

* Sampling Strategies

Probabilistic
Sampling

Non-probabilistic
Sampling

- ① → Simple Random Sampling
- ② → **Cluster** Sampling
- ③ → Systematic Sampling

① Simple Random Sampling :-

- In this all the units of the population have equal chance of being selected in the sample.
- SRS is useful when all population units are similar in nature. (But in reality it need not to be true)

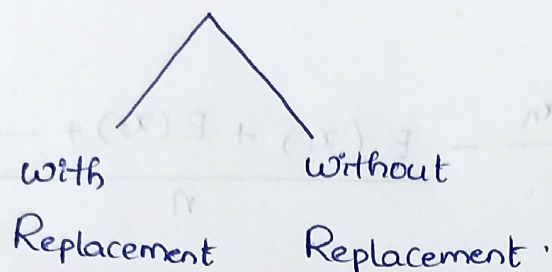
② Cluster Random Sampling

- If the data is heterogeneous in nature, it's better to first cluster them according to its parameter & then take samples from those clusters.

③ Systematic Sampling

In this we first arrange the units linearly & then take out the samples considering whole as population.

④ Simple Random Sampling:-



let us consider any sample : x_1, x_2, \dots, x_n , irrespective of SRSWOR & SRSwithreplacement each x_i takes X_1 or X_2, \dots, X_N with equal probability $\frac{1}{N}$

$$\therefore E(x_i) = \frac{X_1 + X_2 + \dots + X_N}{N}$$

= population mean

= μ

$$\therefore \text{Expected value of Sample mean} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

$$E(\bar{x}) = \frac{E(x_1) + E(x_2) + \dots + E(x_n)}{n}$$

↗ Sample mean

$$= \frac{\mu + \mu + \dots + \mu}{n}$$

$$= \mu$$

$$\therefore E(\text{Sample mean}) = \text{population mean.}$$

→ Variance of the population.

$$\sigma^2 = \frac{1}{N} \left(\sum (x_i - \mu)^2 \right)$$

→ Variance of Sample mean \bar{x}

$$\text{Var}(\bar{x}) = E(\bar{x} - E(\bar{x}))^2$$

$$= E\left(\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} - \frac{E(x_1) + E(x_2) + \dots + E(x_n)}{n}\right)^2$$

$$= \frac{1}{n^2} E\left(E(x_1 + x_2 + x_3 + \dots + x_n - E(x_1) - E(x_2) - \dots - E(x_n))\right)^2$$

$$= \frac{1}{n^2} E\left((x_1 - E(x_1) + x_2 - E(x_2) + \dots + x_n - E(x_n))\right)^2$$

$$= \frac{1}{n^2} E\left(\underbrace{\sum_{i=1}^n (x_i - E(x_i))^2}_{\downarrow} + \underbrace{E\left(\sum_i \sum_{j \neq i} (x_i - E(x_i))(x_j - E(x_j))\right)}_{\downarrow}\right)$$

$$= E(x_i - E(x_i))^2$$

$$= E(x_i - \mu)^2$$

$$= \frac{(x_1 - \mu)^2 + \dots + (x_N - \mu)^2}{N}$$

$$= \frac{\sum (x_i - \mu)^2}{N}$$

$$= \sigma^2$$

$$\therefore \frac{1}{n^2} \sum_i E(x_i - E(x_i))^2$$

$$= \sum \sigma^2 / n^2 = \frac{n\sigma^2}{n^2}$$

$$\frac{1}{n^2} E\left(\sum_i \sum_{j \neq i} (x_i - E(x_i))(x_j - E(x_j))\right)$$

$$= \frac{1}{n^2} \sum_i \sum_{j \neq i} E((x_i - E(x_i))(x_j - E(x_j)))$$

$$= \frac{1}{n^2} \sum_i \sum_{j \neq i} \text{cov}(x_i, x_j)$$

⇒ In Sampling with replacement
then $\text{cov}(x_i, x_j) = 0$

$$= \frac{\sigma^2}{n}$$

So $\text{Var}(\bar{x}) = \frac{\sigma^2}{n}$ if Sampling with Replacement.