Two Port Networks

By
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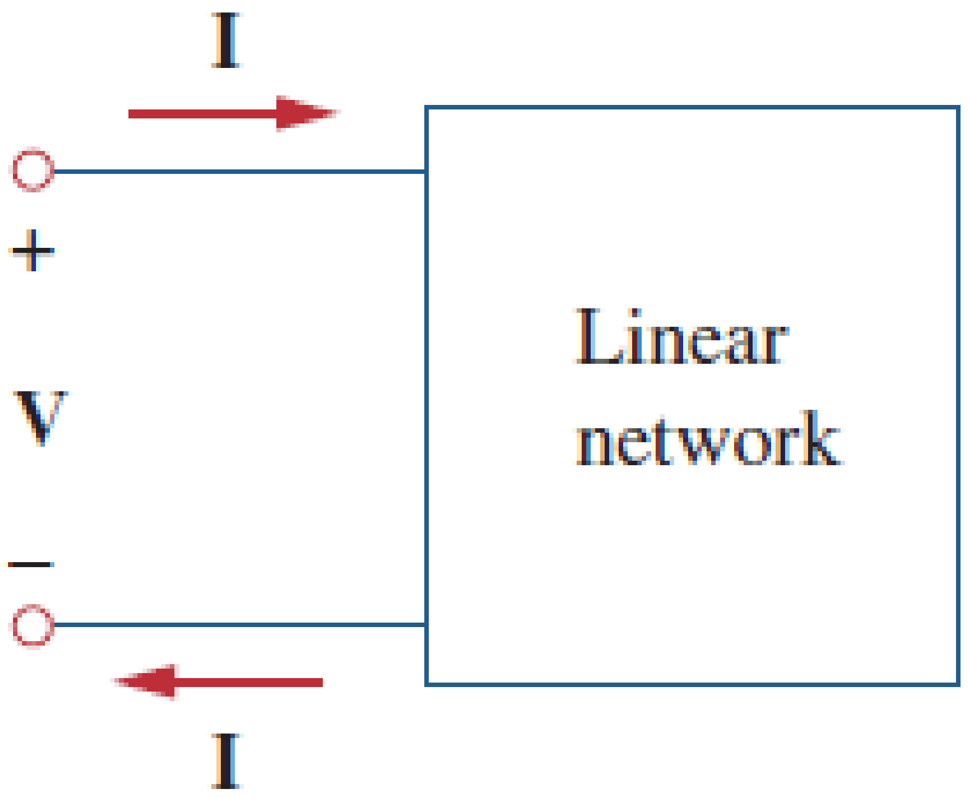
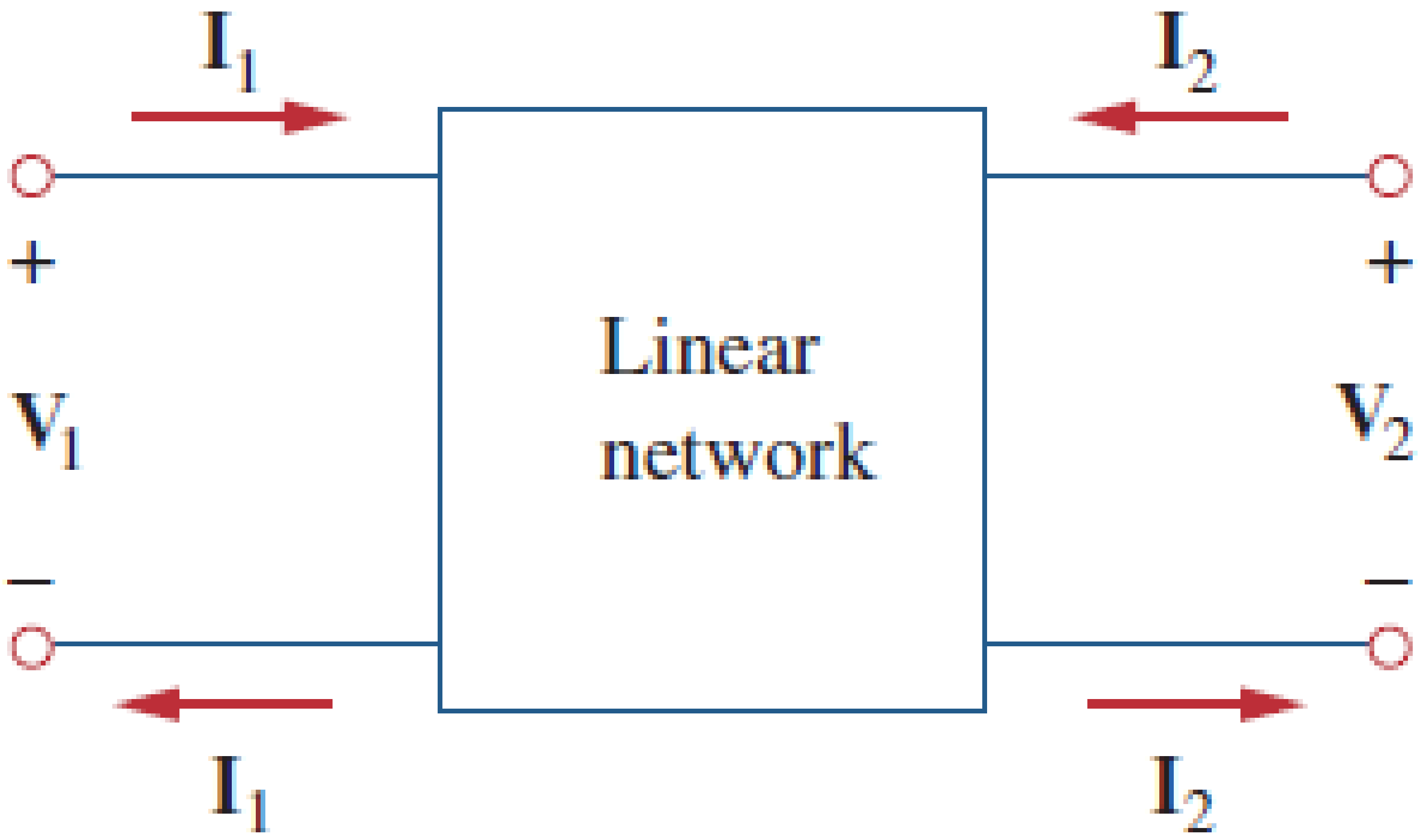
Introduction

- A Pair of terminals through which a current may enter or leave a network is known as a port.
- A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.
- A two-port network is an electrical network with two separate ports for input and output.

Network With One Port



Two Port Networks



Reasons for Studying Two Port Networks

- They are useful in Communications, Electronics, Control Systems Etc
  - Ex: They are used in modelling transistors in transistors
- Knowing the parameters of a two port network enables us to treat it as a "Black Box" when embedded within a larger network

List of Two Port Parameters

- Impedance Parameters
- Admittance Parameters
- Hybrid Parameters
- Transmission Parameters

- * Interconnection of Parameters
- *Conditions of Reciprocity and Symmetry

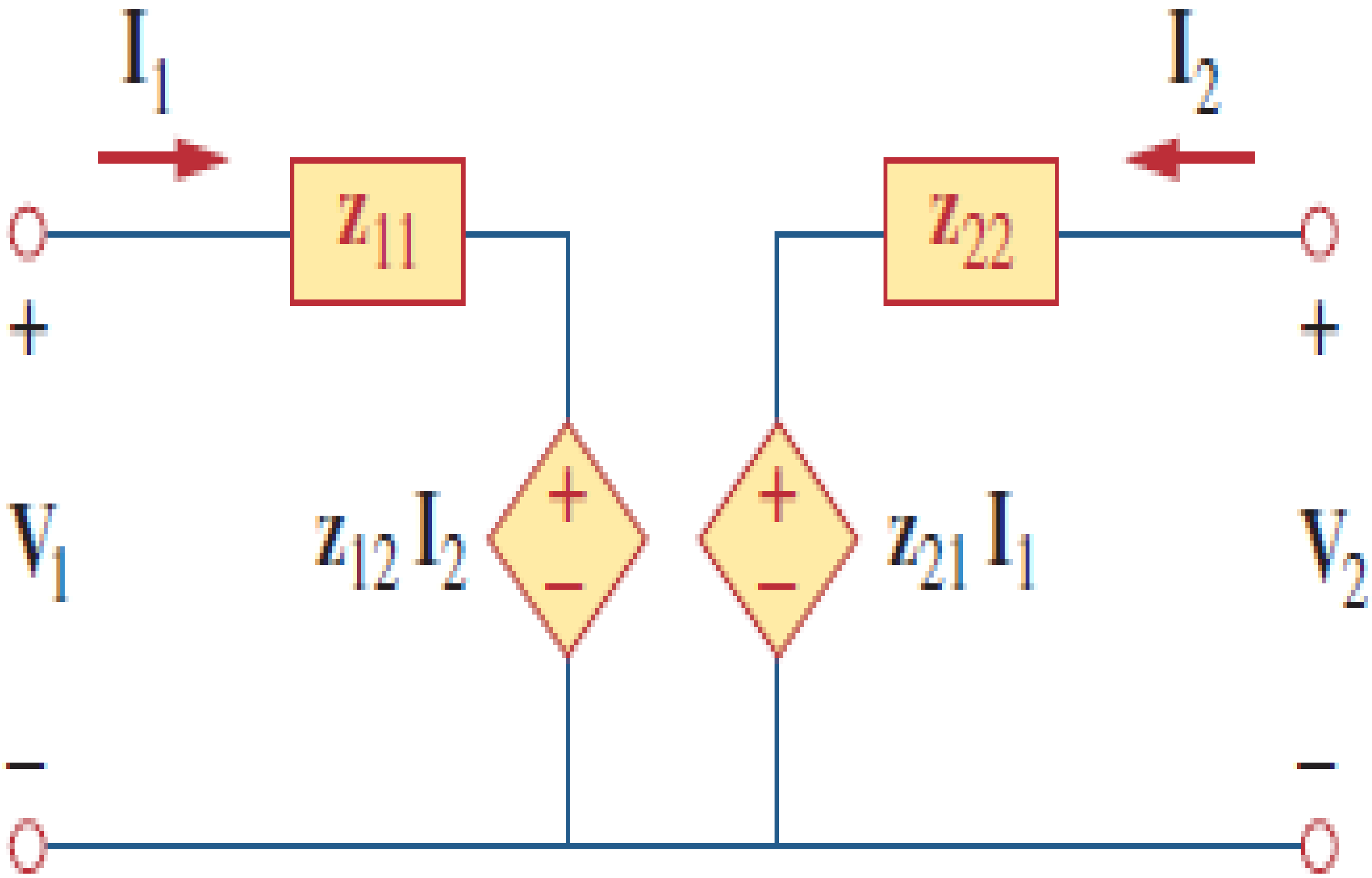
1. Impedance Parameters

- They are commonly used in Synthesis of Filters
- They are useful in Design and Analysis of Impedance Matching Networks and Power Distribution networks
- In all the networks only two variables will be independent and remaining two will be dependent

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

Z Parameters in the form of Matrix

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



I1
z11
I2
z22
+
+
V1
z12 I2
+
−
+
−
z21 I1
V2
−
−

Z Parameters Relationship

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

Name of Z Parameters

$\mathbf{z}_{11}$ = Open-circuit input impedance

$\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2

$\mathbf{z}_{21}$ = Open-circuit transfer impedance from port 2 to port 1

$\mathbf{z}_{22}$ = Open-circuit output impedance

Driving Point Impedance

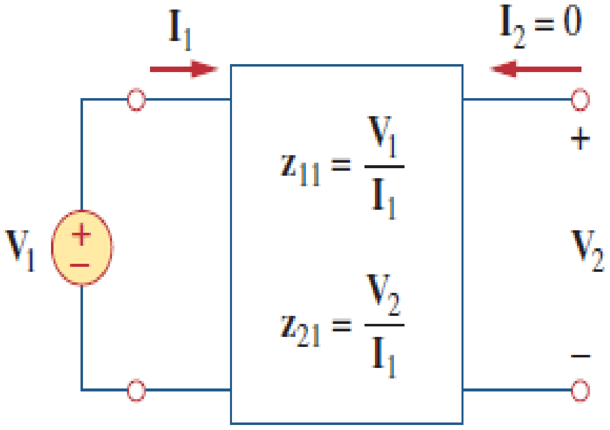
- A driving point impedance is the input impedance of a two terminal device.
- Z_{11} : Input Driving Point Impedance with Output open circuited
- Z_{22} : Output Driving Point Impedance with Input Open Circuited

Transfer Impedances

- Z_{21} and Z_{22} are called Transfer Impedances

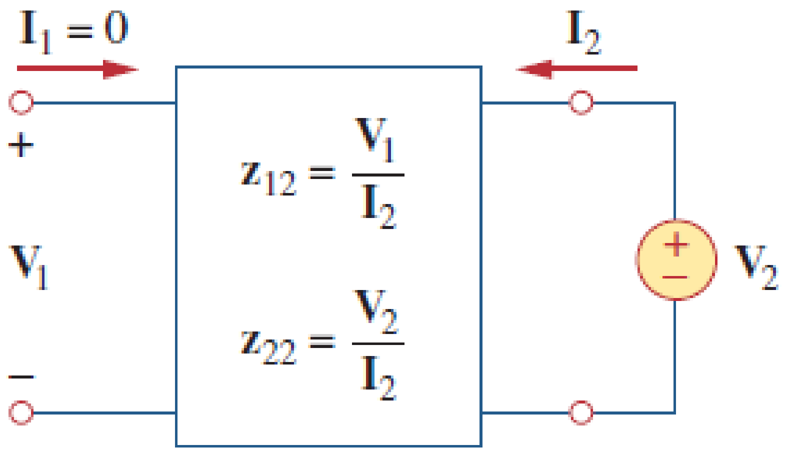
Graphical Approach - I

- Step 1 : Connect V_1 to port 1
- Step 2 : Apply KVL at input Loop
- Step 3 : Find Z_{11} and Z_{21}



Graphical Approach - II

- Step 1 : Connect V_2 to Port 2
- Step 2 : Apply KVL to Output Loop
- Step 3 : Find Z_{12} and Z_{22} .

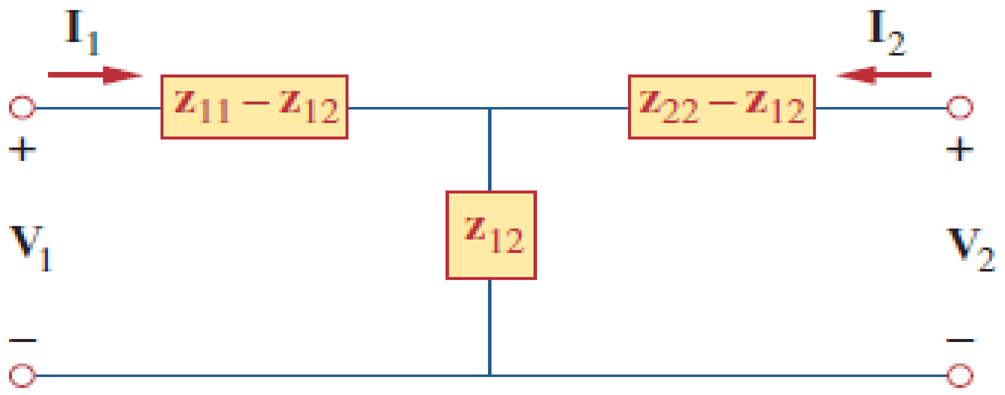


Whether Symmetrical ?

When $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be *symmetrical*. This implies that the network has mirrorlike symmetry about some center line; that is, a line can be found that divides the network into two similar halves.

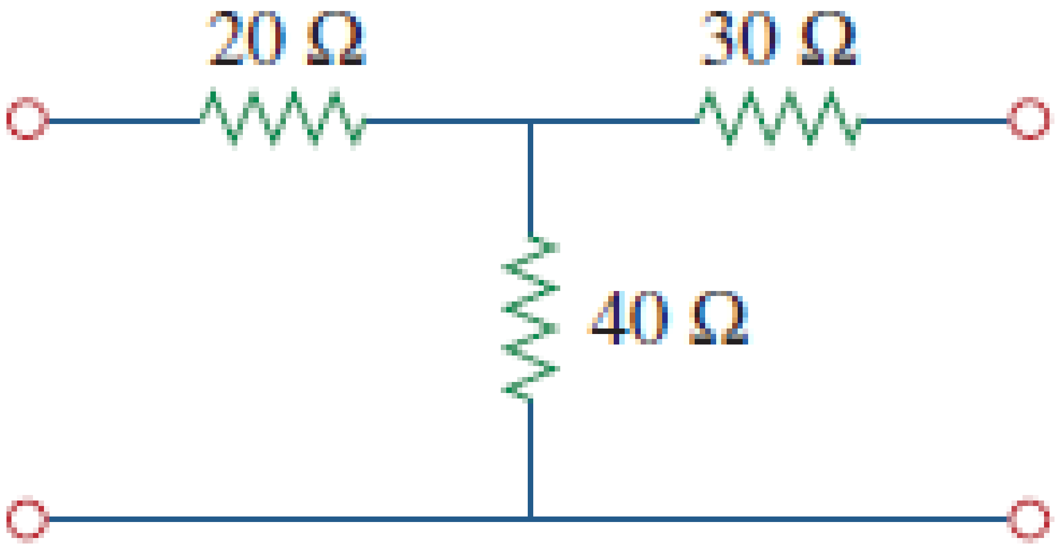
Whether Reciprocal ?

- When the two-port network is linear and has no dependent sources, the transfer impedances are equal (Z12 = Z21) and the two port is said to be *reciprocal*.

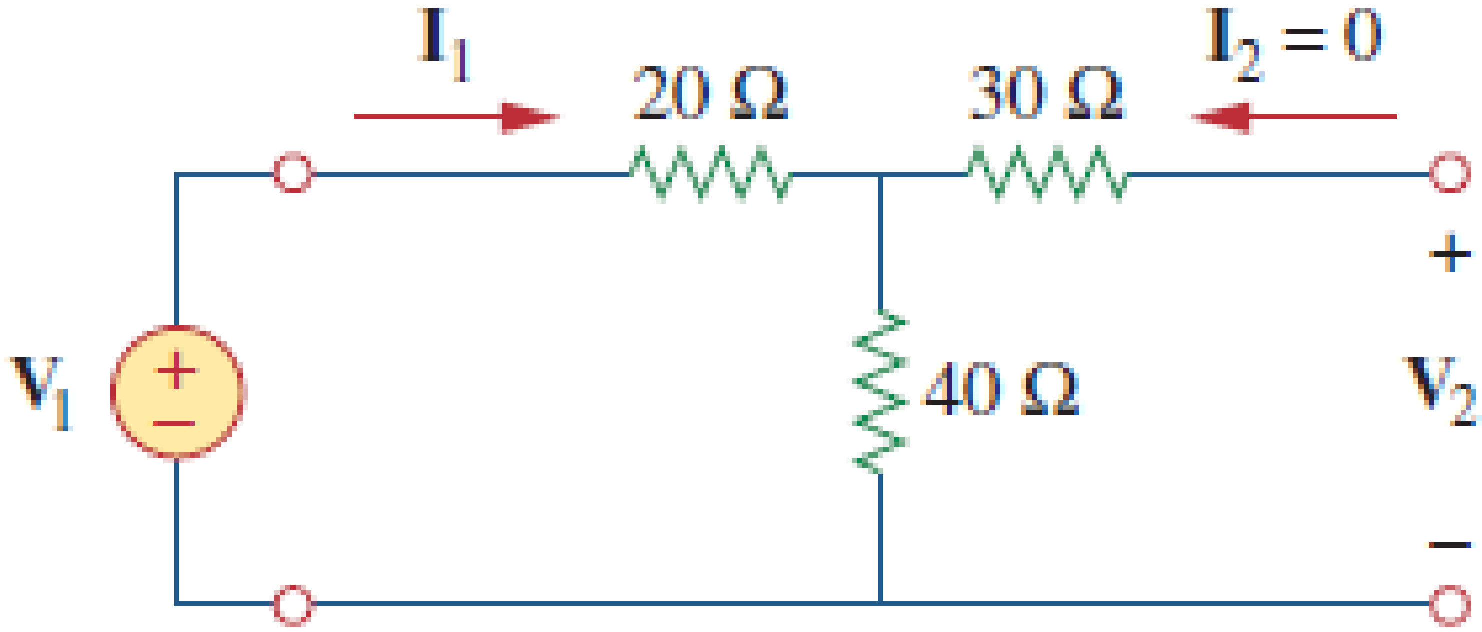


Problem 1

- Determine the Z- Parameters for the below circuit



Circuit 1 for Calculation of Parameters

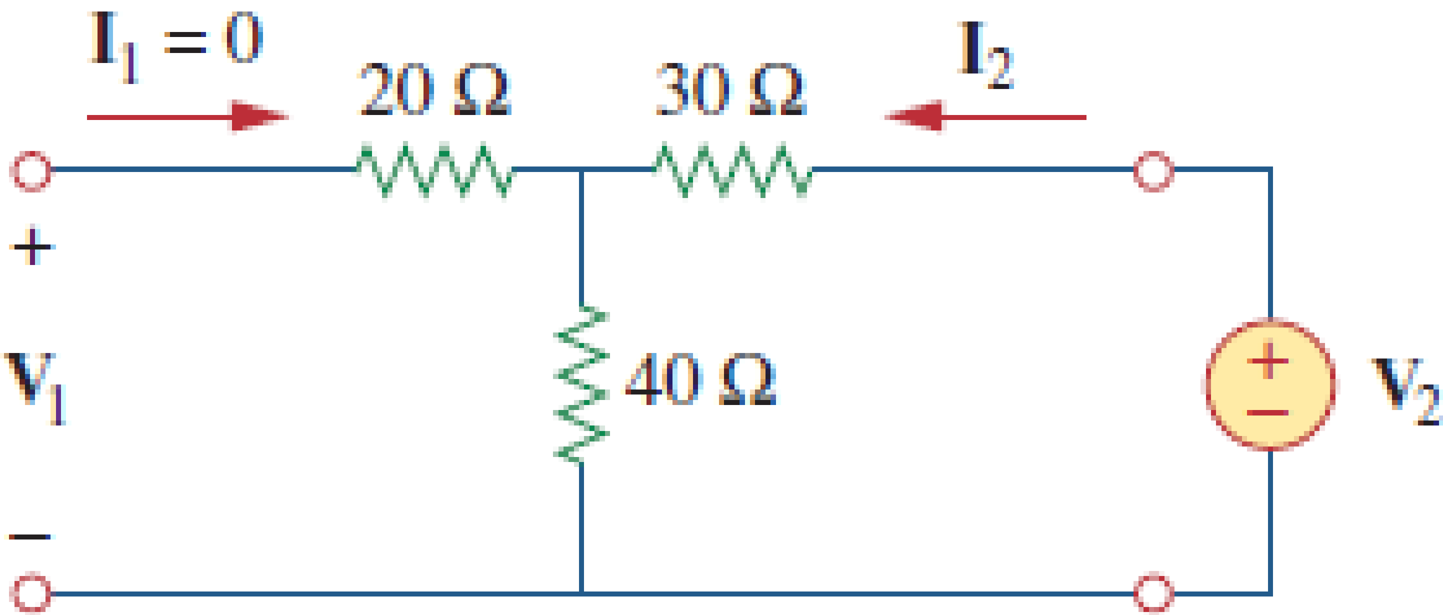


Calculation of Z_{11} and Z_{21}

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{(20 + 40)\mathbf{I}_1}{\mathbf{I}_1} = 60\ \Omega$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40\mathbf{I}_1}{\mathbf{I}_1} = 40\ \Omega$$

Circuit 2 For Calculation of Parameters



Calculation of Z_{12} and Z_{22}

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{40\mathbf{I}_2}{\mathbf{I}_2} = 40\ \Omega,$$

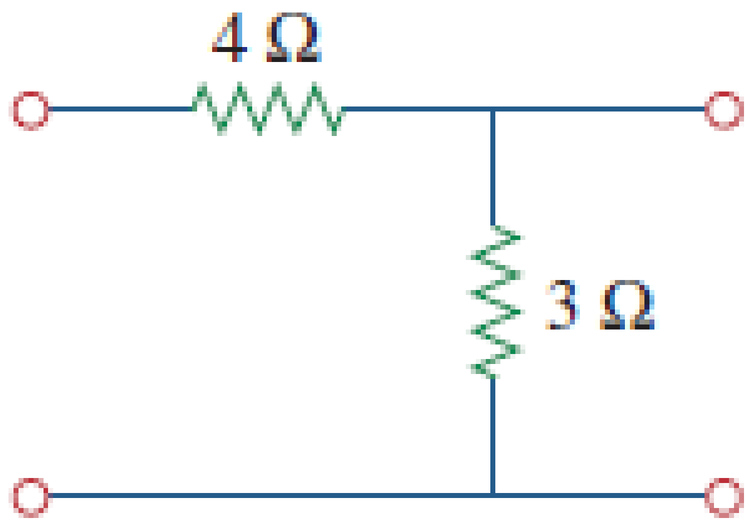
$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{(30 + 40)\mathbf{I}_2}{\mathbf{I}_2} = 70\ \Omega$$

Impedance Matrix

$$[\mathbf{z}] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

Problem 2

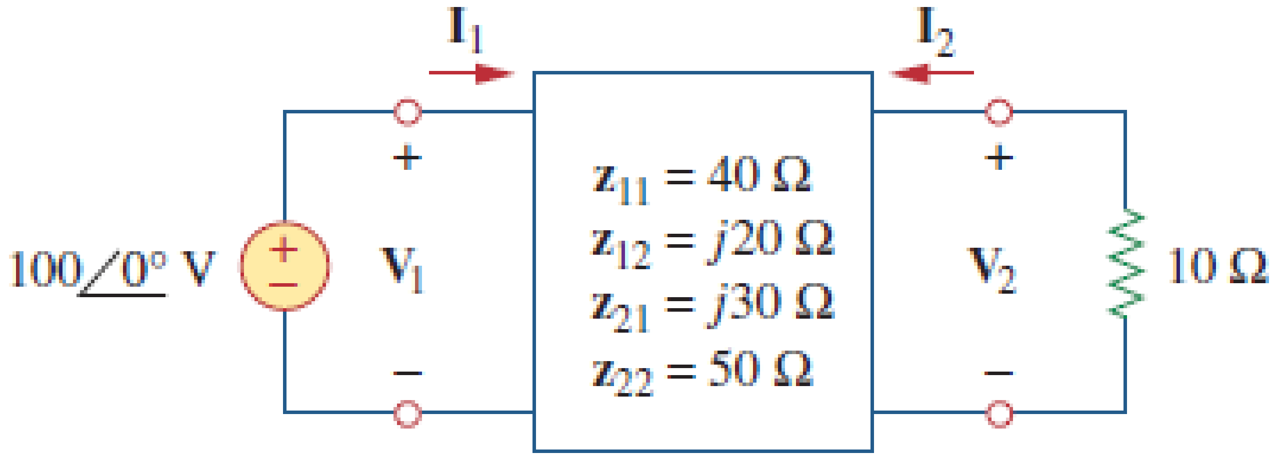
- Find the Z- Parameters for the Below Network



Answer: $z_{11} = 7\ \Omega,\ z_{12} = z_{21} = z_{22} = 3\ \Omega.$

Problem 3

- Find I_1 and I_2



$$\mathbf{I}_1 = 2\angle 0^\circ \text{ A}, \qquad \mathbf{I}_2 = 1\angle -90^\circ \text{ A}$$

Problem 4

- Find the Z-Parameters for the below circuit