Probability for Data Science

What is Probability?

- Probability is a measure of how likely an event is to occur.
- It is expressed as a number between 0 and 1, where 0 indicates that the event will not happen, and 1 indicates that the event will certainly happen.
- The closer the probability is to 1, the more likely the event is, and the closer it is to 0, the less likely it is.

Here's a breakdown:

0: Impossible event (it will not happen).

0.5: Event is equally likely to happen or not happen (50-50 chance).

1: Certain event (it will definitely happen).

$P(A) = \frac{n(A)}{n(S)}$

where:

- $P(A)$ is the probability of event A.
- $n(A)$ is the number of favorable outcomes for event A.
- $n(S)$ is the total number of outcomes in the sample space.

Example: Consider flipping a fair coin. The probability of getting Heads (H) is $P(H) = \frac{1}{2}$ and Tails (T) is also $P(T) = \frac{1}{2}$.

Terminologies

- **Experiment**: Action or set of actions performed. Ex. Flipping a coin.
- **Outcome**: A single possibility from the experiment. Ex. Getting head or tail after a coin is flipped.

- **Sample Space:** The set of all possible outcome. Ex. Head and Tail are two possible outcome of flipping a coin.
- **Event**: Something we can observe. An outcome or occurrence that has a probability assigned to it. Ex. Getting exactly 2 heads when 2 coins are flipped.

Event Types

Mutually Exclusive

If two events are **mutually exclusive**, only **one of the two can occur** or the **occurrence of one excludes the occurrence of other.**

Example

Probability of a random card drawn being a King or a Queen.

A card can be either a King or a Queen but not both. Picking a King excludes the chances of it being a Queen, vice versa.

$P(A\ or\ B) = P(A) + P(B)$

or

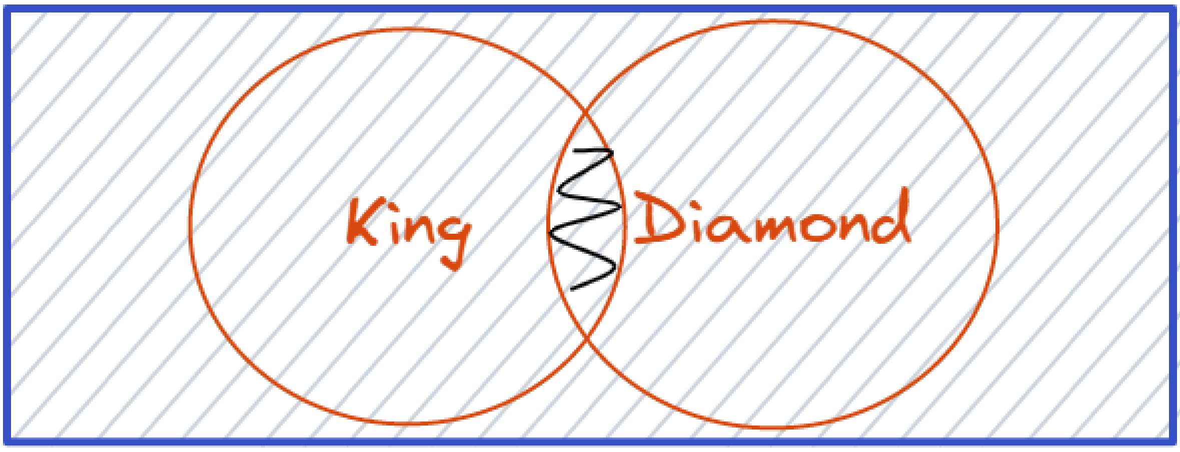
$P(A \bigcup B) = P(A) + P(B)$

Not Mutually Exclusive

If two events are **not mutually exclusive**, both can occur or the **occurrence of one *doesn't* excludes the occurrence of other.**

Example

Probability of a random card drawn being a King or a Diamond.



$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

or

$$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$$

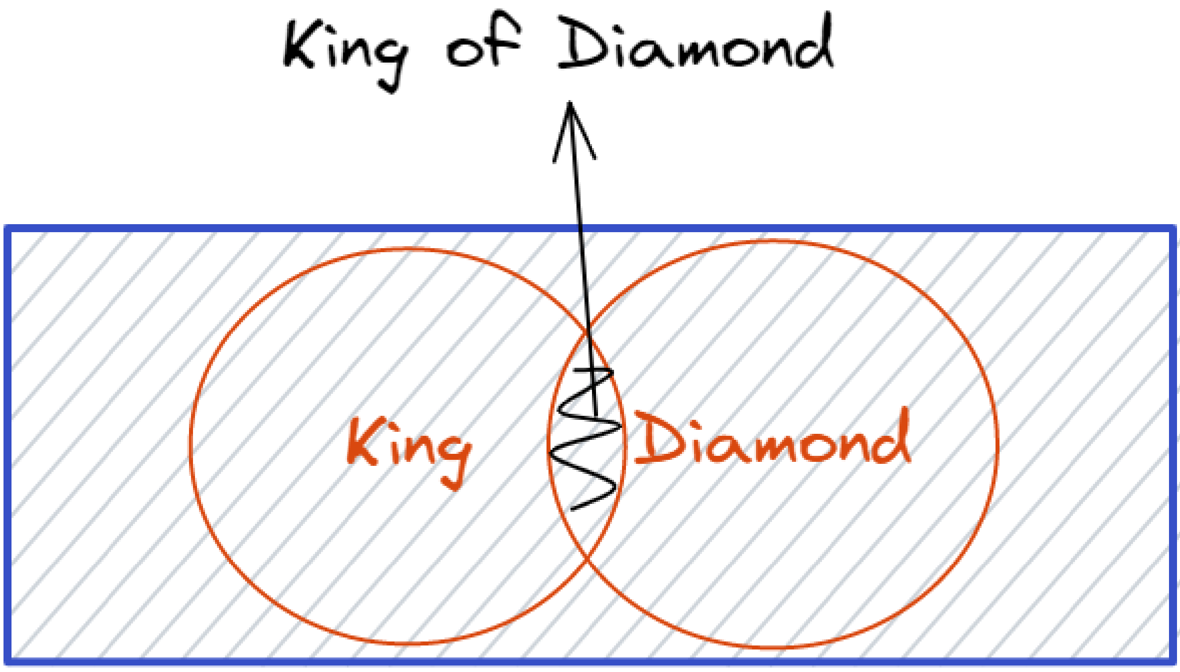
Note:
*If events **A and B** are **exclusive**, then* **P(A∩B) = 0**
*If events **A and B** are **exhaustive**, then* **P(A∪B) = 1**.

Independent

If two events **intersect**, or are **independent**, it's possible they can occur simultaneously.

Example

Probability of a random card drawn being a King or a Diamond.



$P(A \ and \ B) = P(A) * P(B)$

$or,$

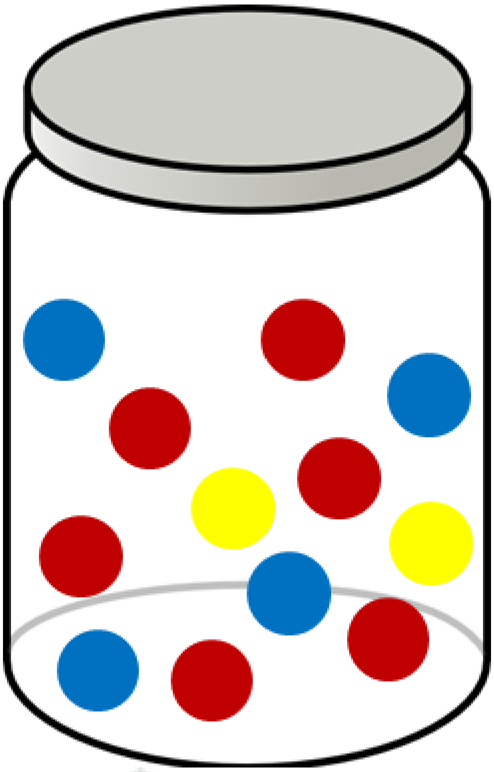
$P(A \ \cap \ B) = P(A) * P(B)$

Dependent

If two events **dependent**, the occurrence of one event affect the probability of occurrence of the other.

Example

Drawing two different color balls from a bag of balls.



$$P(A \bigcap B) = P(A) * P(B|A)$$

$$or$$

$$P(A\ and\ B) = P(A) * P(B|A)$$

AND

$$P(B|A) \rightarrow Conditional\ Probability$$

Conditional Probability:

Conditional probability calculates the likelihood of an event occurring given that another event has already occurred.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

where:

- $P(A|B)$ is the conditional probability of event A given event B.
- $P(A \cap B)$ is the joint probability of both events A and B occurring together.
- $P(B)$ is the probability of event B.

Example: Consider drawing two cards from a standard deck. What is the probability of drawing a Queen as the second card, given that the first card drawn was a King? Let A be the event of drawing a Queen as the second card, and B be the event of drawing a King as the first card. The probability is $P(A|B) = \frac{4}{51}$.

Law of Total Probability

If we have two events A and B, then,

$$P(B) = P(B \cap A) + P(B \cap A') = P(A) * P(B|A) + P(A') * P(B|A')$$

The Law of Total Probability is the denominator of Bayes' Theorem.

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Bayes' Theorem

Describes how the conditional probability of each of a set of possible causes for a given observed outcome can be computed from knowledge of the probability of each cause and the conditional probability of the outcome of each cause.

$$P(A|B) = \frac{P(A) * P(B|A)}{P(A) * P(B|A) + P(A') * P(B|A')}$$

Bayes' Theorem

Therefore,

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

$$Where,$$

$$P(A|B) \rightarrow Posterior\ Probability$$

$$P(B|A) \rightarrow Likelihood$$

$$P(A) \rightarrow Class\ Prior\ Probability$$

$$P(B) \rightarrow Predictor\ Prior\ Probability$$

Expectation and Variance:

Expectation (mean) and variance are statistical measures that provide valuable insights about a random variable.

$$E[X] = \sum_x x \cdot P(X = x)$$

The variance (Var[X]) is calculated as:

$$Var[X] = E[X^2] - (E[X])^2$$

Example: Consider the random variable X representing the number obtained when rolling a fair six-sided die. The expectation is $E[X] = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = \frac{7}{2}$ and the variance is $Var[X] = \frac{35}{12}$.

Joint Probability and Marginal Probability:

Joint probability deals with the probability of two or more events occurring together. On the other hand, marginal probability involves the probability of a single event regardless of the occurrence of other events.

Example: Consider tossing two coins. The joint probability of getting Heads on both coins is $P(H \cap H) = \frac{1}{4}$, and the marginal probability of getting at least one Head is $P(H \cup T) = \frac{3}{4}$.

Random Variables

Discrete Random Variables

- Probability Mass Function (PMF)
- Likelihood Functions
- Maximum Likelihood Estimate
- Bernoulli distribution
- Geometric Distribution
- Binomial Distribution
- Poisson distribution

Continuous Random Variables

- Cumulative Distribution Function (CDF)
- Probability Distribution Function (PDF)
- Kernel Density Estimate (KDE)
- Likelihood Functions
- Uniform Distribution
- Exponential Distribution
- Gaussian distribution

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