Backpropagation

In this assignment, you will implement Backpropagation from scratch. You will then verify the correctness of the your implementation using a "grader" function/cell (provided by us) which will match your implementation.

The grader fucntion would help you validate the correctness of your code.

Please submit the final Colab notebook in the classroom ONLY after you have verified your code using the grader function/cell.

Loading data

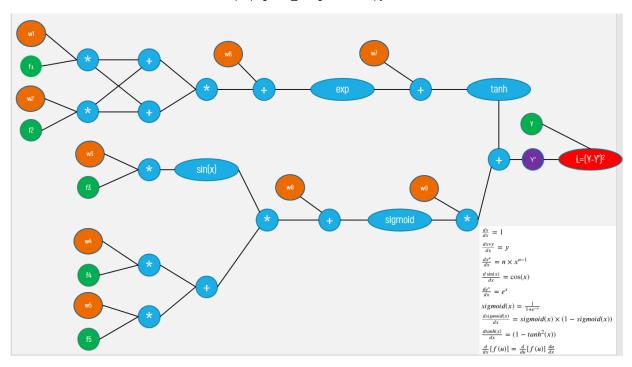
```
In [1]: import pickle
   import numpy as np
   from tqdm import tqdm
   import matplotlib.pyplot as plt

with open('data.pkl', 'rb') as f:
        data = pickle.load(f)
   print(data.shape)
   X = data[:, :5]
   y = data[:, -1]
   print(X.shape, y.shape)
(506, 6)
(506, 5) (506,)
```

Check this video for better understanding of the computational graphs and back propagation

```
In [2]: from IPython.display import YouTubeVideo
        YouTubeVideo('i940vYb6noo',width="1000",height="500")
Out[2]:
                CS231n Winter 2016: Lecture 4: Backpropagation, Neural Networks 1
```

Computational graph



- If you observe the graph, we are having input features [f1, f2, f3, f4, f5] and 9 weights [w1, w2, w3, w4, w5, w6, w7, w8, w9].
- The final output of this graph is a value L which is computed as (Y-Y')^2

Task 1: Implementing Forward propagation, Backpropagation and Gradient checking

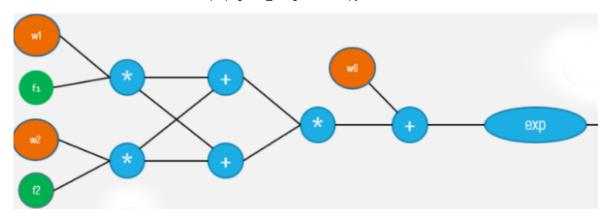
Task 1.1

Forward propagation

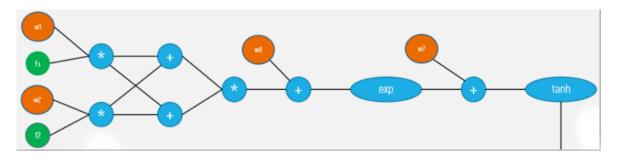
Forward propagation(Write your code in def forward_propagation())

For easy debugging, we will break the computational graph into 3 parts.

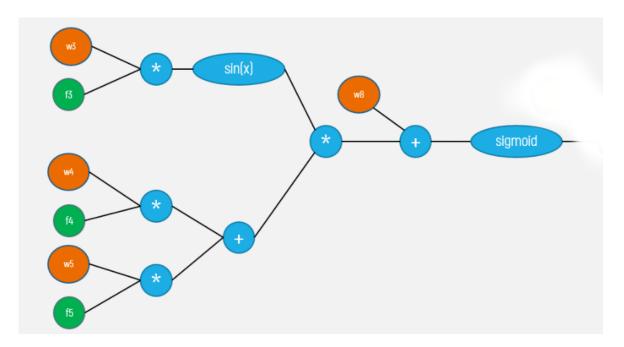
Part 1



Part 2



Part 3



```
In [3]: import math
   import numpy as np
   def sigmoid(z):

# we can use this function in forward and backward propagation
# write the code to compute the sigmoid value of z and return that valuen
        return 1/(1+math.exp(-z))
```

```
In [4]: def grader sigmoid(z):
           #if you have written the code correctly then the grader function will output the
           val=sigmoid(z)
           assert(val==0.8807970779778823)
           return True
         grader_sigmoid(2)
 Out[4]: True
 In [5]: def forward_propagation(x, y, w):
         # X: input data point, note that in this assignment you are having 5-d data point
         # y: output varible
         # W: weight array, its of Length 9, W[0] corresponds to w1 in graph, W[1] corresp
         # you have to return the following variables
         # exp= part1 (compute the forward propagation until exp and then store the values
         # tanh =part2(compute the forward propagation until tanh and then store the value
         # sig = part3(compute the forward propagation until sigmoid and then store the vd
         # we are computing one of the values for better understanding
             exp = np.exp((((x[0]*w[0])+(x[1]*w[1])) * ((x[0]*w[0])+ (x[1]*w[1]))) + w[5])
             tanh = np.tanh(exp+w[6])
             sig = sigmoid((np.sin(x[2]*w[2]) * ((x[3]*w[3])+(x[4]*w[4]))) + w[7])
             y_bar = sig*w[8] + tanh
             loss = (y-y_bar)**2
             dy pr = 2*(y bar-y)
             return {'exp':exp,'tanh':tanh,'sigmoid':sig,'loss':loss,'dy_pr':dy_pr}
In [28]: def grader forwardprop(data):
             dl = (data['dy pr']==-1.9285278284819143)
             loss=(data['loss']==0.9298048963072919)
             part1=(data['exp']==1.1272967040973583)
             part2=(data['tanh']==0.8417934192562146)
             part3=(data['sigmoid']==0.5279179387419721)
             assert(dl and loss and part1 and part2 and part3)
             return True
```

Out[28]: True

Grader Function

d1=forward_propagation(X[0],y[0],w)

w=np.ones(9)*0.1

grader forwardprop(d1)

```
In [7]: def grader_sigmoid(z):
     val=sigmoid(z)
     assert(val==0.8807970779778823)
     return True
     grader_sigmoid(2)
```

Out[7]: True

Grader Function-2

```
In [9]: def grader_forwardprop(data):
    dl = (data['dy_pr']==-1.9285278284819143)
    loss=(data['loss']==0.9298048963072919)
    part1=(data['exp']==1.1272967040973583)
    part2=(data['tanh']==0.8417934192562146)
    part3=(data['sigmoid']==0.5279179387419721)
    assert(dl and loss and part1 and part2 and part3)
    return True
    w=np.ones(9)*0.1
    d1=forward_propagation(X[0],y[0],w)
    grader_forwardprop(d1)
```

Out[9]: True

Task 1.2

Backward propagation

```
In [10]: def backward propagation(X,W,dict):
         #'''In this function, we will compute the backward propagation '''
         # L: the loss we calculated for the current point
         # dictionary: the outputs of the forward propagation() function
         # write code to compute the gradients of each weight [w1,w2,w3,...,w9]
         # Hint: you can use dict type to store the required variables
         # dw1 = # in dw1 compute derivative of L w.r.to w1
         # dw2 = # in dw2 compute derivative of L w.r.to w2
         # dw3 = # in dw3 compute derivative of L w.r.to w3
         # dw4 = # in dw4 compute derivative of L w.r.to w4
         # dw5 = # in dw5 compute derivative of L w.r.to w5
         # dw6 = # in dw6 compute derivative of L w.r.to w6
         # dw7 = # in dw7 compute derivative of L w.r.to w7
         # dw8 = # in dw8 compute derivative of L w.r.to w8
         # dw9 = # in dw9 compute derivative of L w.r.to w9
             exp = dict['exp']
             tanh = dict['tanh']
             sig = dict['sigmoid']
             loss = dict['loss']
             dv pr = dict['dv pr']
             dw1 = dy pr*(1-np.tanh(exp+W[6])**2)*np.exp(W[5]+((X[0]*W[0])+(X[1]*W[1]))*(
         (X[1]*W[1]))*X[0])+(((X[0]*W[0])+(X[1]*W[1]))*X[0]))
             dw2 = dy pr*(1-np.tanh(exp+W[6])**2)*np.exp(W[5]+(((X[0]*W[0])+(X[1]*W[1]))*(
         (X[1]*W[1]))*X[1])+(((X[0]*W[0])+(X[1]*W[1]))*X[1]))
             dw3 = dy_pr*W[8]*sigmoid(W[7]+(np.sin(X[2]*W[2])*((X[3]*W[3])+(X[4]*W[4]))))*
         ((X[3]*W[3])+(X[4]*W[4])))))*((X[3]*W[3])+(X[4]*W[4]))*np.cos(W[2]*X[2])*X[2]
             dw4 = dy pr*W[8]*sigmoid(W[7]+(np.sin(X[2]*W[2])*((X[3]*W[3])+(X[4]*W[4]))))*
         ((X[3]*W[3])+(X[4]*W[4]))))*np.sin(X[2]*W[2])*X[3]
             dw5 = dy pr*W[8]*sigmoid(W[7]+(np.sin(X[2]*W[2])*((X[3]*W[3])+(X[4]*W[4]))))*
         ((X[3]*W[3])+(X[4]*W[4]))))*np.sin(X[2]*W[2])*X[4]
             dw6 = dy_pr*(1-np.tanh(W[6]+exp)**2)*np.exp(W[5]+(((X[0]*W[0])+(X[1]*W[1]))*(
             dw7 = dy pr*(1-np.tanh(exp+W[6])**2)
             dw8 = dy pr*W[8]*sigmoid(W[7]+(np.sin(X[2]*W[2])*((X[3]*W[3])+(X[4]*W[4]))))*
         ((X[3]*W[3])+(X[4]*W[4])))))
             dw9 = dy_pr * sig
         #store the variables dw1,dw2 etc. in a dict as backward dict['dw1']= dw1,backward
         # return dW, dW is a dictionary with gradients of all the weights
             return {'dw1':dw1,'dw2':dw2,'dw3':dw3,'dw4':dw4,'dw5':dw5,'dw6':dw6,'dw7':dw7
```

```
In [31]: def grader backprop(data):
             dw1=(data['dw1']==-0.22973323498702003)
             dw2=(data['dw2']==-0.021407614717752925)
             dw3=(data['dw3']==-0.005625405580266319)
             dw4=(data['dw4']==-0.004657941222712423)
             dw5=(data['dw5']==-0.0010077228498574246)
             dw6=(data['dw6']==-0.6334751873437471)
             dw7=(data['dw7']==-0.561941842854033)
             dw8=(data['dw8']==-0.04806288407316516)
             dw9=(data['dw9']==-1.0181044360187037)
             assert(dw1 and dw2 and dw3 and dw4 and dw5 and dw6 and dw7 and dw8 and dw9)
             return True
         w=np.ones(9)*0.1
         d1=forward propagation(X[0],y[0],w)
         d1=backward_propagation(X[0],w,d1)
         grader backprop(d1)
Out[31]: True
```

```
In [32]: def grader backprop(data):
             dw1=(np.round(data['dw1'],6)==-0.229733)
             dw2=(np.round(data['dw2'],6)==-0.021408)
             dw3=(np.round(data['dw3'],6)==-0.005625)
             dw4=(np.round(data['dw4'],6)==-0.004658)
             dw5=(np.round(data['dw5'],6)==-0.001008)
             dw6=(np.round(data['dw6'],6)==-0.633475)
             dw7=(np.round(data['dw7'],6)==-0.561942)
             dw8=(np.round(data['dw8'],6)==-0.048063)
             dw9=(np.round(data['dw9'],6)==-1.018104)
             assert(dw1 and dw2 and dw3 and dw4 and dw5 and dw6 and dw7 and dw8 and dw9)
             return True
         w=np.ones(9)*0.1
         forward dict=forward propagation(X[0],y[0],w)
         backward_dict=backward_propagation(X[0],w,forward_dict)
         grader backprop(backward dict)
```

Out[32]: True

Task 1.3

Gradient clipping

Check this <u>blog link (https://towardsdatascience.com/how-to-debug-a-neural-network-with-gradient-checking-41deec0357a9)</u> for more details on Gradient clipping

we know that the derivative of any function is

$$\lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x-\epsilon)}{2\epsilon}$$

- The definition above can be used as a numerical approximation of the derivative. Taking an
 epsilon small enough, the calculated approximation will have an error in the range of epsilon
 squared.
- In other words, if epsilon is 0.001, the approximation will be off by 0.00001.

Therefore, we can use this to approximate the gradient, and in turn make sure that backpropagation is implemented properly. This forms the basis of **gradient checking!**

Gradient checking example

lets understand the concept with a simple example: $f(w1, w2, x1, x2) = w_1^2 \cdot x_1 + w_2 \cdot x_2$

from the above function , lets assume $w_1=1,\,w_2=2,\,x_1=3,\,x_2=4$ the gradient of f w.r.t w_1 is

$$\frac{df}{dw_1} = dw_1 = 2.w_1.x_1 = 2.1.3 = 6$$

let calculate the aproximate gradient of w_1 as mentinoned in the above formula and considering $\epsilon=0.0001$

Then, we apply the following formula for gradient check: $gradient_check = \frac{\|(dW - dW^{approx})\|_2}{\|(dW)\|_2 + \|(dW^{approx})\|_2}$

The equation above is basically the Euclidean distance normalized by the sum of the norm of the vectors. We use normalization in case that one of the vectors is very small. As a value for epsilon, we usually opt for 1e-7. Therefore, if gradient check return a value less than 1e-7, then it means that backpropagation was implemented correctly. Otherwise, there is potentially a mistake in your implementation. If the value exceeds 1e-3, then you are sure that the code is not correct.

in our example:
$$gradient_check = \frac{(6-5.99999999994898)}{(6+5.99999999994898)} = 4.2514140356330737e^{-13}$$

you can mathamatically derive the same thing like this

$$dw_{1}^{approx} = \frac{f(w1+\epsilon,w2,x1,x2)-f(w1-\epsilon,w2,x1,x2)}{2\epsilon}$$

$$= \frac{((w_{1}+\epsilon)^{2}.x_{1}+w_{2}.x_{2})-((w_{1}-\epsilon)^{2}.x_{1}+w_{2}.x_{2})}{2\epsilon}$$

$$= \frac{4.\epsilon.w_{1}.x_{1}}{2\epsilon}$$

$$= 2.w_{1}.x_{1}$$

Implement Gradient checking

(Write your code in def gradient_checking())

Algorithm

```
W = initilize randomly
def gradient checking(data point, W):
    # compute the L value using forward propagation()
    # compute the gradients of W using backword_propagation()
    approx_gradients = []
    for each wi weight value in W:
        # add a small value to weight wi, and then find the values of L
 with the updated weights
        # subtract a small value to weight wi, and then find the values
 of L with the updated weights
        # compute the approximation gradients of weight wi
        approx_gradients.append(approximation gradients of weight wi)
    # compare the gradient of weights W from backword propagation() with
the aproximation gradients of weights with
  gradient_check formula
    return gradient_check
NOTE: you can do sanity check by checking all the return values of gradi
ent checking(),
 they have to be zero. if not you have bug in your code
```

```
In [12]: W = np.ones(9)*0.1
         e = 0.001
         def gradient checking(data point, W):
         # compute the L value using forward propagation()
         # compute the gradients of W using backword propagation()
             d1=forward propagation(X[0],y[0],W)
             d2=backward propagation(X[0],W,d1)
             #we are storing the original gradients for the given datapoints in a list
             approx gradients = []
             for i in range(len(W)):
                 weight = W.copy()
                 weight[i] = weight[i]+e
                 forw = forward propagation(X[0],y[0],weight)
                 first loss = forw['loss']
             # make sure that the order is correct i.e. first element in the list correspo
             # you can use reverse function if the values are in reverse order
             #now we have to write code for approx gradients, here you have to make sure t
             #write your code here and append the approximate gradient value for each weid
                 weight = W.copy()
                 weight[i] = weight[i]-e
                 forw = forward propagation(X[0],y[0],weight)
                 second loss = forw['loss']
                 approximate gradient = (first loss - second loss)/(2*e)
         # add a small value to weight wi, and then find the values of L with the updated
         # subtract a small value to weight wi, and then find the values of L with the upd
         # compute the approximation gradients of weight wi
                 approx gradients.append(approximate gradient)
         # compare the gradient of weights W from backword propagation() with the aproxima
             return approx gradients
In [13]: | approx gradients = gradient checking(X[0],W)
In [14]: approx gradients
Out[14]: [-0.2297327584170894,
          -0.021407614332225045,
          -0.005625403583953137,
          -0.004657941219121664,
          -0.0010077228498328594,
          -0.6334750907762698,
          -0.5619421966853166,
          -0.048062880140031794,
          -1.0181044360186853]
```

```
In [15]: list(d1.values())
Out[15]: [-0.22973323498702003,
          -0.021407614717752925,
          -0.005625405580266319,
          -0.004657941222712423,
          -0.0010077228498574246,
          -0.6334751873437471,
          -0.561941842854033,
          -0.04806288407316516,
          -1.0181044360187037]
In [34]: # Euclidean distance normalized by the sum of the norm of the vectors
         np.linalg.norm(np.array(list(d1.values())) - np.array(approx gradients))/( (np.li
         np.linalg.norm(approx gradients) )
Out[34]: 2.235509031122564e-07
In [39]: def grader grad check(value):
             print(value)
             assert(np.all(value <= 10**-3))</pre>
             return True
         w=[0.00271756, 0.01260512, 0.00167639, -0.00207756, 0.00720768,
            0.00114524, 0.00684168, 0.02242521, 0.01296444]
         eps=10**-7
         value= gradient_checking(X[0],w)
         grader_grad_check(value)
         [-0.011663033554154545, -0.001086817516426919, 3.552009464335981e-06, -1.149056
         483296107e-05, -2.4859275749022913e-06, -0.903275579098084, -0.902219744669197
         1, -0.007047590417363914, -1.0995465311666175]
                                                    Traceback (most recent call last)
         C:\Users\ADM-VA~1\AppData\Local\Temp/ipykernel_2376/3542544610.py in <module>
               9 eps=10**-7
              10 value= gradient checking(X[0],w)
         ---> 11 grader_grad_check(value)
         C:\Users\ADM-VA~1\AppData\Local\Temp/ipykernel 2376/3542544610.py in grader gra
         d check(value)
               1 def grader_grad_check(value):
                     print(value)
                     assert(np.all(value <= 10**-3))</pre>
          ---> 3
                     return True
               4
               5
         TypeError: '<=' not supported between instances of 'list' and 'float'</pre>
```

Task 2: Optimizers

- As a part of this task, you will be implementing 2 optimizers(methods to update weight)
- Use the same computational graph that was mentioned above to do this task
- The weights have been initialized from normal distribution with mean=0 and std=0.01. The
 initialization of weights is very important otherwiswe you can face vanishing gradient and
 exploding gradients problem.

Check below video for reference purpose

```
In [36]: from IPython.display import YouTubeVideo
         YouTubeVideo('gYpoJMlgyXA',width="1000",height="500")
Out[36]:
                 CS231n Winter 2016: Lecture 5: Neural Networks Part 2
```

Algorithm

```
for each epoch(1-20):
    for each data point in your data:
        using the functions forward_propagation() and backword_propagation() compute the gradients of weights
        update the weigts with help of gradients
```

Implement below tasks

- Task 2.1: you will be implementing the above algorithm with Vanilla update of weights
- Task 2.2: you will be implementing the above algorithm with Momentum update of weights
- Task 2.3: you will be implementing the above algorithm with Adam update of weights

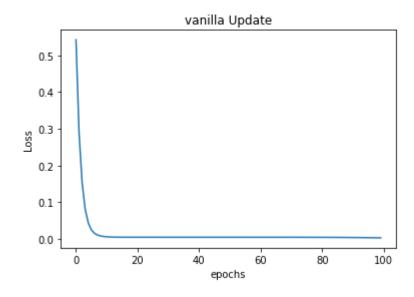
Note: If you get any assertion error while running grader functions, please print the variables in grader functions and check which variable is returning False. Recheck your logic for that variable.

2.1 Algorithm with Vanilla update of weights

```
w = np.random.normal(loc = 0.0, scale= 0.01, size=9)
In [18]:
         epochs = 100
         lr = 0.001
         gradient_losses = []
         v1 = []
         dw = []
         gamma = 0
         for i in range(epochs):
             for j in range(len(X)):
                  if len(dw) == 0:
                      d1 = forward_propagation(X[j],y[j],w)
                      d1 = backward propagation(X[j],w,d1)
                      for k in d1.values():
                          dw.append(k)
                  else:
                      d1 = forward_propagation(X[j],y[j],w)
                      d1 = backward_propagation(X[j],w,d1)
                      for i in range(len(d1.values())):
                          dw[i] = list(d1.values())[i]
                  for 1 in range(len(dw)):
                      w[1] = w[1] - lr*dw[1]
             11 = forward propagation(X[0],y[0],w)
             loss = l1['loss']
             gradient_losses.append(loss)
```

```
In [19]: plt.plot(range(epochs),gradient_losses)
    plt.ylabel('Loss')
    plt.xlabel('epochs')
    plt.title('vanilla Update')
```

Out[19]: Text(0.5, 1.0, 'vanilla Update')



In [37]: #as we see we have Plotted the plot with loss epochs.
#aw we observe the loss is getting rapidly decreasing as the epoch increases

2.2 Algorithm with Momentum update of weights

Momentum based Gradient Descent Update Rule

$$egin{aligned} v_t &= \gamma * v_{t-1} + \eta
abla w_t \ w_{t+1} &= w_t - v_t \end{aligned}$$

Here Gamma referes to the momentum coefficient, eta is leaning rate and v_t is moving average of our gradients at timestep t

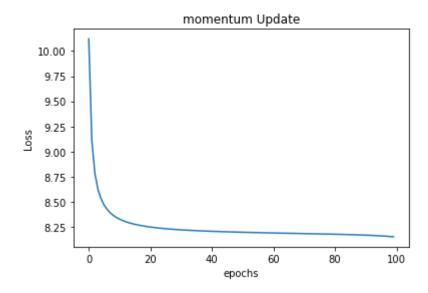
Type *Markdown* and LaTeX: α^2

```
In [20]: w = np.random.normal(loc = 0.0, scale= 0.01, size=9)
         epochs = 100
         lr = 0.001
         momentum losses = []
         v1 = []
         dw = []
         gamma = 0.95
         for i in range(epochs):
             for j in range(len(X)):
                 d1 = forward_propagation(X[j],y[j],w)
                 data = backward propagation(X[j],w,d1)
                 if len(dw) == 0:
                      for k in data.values():
                          dw.append(k)
                  if len(v1) == 0:
                     for 1 in data.values():
                          v1.append(1)
                      for m in range(len(w)):
                          w[m] = w[m] - lr*dw[m]
                      d1 = forward_propagation(X[j],y[j],w)
                      data = backward_propagation(X[j],w,d1)
                      for k in range(len(data.values())):
                          dw[k] = list(data.values())[k]
                      for n in range(len(v1)):
                          v1[n] = (gamma**j)*v1[n] - lr*dw[n]
                      for o in range(len(w)):
                          w[o] = w[o] + v1[o]
             d1 = forward_propagation(X[0],y[0],w)
             loss = d1['loss']
             momentum_losses.append(loss)
```

Plot between epochs and loss

```
In [22]: plt.plot(range(epochs), momentum_losses)
    plt.ylabel('Loss')
    plt.xlabel('epochs')
    plt.title('momentum Update')
```

Out[22]: Text(0.5, 1.0, 'momentum Update')



In []: #compared to this vanilla update of weights has more loss redumption w.t.to epoch

2.3 Algorithm with Adam update of weights

$$m_{t} = \beta_{1} * m_{t-1} + (1 - \beta_{1}) * \nabla w_{t}$$

$$v_{t} = \beta_{2} * v_{t-1} + (1 - \beta_{2}) * (\nabla w_{t})^{2}$$

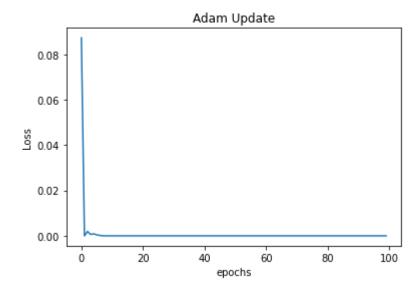
$$\hat{m}_{t} = \frac{m_{t}}{1 - \beta_{1}^{t}} \qquad \hat{v}_{t} = \frac{v_{t}}{1 - \beta_{2}^{t}}$$

$$w_{t+1} = w_{t} - \frac{\eta}{\sqrt{\hat{v}_{t} + \epsilon}} * \hat{m}_{t}$$

```
In [23]: import mpmath
         w = np.random.normal(loc = 0.0, scale= 0.01, size=9)
         epochs = 100
         alpha = 0.001
         adam losses = []
         vt = []
         mt = []
         mcapt = []
         vcapt = []
         dw = []
         beta1 = 0.9
         beta2 = 0.99
         epsilon = 0.0001
         for i in range(epochs):
             for j in range(len(X)):
                  d1 = forward_propagation(X[j],y[j],w)
                  data = backward propagation(X[j],w,d1)
                  if len(dw) == 0:
                      for k in data.values():
                          dw.append(k)
                      for k in range(len(w)):
                          mt.append((1-beta1)*dw[k])
                      for k in range(len(w)):
                          vt.append((1-beta2)*(dw[k])**2)
                      #for k in range(len(w)):
                      \#mcapt.append(mt[k]/(1 - beta1))
                      #for k in range(len(w)):
                      \#vcapt.append(vt[k]/(1-beta2))
                      for k in range(len(w)):
                          w[k] = w[k] - ((alpha*mt[k])/(np.sqrt(vt[k]) + epsilon))
                  else:
                      \#d1 = forward propagation(X[j],y[j],w)
                      \#data = backward propagation(X[j], w, d1)
                      for k in range(len(data.values())):
                          dw[k] = list(data.values())[k]
                      for k in range(len(mt)):
                          mt[k] = beta1*mt[k] + (1-beta1)*dw[k]
                      for k in range(len(vt)):
                          vt[k] = beta2*vt[k] + (1-beta2)*(dw[k])**2
                      #for k in range(len(mcapt)):
                      \#mcapt[k] = mt[k]/(1-beta1**j)
                      #for k in range(len(vcapt)):
                      \#vcapt[k] = vt[k]/(1-beta2**j)
                      for k in range(len(mt)):
                          w[k] = w[k] - ((alpha*mt[k])/(mpmath.sqrt(vt[k]) + epsilon))
                          #w[k] += -alpha*mcapt[k] / ( mpmath.sqrt(vcapt[k]) + epsilon )
             d1 = forward_propagation(X[j],y[j],w)
              loss = d1['loss']
             adam losses.append(loss)
```

```
In [24]: plt.plot(range(epochs),adam_losses)
    plt.ylabel('Loss')
    plt.xlabel('epochs')
    plt.title('Adam Update')
```

Out[24]: Text(0.5, 1.0, 'Adam Update')



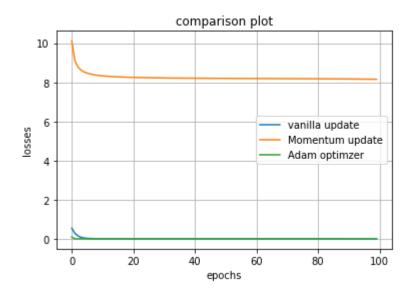
In []: #compared to vanilla and Momentum update of weights Adam is much more better as

Comparision plot between epochs and loss with different optimizers. Make sure that loss is conerging with increaing epochs

In [26]: #plot the graph between loss vs epochs for all 3 optimizers.

```
In [27]: plt.plot(range(epochs),gradient_losses, label = 'vanilla update')
    plt.plot(range(epochs),momentum_losses, label = 'Momentum update')
    plt.plot(range(epochs),adam_losses,label = 'Adam optimzer')
    plt.xlabel('epochs')
    plt.ylabel('losses')
    plt.title('comparison plot')
    plt.legend()
    plt.grid()
    plt.plot()
```

Out[27]: []



In []: #compared all the 3 optimizers and plotted those comparative results

You can go through the following blog to understand the implementation of other optimizers .

Gradients update blog (https://cs231n.github.io/neural-networks-3/)

In []: