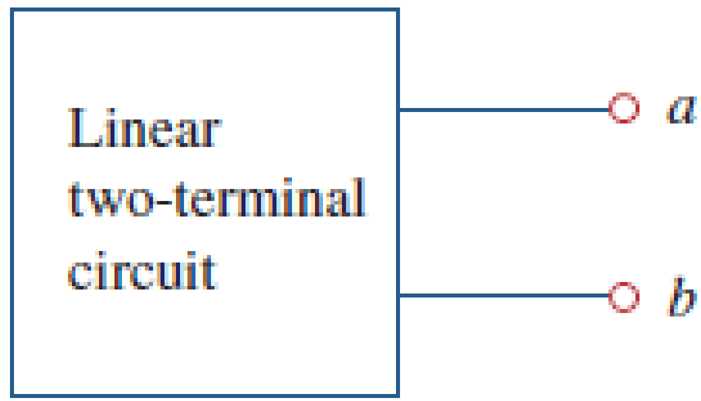
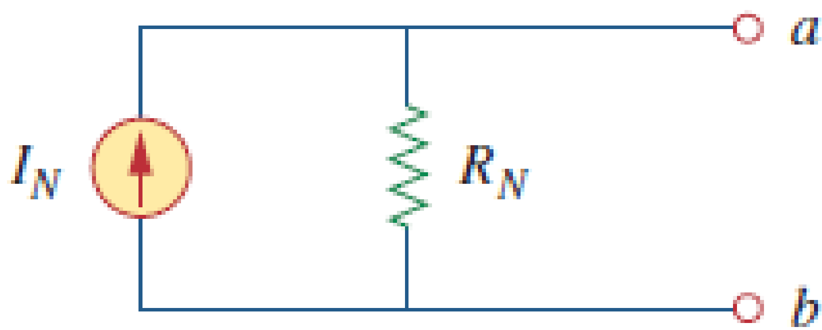
Norton’s Theorem

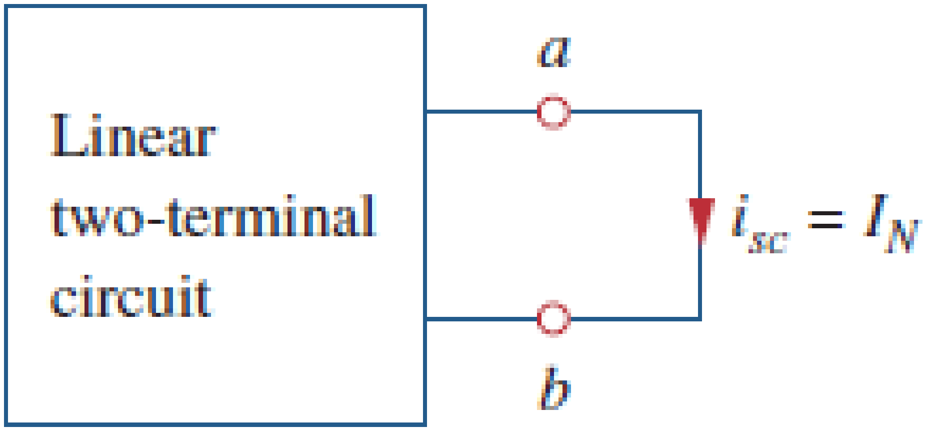
In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



(a)





$$\boxed{R_N = R_{Th}}$$

$$V_{Th} = v_{oc}$$

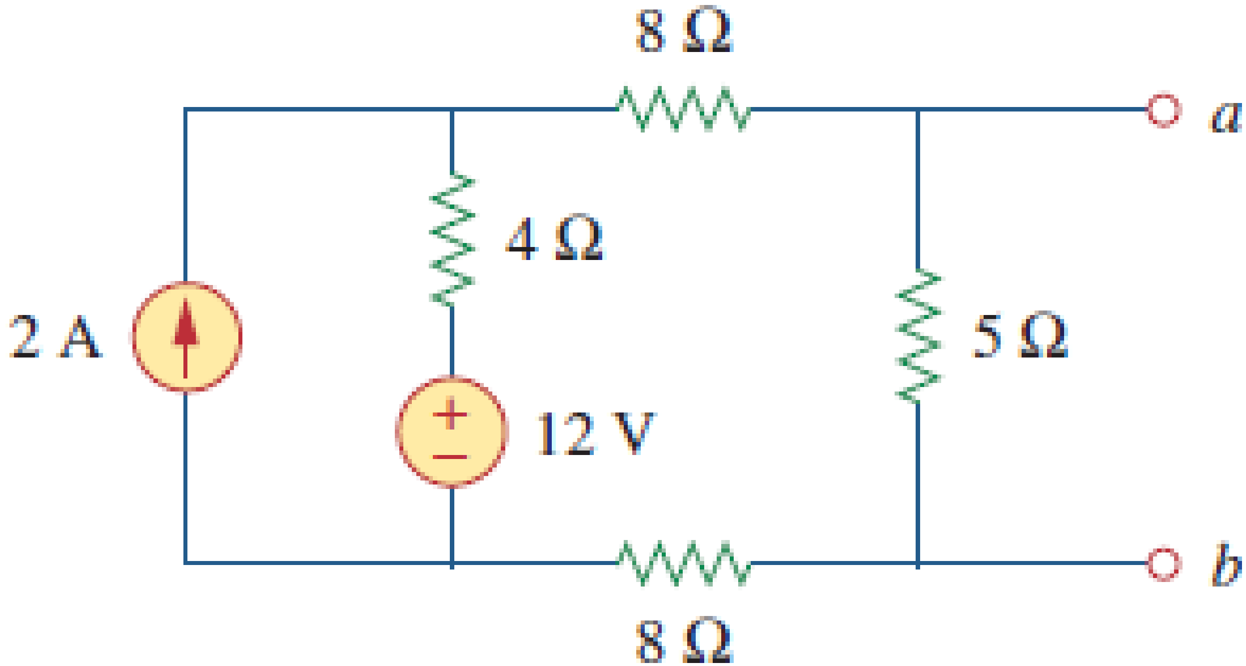
$$I_N = i_{sc}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

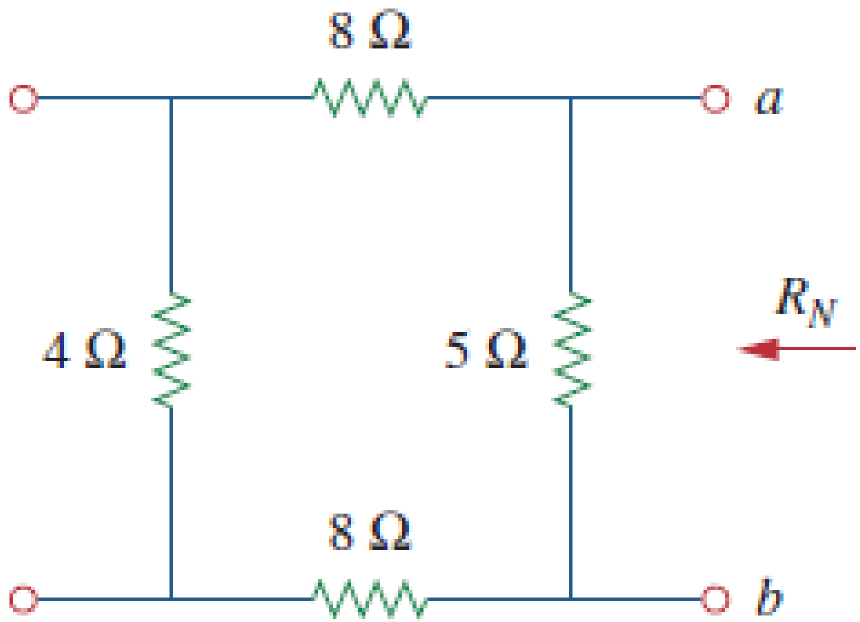
$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

Problem 1

- Find the Norton's Equivalent of the Below Circuit

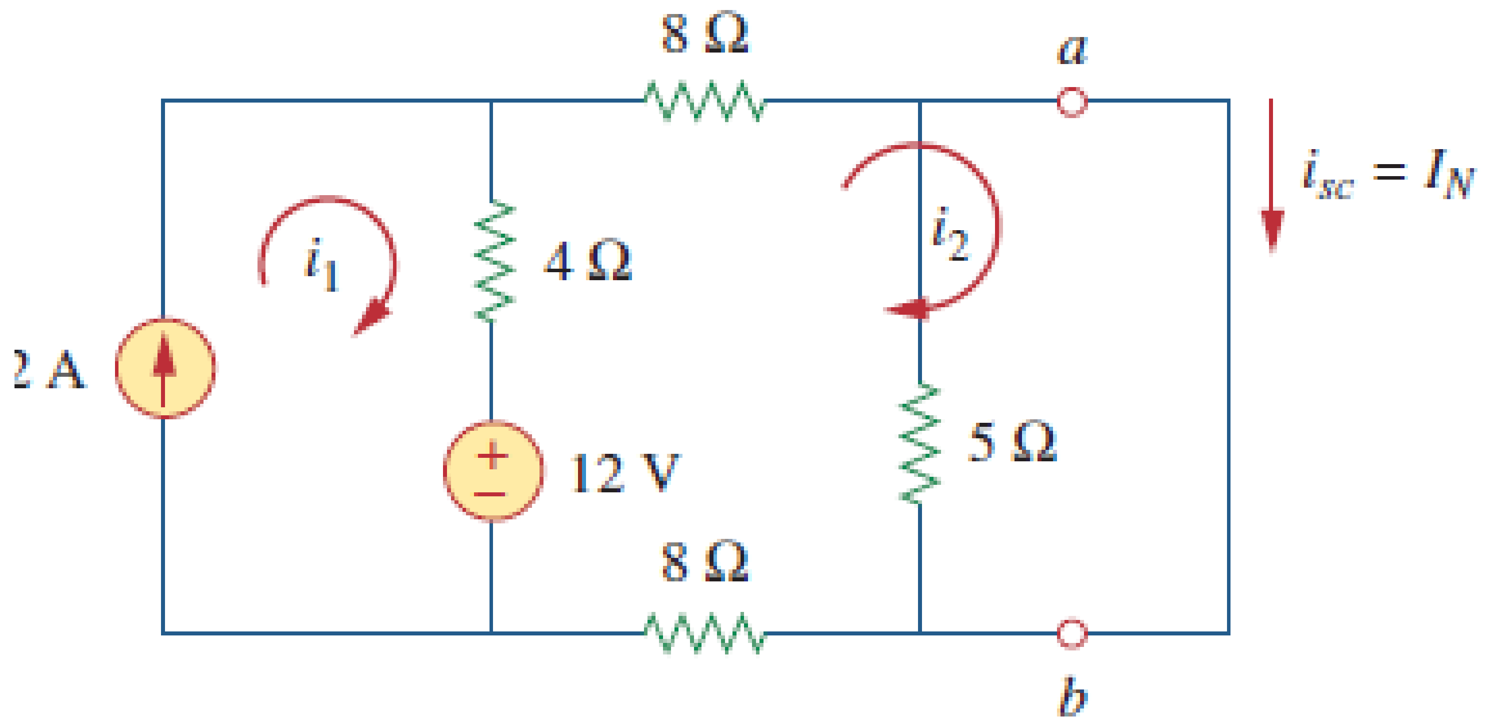


Finding the R_N



$$R_N = 5 \,\|\, (8 + 4 + 8) = 5 \,\|\, 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

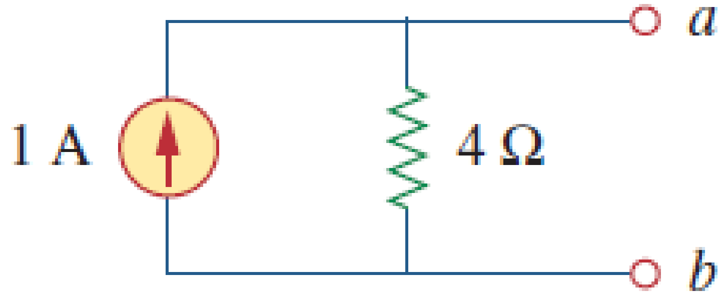
Finding I_N

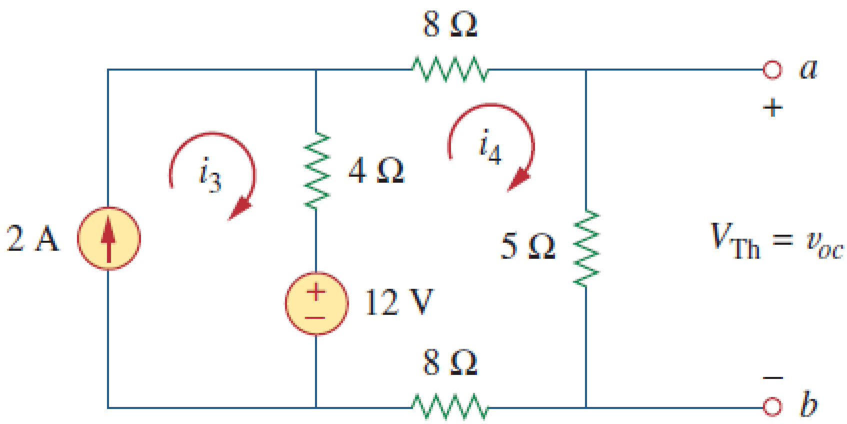


IN=1A

COMPARISION

- Find Vth and Rth for the above circuit and Compare it with Norton's Value





(c)

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

and

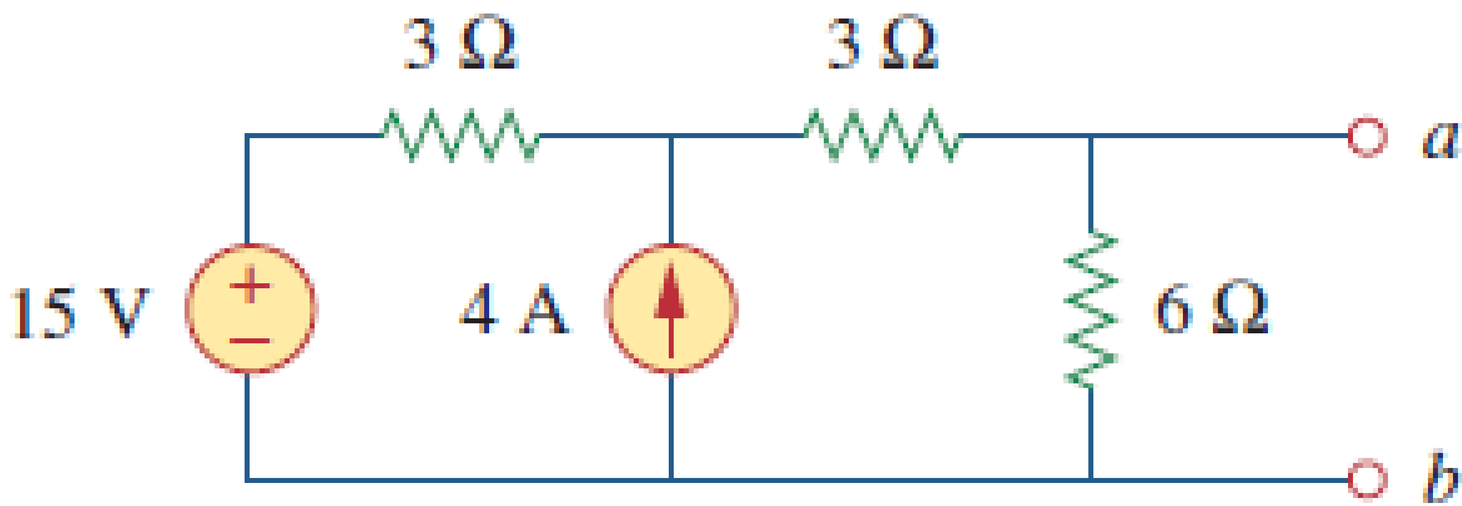
$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

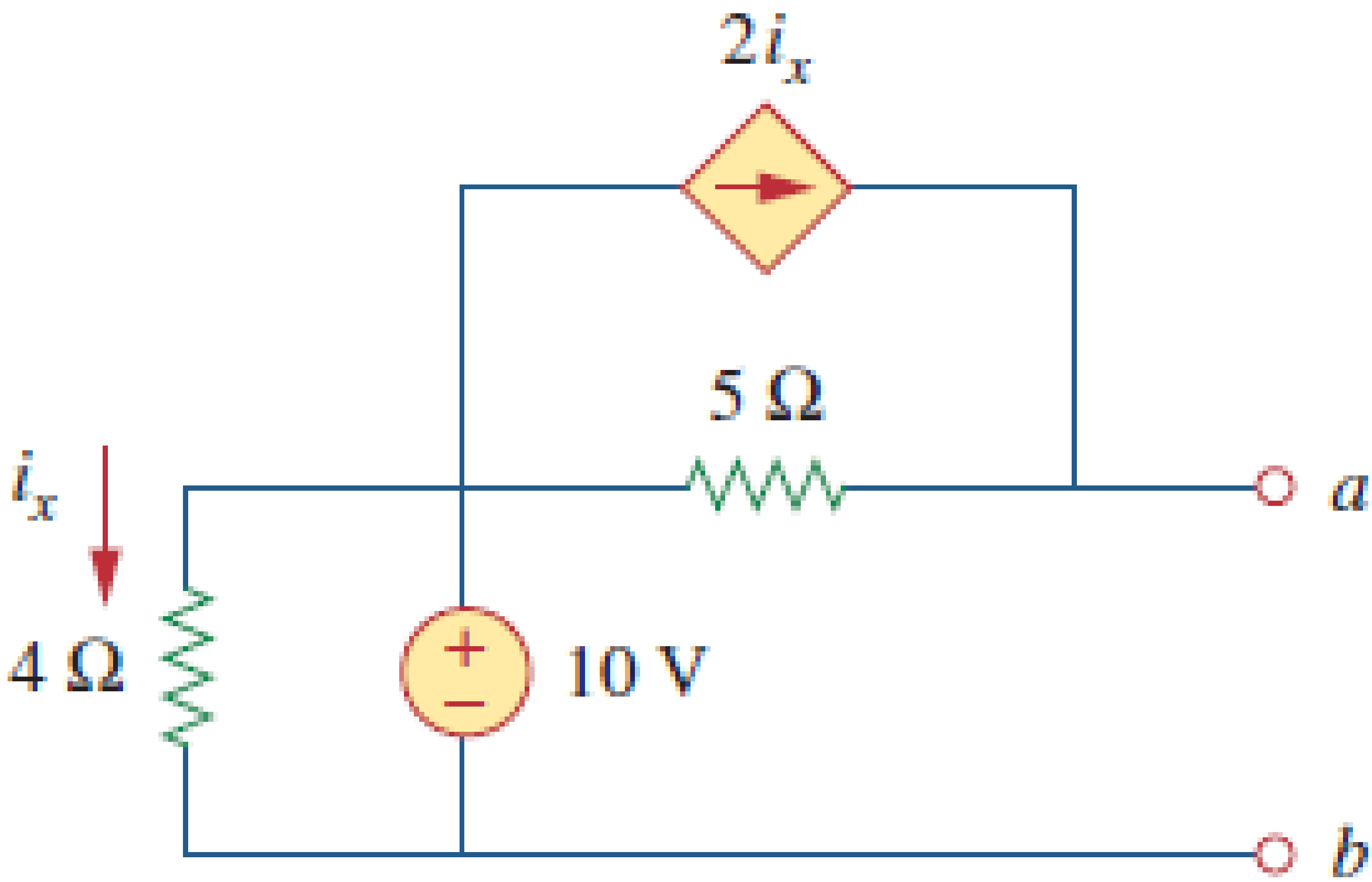
as obtained previously. This also serves to confirm Eq. (4.12c) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

Problem 2
Find the Norton’s Equivalent Circuit and Compare it with Thevenin’s Theorem



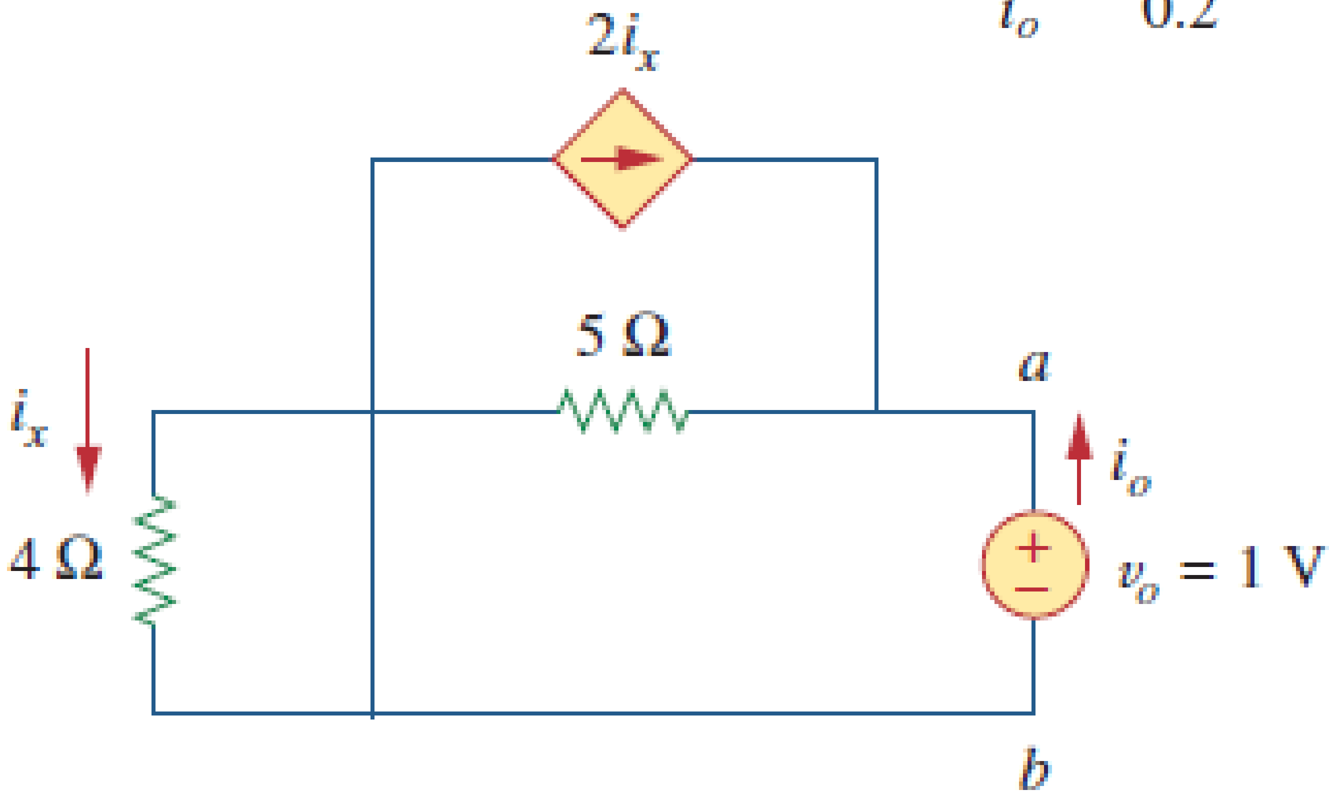
$R_N = 3\ \Omega,\ I_N = 4.5\ \text{A}.$

Problem 3
Find the Norton Equivalent of the below circuit



Circuit to find R_N

$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$



$$i_x = \frac{10}{4} = 2.5 \text{ A}$$

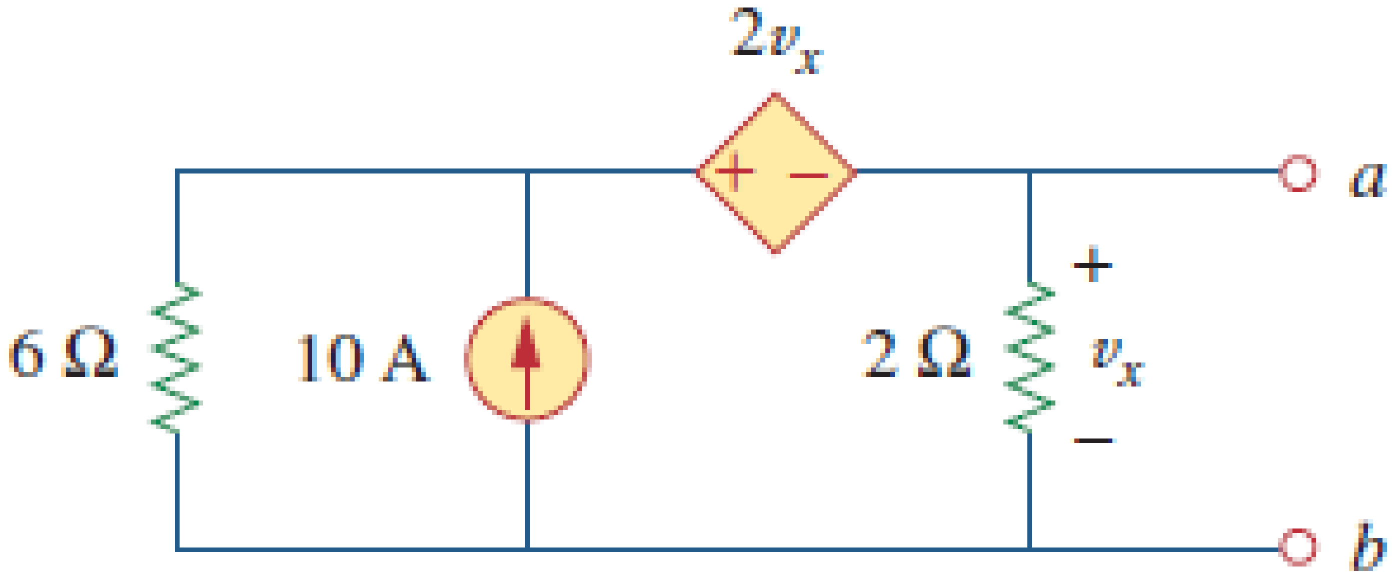
At node a, KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7 \text{ A}$$

Thus,

$$I_N = 7 \text{ A}$$

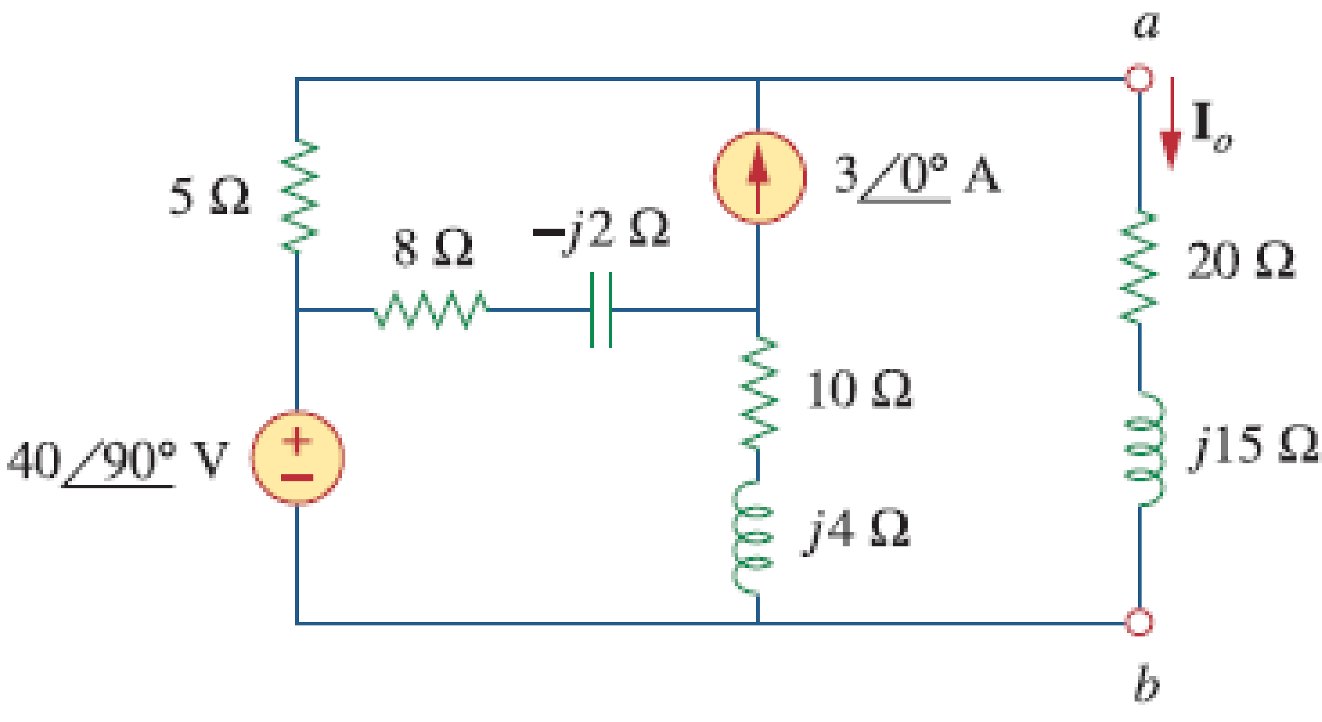
Problem 4



$R_N = 1\ \Omega$, $I_N = 10$ A.

Problem 5

- Find the I_0 From the below circuit using Norton's Theorem



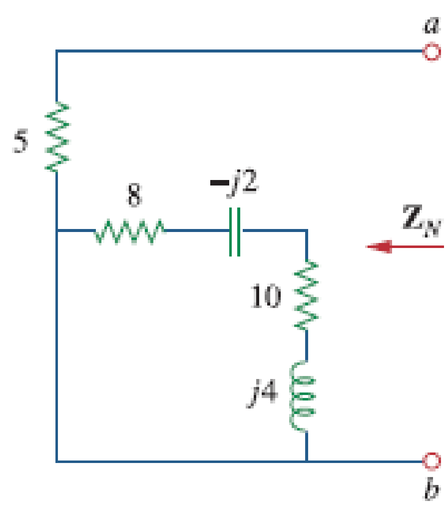
Solution:

Our first objective is to find the Norton equivalent at terminals *a-b*. $\mathbf{Z}_N$ is found in the same way as $\mathbf{Z}_{\text{Th}}$. We set the sources to zero as shown in Fig. 10.29(a). As evident from the figure, the $(8 - j2)$ and $(10 + j4)$ impedances are short-circuited, so that

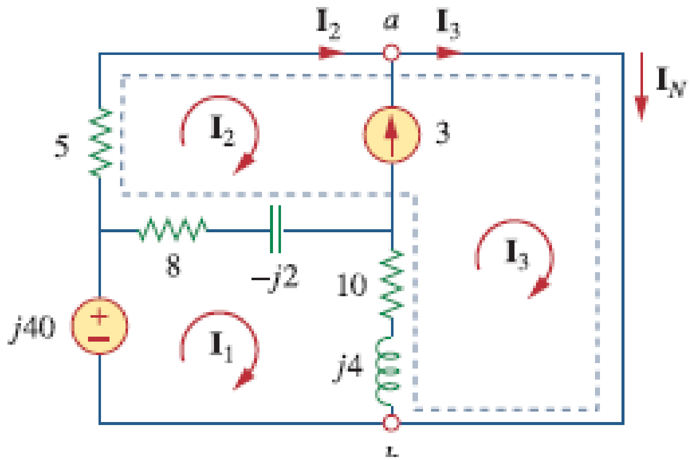
$$\mathbf{Z}_N = 5\ \Omega$$

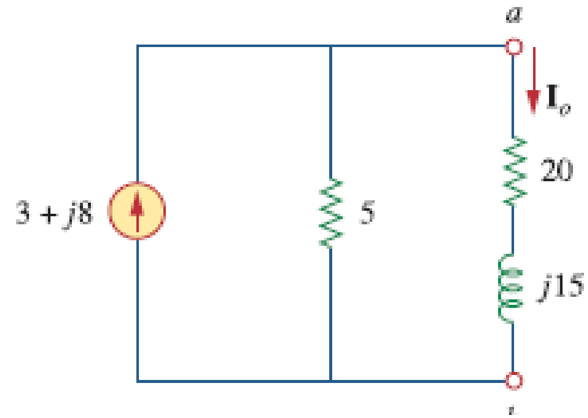
To get $\mathbf{I}_N$, we short-circuit terminals *a-b* as in Fig. 10.29(b) and apply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad \textbf{(10.10.1)}$$



(a)





For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad \textbf{(10.10.2)}$$

At node a, due to the current source between meshes 2 and 3,

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 \qquad \textbf{(10.10.3)}$$

Adding Eqs. (10.10.1) and (10.10.2) gives

$$-j40 + 5\mathbf{I}_2 = 0 \quad \Rightarrow \quad \mathbf{I}_2 = j8$$

From Eq. (10.10.3),

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$

The Norton current is

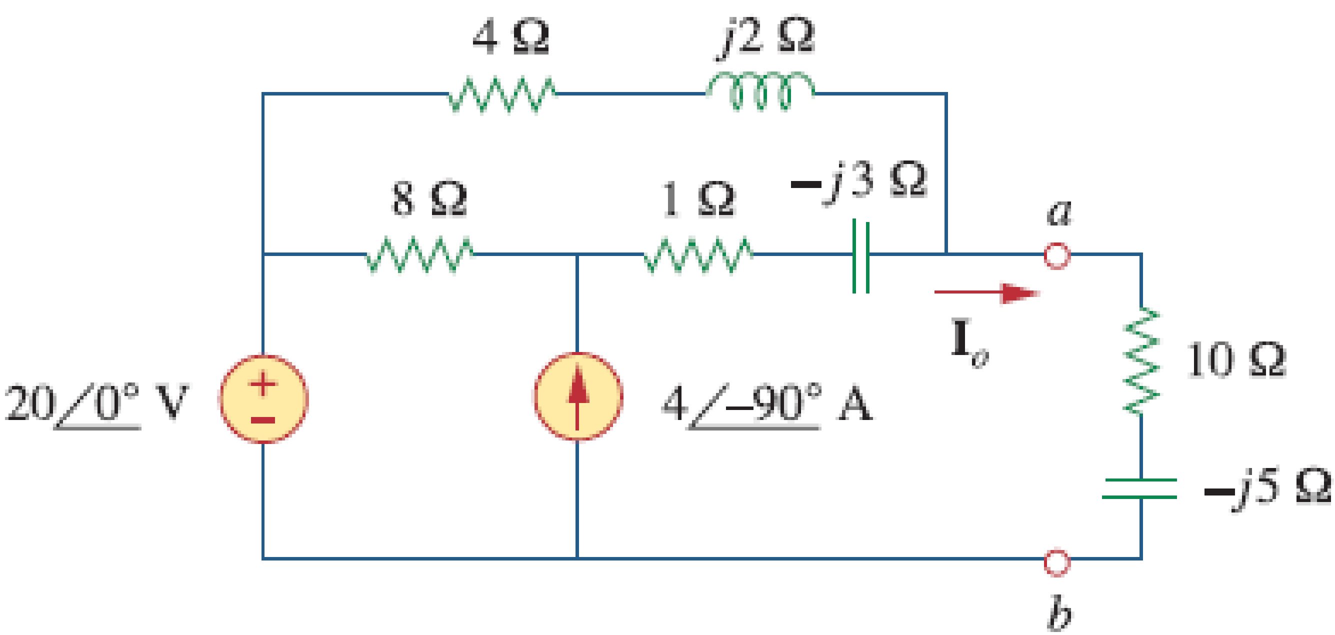
$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\text{ A}$$

Figure 10.29(c) shows the Norton equivalent circuit along with the impedance at terminals a-b. By current division,

$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = \underline{1.465\angle 38.48^\circ\text{ A}}$$

Problem 6

- Find the current flowing through 10-*j*5 Load.



10.67 Find the Thevenin and Norton equivalent circuits at terminals *a-b* in the circuit of Fig. 10.110.

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