Sampling Strategies

- Probabilistic Sampling
  1. Simple Random Sampling
  2. Cluster Sampling
  3. Systematic Sampling
- Non-probabilistic Sampling

① Simple Random Sampling :

- In this all the units of the population have equal Chance of being selected in the sample.
- SRS is useful when all population units are similar in nature. (But in reality it need not to be true)

② Cluster Random Sampling

- If the data is heterogeneous in nature, it better to first cluster them according to its parameter & then take samples from those clusters.

③ Systematic Sampling

In this we first arrange the units linearly & then take out the samples considering whole as population.

④ Simple Random Sampling:-

with Replacement | without Replacement.

let us consider any sample : $x_1, x_2 -- x_n$, irrespective of SRSWOR & SRSwithReplacement each x_i takes X_1 or $X_2 -- X_N$ with equal probability. $\frac{1}{N}$

$$\therefore E(x_i) = \frac{X_1 + X_2 + -- + X_N}{N}$$

= population mean

$= \mu$

$\therefore$ Expected value of sample mean = $\frac{x_1 + x_2 + -- x_n}{n}$

$$\therefore \overset{\text{sample mean}}{E(\bar{x})} = \frac{E(x_1) + E(x_2) + --- + E(x_n)}{n}$$

$$= \frac{\mu + \mu + --- + \mu}{n}$$

$= \mu$

$\therefore$ E(Sample mean) = population mean.

* Variance of the population.

$$\sigma^2 = \frac{1}{N}\left(\Sigma (x_i - \mu)^2\right)$$

* Variance of Sample mean $\bar{x}$

$$Var(\bar{x}) = E(\bar{x} - E(\bar{x}))^2$$

$$= E\left(\frac{x_1 + x_2 + x_3 - - + x_n}{n} - \frac{E(x_1) + E(x_2) + - - + E(x_n)}{n}\right)^2$$

$$= \frac{1}{n^2}\left(E(x_1 + x_2 + x_3 - - + x_n - E(x_1) - E(x_2) - - - - E(x_n)\right)^2$$

$$= \frac{1}{n^2} E\left((x_1 - E(x_1) + x_2 - E(x_2) + - (x_n - E(x_n)\right)^2$$

$$= \frac{1}{n^2} E\left(\sum_{i=1}^{n}(x_i - E(x_i))^2 + E\left(\sum_i \sum_{j \neq i}(x_i - E(x_i))(x_j - E(x_j)\right)\right)$$

$$= E(x_i - E(x_i))^2$$

$$= E(x_i - \mu)^2$$

$$= \frac{(X_1 - \mu)^2 + - - - + (X_N - \mu)^2}{N}$$

$$= \frac{\Sigma (X_i - \mu)^2}{N}$$

$$= \sigma^2$$

$$\therefore \frac{1}{n^2}\sum_i E(x_i - E(x_i))^2$$

$$= \sum_i \sigma^2/n^2 = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$$\frac{1}{n^2} E\left(\sum_i \sum_{j \neq i}(x_i - E(x_i))(x_j - E(x_j))\right)$$

$$= \frac{1}{n^2}\sum_i \sum_{j \neq i} E((x_i - E(x_i))(x_j - E(x_j)))$$

$$= \frac{1}{n^2}\sum_i \sum_{j \neq i} cov(x_i, x_j)$$

⇒ In Sampling with replacement then $\cancel{cov(x_i, x_j)}$ 0

$\therefore 0$

So $Var(\bar{x}) = \frac{\sigma^2}{n}$ if Sampling with Replacement.