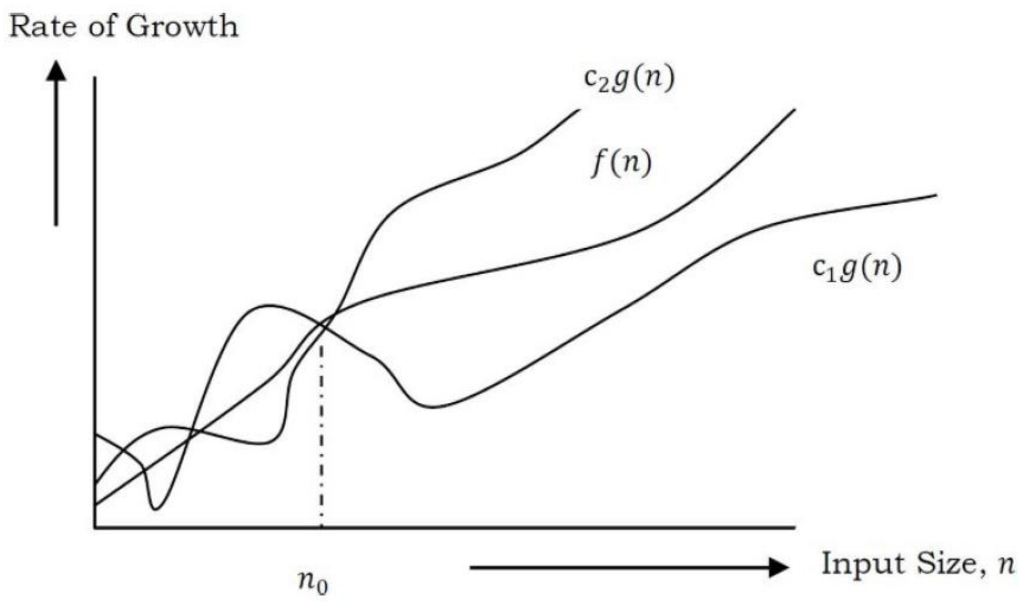
- **Example-1:** Find lower bound for $f(n) = 5n^2$.

- **Example-2:** Find lower bound for $f(n) = 10n^2 + 4n + 2$
- **Example-3:** Find lower bound for $f(n) = 6 * 2^n + n^2$
- **Example-4:** Prove $f(n) = 100n + 5 \neq \Omega(n^2)$.
- **Example-5:** Prove that $2n = \Omega(n)$, $n^3 = \Omega(n^3)$, $n^3 = O(logn)$.

θ (Theta) Notation

- The notation θ(n) is the formal way to express both the lower bound and the upper bound of an algorithm's running time.
- **Definition:** The function f (n) = θ (g (n)) [read as "f is the theta of g of n"] if and only if there exists positive constant c_1, c_2 and n_0 such that

$$c_1 * g(n) \leq f(n) \leq c_2 * g(n) \text{ for all } n, n \geq n_0$$



Examples on θ (Theta) Notation

- Prove that $3n+2 = \theta(n)$

- Prove that $10n^2+4n+2 = \theta(n^2)$
- Prove that $n^3 + 106n^2 = \theta(n^3)$
- Prove that the following is incorrect:

$$n^2/\log n = \theta(n^2)$$

Little oh and Little omega Notation

Definition 1.7 [Little "oh"] The function $f(n) = o(g(n))$ (read as "f of n is little oh of g of n") iff

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

Example: The function 3n +2 = o(n^2)

Definition 1.8 [Little omega] The function $f(n) = \omega(g(n))$ (read as "f of n is little omega of g of n") iff

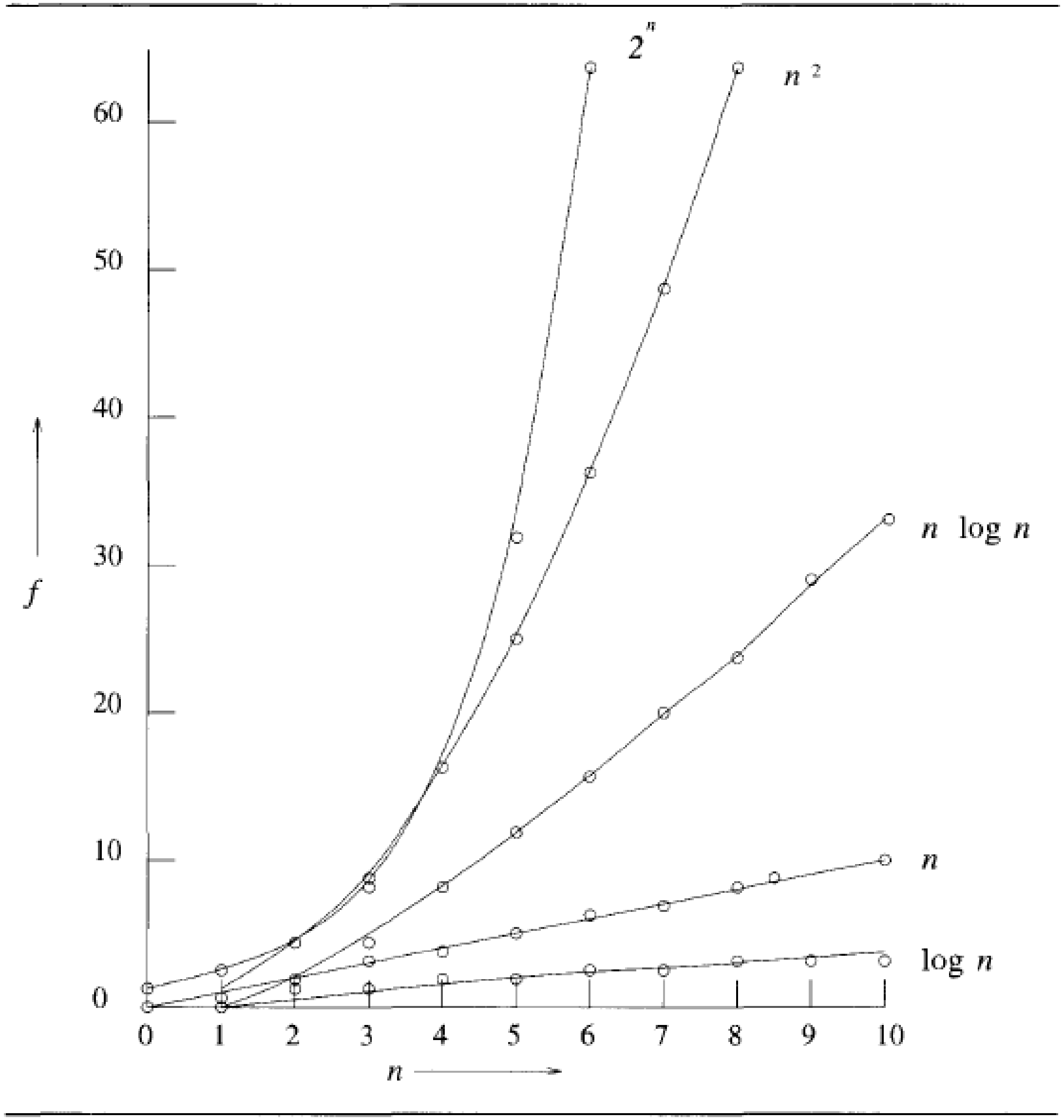
$$\lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$$

Why is it called Asymptotic Analysis?

- From the discussion above, we can easily understand that, in every case for a given function $f(n)$ we are trying to find another function $g(n)$ which approximates $f(n)$ at higher values of n.

- In mathematics we call such a curve an *asymptotic curve*. In other terms, $g(n)$ is the asymptotic curve for $f(n)$. For this reason, we call algorithm analysis *asymptotic analysis.*

Different Asymptotic functions



Guidelines for Asymptotic Analysis

- 1) **Loops:** The running time of a loop is, at most, the running time of the statements inside the loop (including tests) multiplied by the number of iterations.

```
// executes n times
for (i=1; i<=n; i++)
    m = m + 2; // constant time, c
```

- 2) **Nested loops:** Analyze from the inside out. Total running time is the product of the sizes of all the loops.

```
//outer loop executed n times
for (i=1; i<=n; i++) {
    // inner loop executes n times
    for (j=1; j<=n; j++)
        k = k+1; //constant time
}
```

Guidelines for Asymptotic Analysis

- **3. Consecutive statements:** Add the time complexities of each statement.

```
x = x +1; //constant time
// executes n times
for (i=1; i<=n; i++)
    m = m + 2; //constant time
//outer loop executes n times
for (i=1; i<=n; i++) {
    //inner loop executed n times
    for (j=1; j<=n; j++)
        k = k+1; //constant time
}
```

Guidelines for Asymptotic Analysis

- 4) **If-then-else statements:** Worst-case running time: **the test, plus *either* the *then* part *or* the *else* part (whichever is the larger).**

```
//test: constant
if(length( ) == 0 ) {
    return false; //then part: constant
}
else {// else part: (constant + constant) * n
    for (int n = 0; n < length( ); n++) {
      // another if : constant + constant (no else part)
      if(!list[n].equals(otherList.list[n]))
          //constant
        return false;
    }
}
```

Guidelines for Asymptotic Analysis

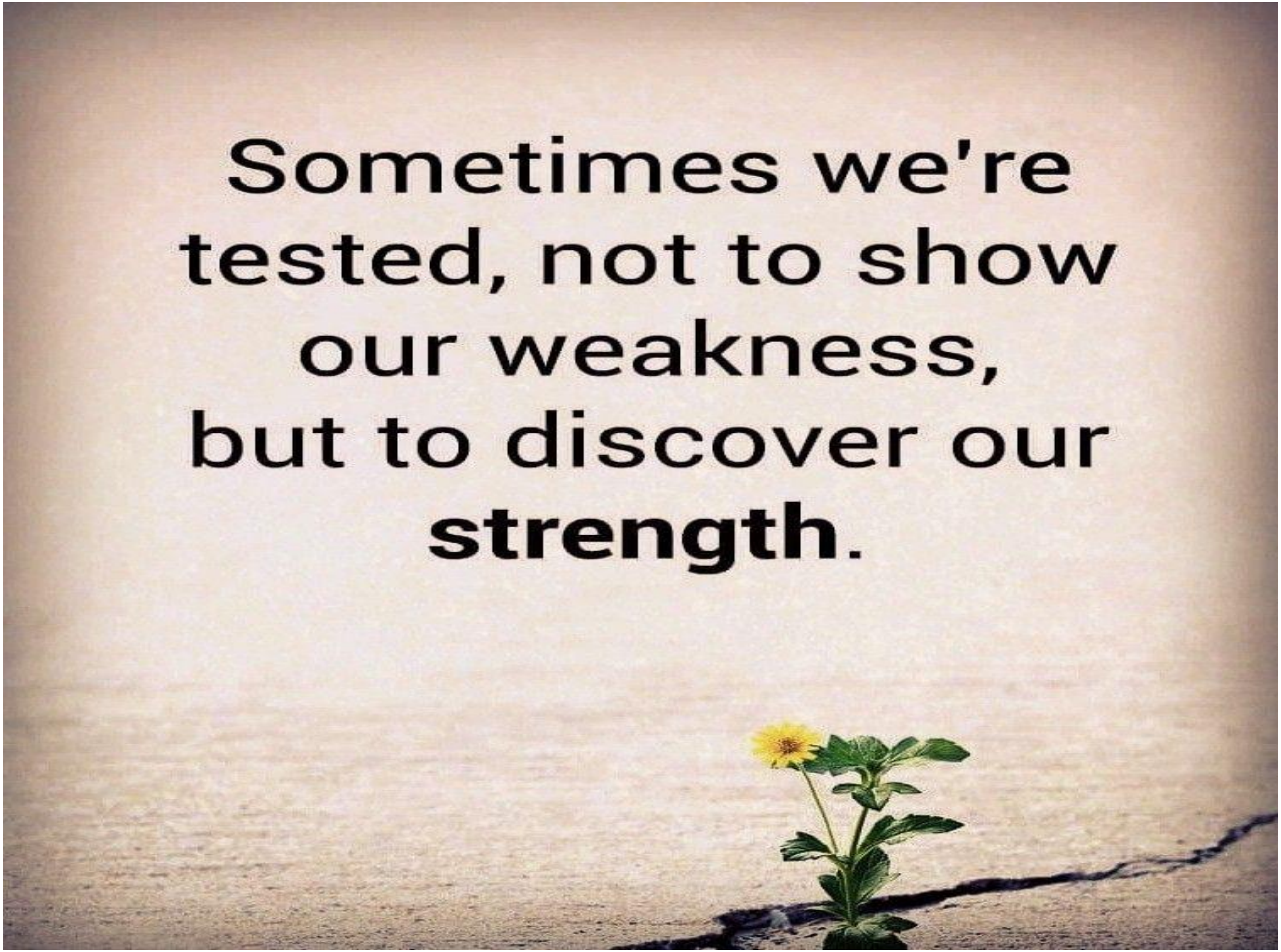
- 5) **Logarithmic complexity:** An algorithm is O(*logn*) if it takes a constant time to cut the problem size by a fraction (usually by ½).

```
for (i=1; i<=n;)
    i = i*2;
```

```
for (i=n; i>=1;)
    i = i/2;
```

Examples on Performance Analysis

- https://www.geeksforgeeks.org/miscellaneous-problems-of-time-complexity/
- https://www.geeksforgeeks.org/analysis-algorithms-set-5-practice-problems/?ref=rp


Sometimes we're
tested, not to show
our weakness,
but to discover our
strength.