Final Project Draft

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1 Introduction

When a great threat comes to earth, how does humanity react. Do they collaborate and find ways to fight back, or do humans become self interested, seeking to maximize their short term well being, at a cost of weakening humanity's chances of fighting back?

This project seems to analyze this problem from an evolutionary game theory perspective. Given the choices of humanity to collaborate or act in self interest, and given an Invader's choice to be aggressive in their actions or passive, can patterns be found that allow humanity to survive.

2 Theoretical Setup

2.1 Invader Game Design

To analyze this scenario, part of this project designs a formal tripartite game to describe how the choices of humanity and a sentient threat agent could lead to some payoff. This type of game design is based on prior work analyzing the interaction between customers, restaurants, and delivery platforms Wang [2023].

Below is how the proposed game will be designed at a high level. The variables a#, b#, c# represents the payoffs for Human 1, Human 2, and the Invader.

		Invader			
		Passive		Active	
		Human 2			
		Collaborate	Self-Interest	Collaborate	Self-Interest
Human 1	Collaborate	a1, b1, c1	a2, b2, c2	a3, b3, c3	a4, b4, c4
	Self-Interest	a5, b5, c5	a6, b6, c6	a7, b7, c7	a8, b8, c8

2.2 Payoff Calculations

Calculating these payoffs will be build from the below variables

Variable List:

- **V** - Resource to strengthen humanity to defend from the invader
- **C** - Cost of self interest
- **S** - Synergistic Factor of Collaboration. The assumption here being that when more people collaborate, the better they can do to defend from the invader
- **D** - Damage from a aggressive Invader
- **A** - Cost of an Invader's Attack
- **B** - How much Damaged is Reduced in Attack against a collaborative humanity

Each of the player's payoffs can be computed, using the above variables strategy set, as defined below:

Collaborative - Collaborative - Passive

Human 1	V/2 * S
Human 2	V/2 * S
Invader	-VS

Collaborative - Self Interested - Passive

Human 1	0
Human 2	V
Invader	-V

Self Interested - Collaborative - Passive

Human 1	V
Human 2	0
Invader	-V

Self Interested - Self Interested - Passive

Human 1	(V-C)/2
Human 2	(V-C)/2
Invader	-(V-C)

Collaborative - Collaborative - Active

Human 1	(V/2 * S) - B
Human 2	(V/2 * S) - B
Invader	-(VS - 2B + A)

Collaborative - Self Interested - Active

Human 1	-D
Human 2	V-D
Invader	-(V-2D+A)

Self Interested - Collaborative - Active

Human 1	V-D
Human 2	-D
Invader	-(V-2D+A)

Self Interested - Self Interested - Active

Human 1	V/2 -C -D
Human 2	V/2 -C -D
Invader	-(V - 2C - 2D +A)

One thing of note with these payoff definitions is the symmetry in some of these cases. For example, the Collaborative - Self Interested - Passive case is symmetric to the Self Interested - Collaborative - Passive case. This is the case because the ordering of how the human's behave does not effect their payoff. Similarly the invader does not differentiates between the ordering of the types of humans they are against.

Another thing of note is if the overall payoff matrix was split into two, one focusing on if the invader was passive and the other on if the invader was active. The passive matrix would actually be a zero sum game between the two humans and the invader, while the active matrix would be a -A sum game. This -A representing the amount of resources a invader has to put into the game itself, in order to be active and do damage

2.3 Specific Game Used in Project

As of now, this project is focused on one specific set of variables to analyze. Below are those variables, and the payoff matrix that they create:

Variable List:	
V	15
C	10
S	1.5
D	20
A	5
B	2

		Invader			
		Passive		Active	
		Human 2			
		Collaborate	Self-Interest	Collaborate	Self-Interest
Human 1	Collaborate	11.25, 11.25, -22.5	0, 15, -15	9.25, 9.25, -23.5	-20, -5, 20
	Self-Interest	15, 0, -15	2.5, 2.5, -5	-5, -20, 20	-22.5, -22.5, 40

This variable set was chosen such that no one player has a dominant strategy. In not all cases will it be better for a threat to be Active in their attacks. Similarly for humans, the choice to be collaborative or self-interested is not always clear. The invader's goal here is clear, since its costly to do an attack, they want the humans to be as self-interested as possible in order to minimize the amount of times they need to attack to take over.

This type of game design lends it self well to see how players strategies change over time, as well as how populations evolve with respect to these payoffs.

3 Simulation

3.1 Project Materials

All code and original images for this project is located in the defend-from-invader repository. Driver files to produce the results for the below simulation sections can be found at the main level of the repository.

3.2 Iterative Simulation

In the first phase of the project, we analyze how different learning rates effect outcomes in an iterative game. In this game there are only 3 players, 2 Humans and 1 Invader. Each player in the game will have a probability P_i of doing one action (being collaborative for humans, and passive for the invader) and probability $1 - P_i$ of doing their other action (being self-interested for humans, and aggressive for the invader).

Amongst these 3 players, we run the game 10,000 times, where in between each iteration, the players have a chance to update their probability P_i. Say a human was collaborative or the invader was being passive, if this action yielded a higher payoff than being self-interested or active, then that players P_i would get increased by a player's learning rate, otherwise it would get decreased. Similarly, if a human was being self-interested or an invader was being active, and these actions yielded a higher payoff that the player's alternative action, P_i would then be decreased, such that those actions would appear with higher probability. Between rounds, each increase/decrease would be of the same amount, which is equal to each player's learning rate.

Multiple learning rate combinations for the 3 players were tested out. Abstractly, a combination of Slow, Medium, and Fast Learning Rates were applied. Initial probabilities were also assigned and were made constant throughout the various learning rate combination experiments. For the 2 humans, one human was made to be initially more collaborative, having a P_i = .9, the other human was made to be more self-interested initially, having a P_i = .2. The invader was made to be initially passive, having a P_i = .9. Below are graphs of the three player's probabilities of actions across the 10,000 iterations. Varying the learning rates between the 3 players lead to interesting results which will be analyzed for each graph.

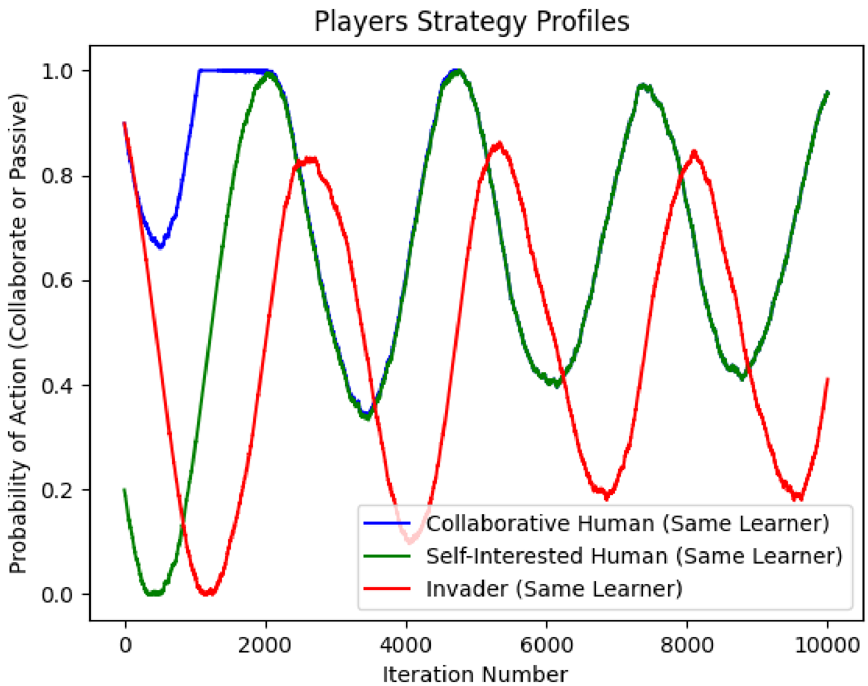
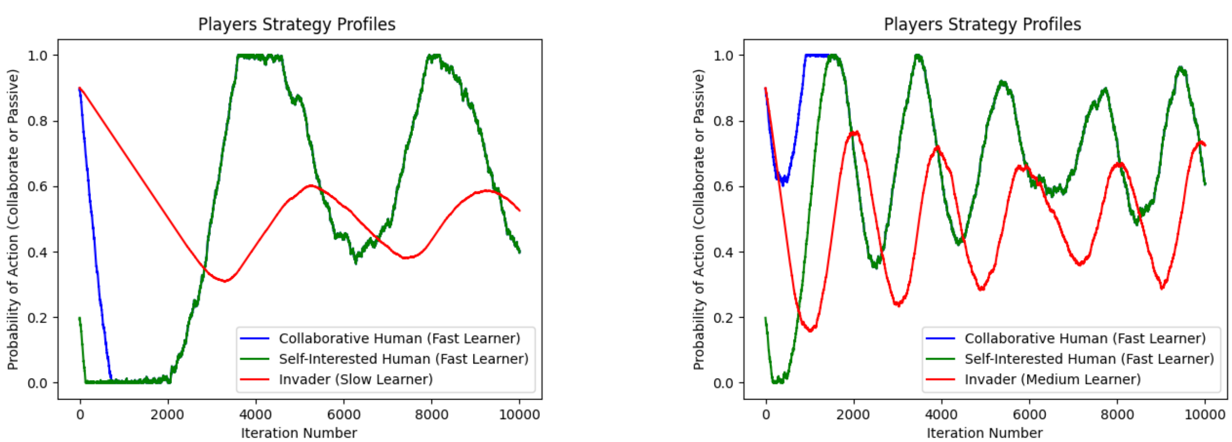


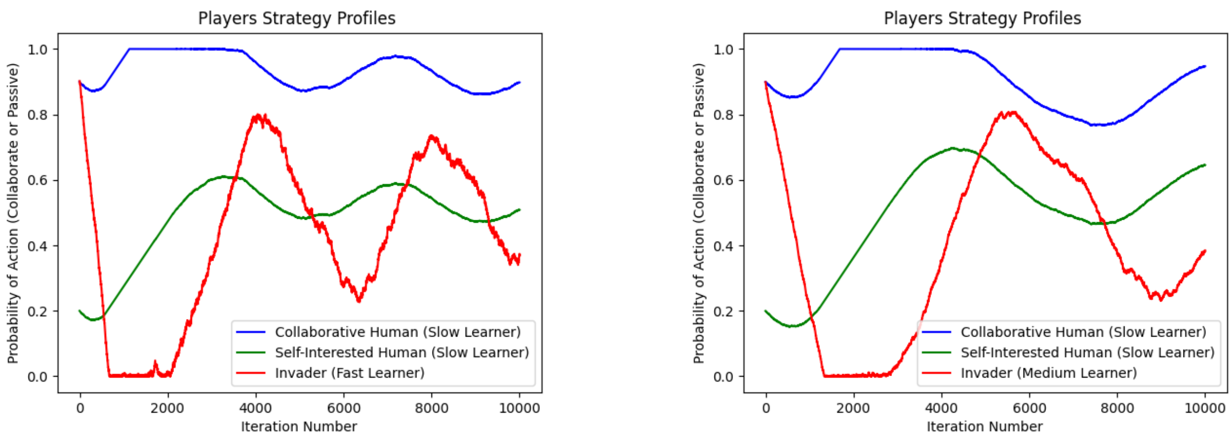
Figure 1: This scenario showcases the base case of all 3 players having the same learning rate. The two humans, despite having different initial conditions converge rather quickly to have the same cyclic pattern to their probability of being collaborative. Interestingly enough when compared to the invader, the probability waves are offset from each other, showcasing that the humans become more collaborative when the invader is active and more self-interested when the invader is passive. Another thing to notice here is that the amplitudes for all players are very similar, mostly because their learning rates are exactly the same



(a)

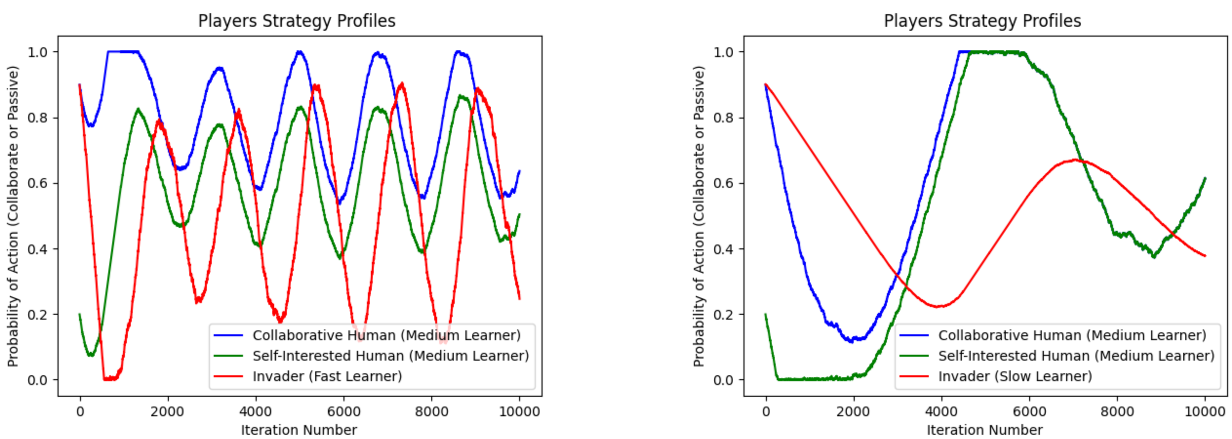
(b)

Figure 2: These two figures showcase when the humans have a fast learning rate, and the invader varies their learning rate. Since the humans are quick to adapt their strategies, they soon become in-sync similar to when all players have the same learning rate.



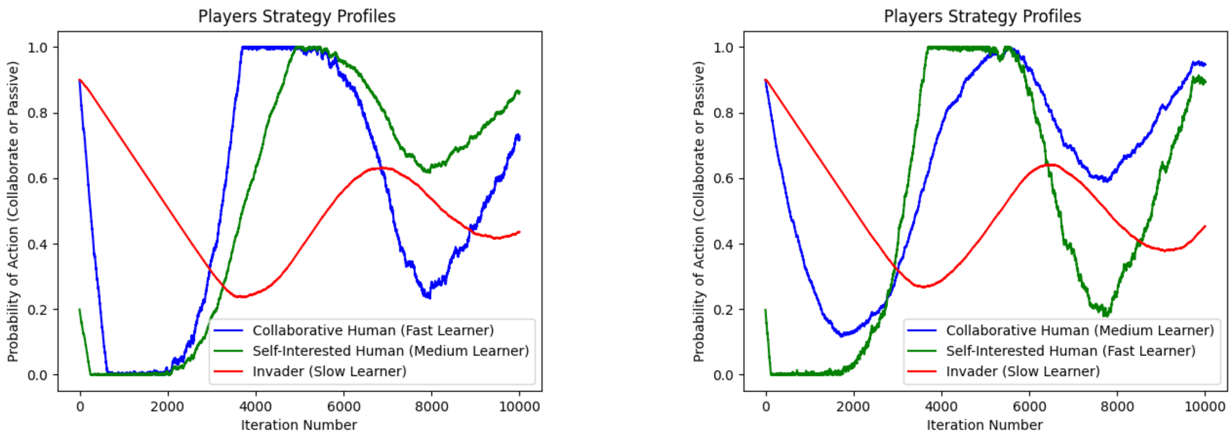
(a) (b)

Figure 3: These two figures showcase when the humans have a slow learning rate, and the invader varies their learning rate. With the invader having a faster learning than the humans, they are able to quickly adapt to any strategy. However, the humans can not react fast enough, and thus oscillate around their initial positions, not converging with one another.



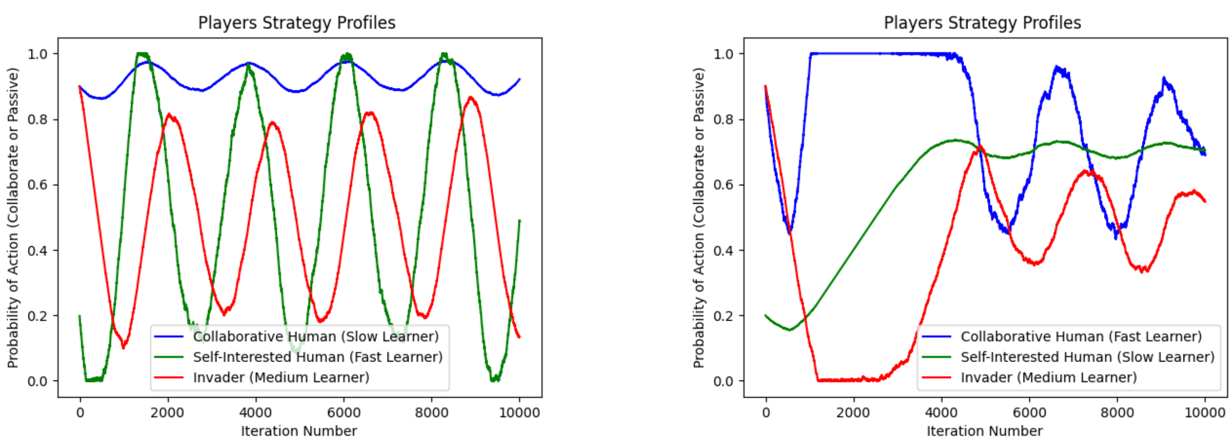
(a) (b)

Figure 4: These two figures showcase when the humans have a medium learning rate, and the invader varies their learning rate. Interestingly enough the behavior changes depending on what the invader's learning rate is. If the invader has a slower learning rate, the humans have time to reach and converge their strategies. However if the invader has a faster learning rate, the humans are not able to react fast enough and converge to each other.



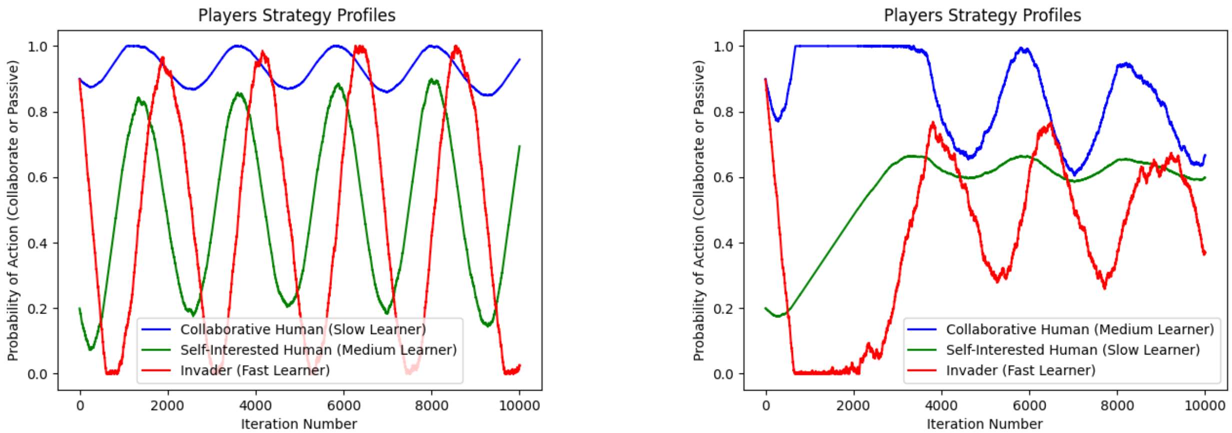
(a) (b)

Figure 5: These figures showcase when the invader has a slow learning rate, and where the humans switch off between being a fast and medium learner. The humans now having different learning rates, no longer converge like in the same learning rates case. With the invader being slow to adjust their strategy, the faster humans are able to react quickly enough to collaborate with each other. A common theme in all of these figures is looking at the cyclical nature of the strategy sets, as well as the amplitude. Faster learners are able to more quickly adjust back and forth between their two action sets.



(a) (b)

Figure 6: These figures showcase when the invader has a medium learning rate, with the humans varying between a slow and fast learning. Slow learning will have a smaller amplitude. Interestingly, in figure (a), the human who was initially collaborative and is a slow learner stays relatively collaborative. However, in figure (b), the self-interested human, actually raises their likelyhood to collaborate quite a bit before stabilizing at around .7. These two figures also showcase how not only initial human strategies impacts how the invader itself adjusts their strategy, but also how the learning rates of the humans can also impact it as well. In figure (b) With the collaborative human being quick at learning, they can begin to adopt a pure strategy of being collaborative, while waiting for the self-interested human to catch up. In contrast, in figure (a), with the self interested human being more quick to learn, they can end up collaborating with the other human, once the invader become more aggressive



(a) (b)

Figure 7: These figures showcase when the invader is a fast learner while the humans vary between being slow and medium learners. Interestingly enough the behaviors in these figures mirrors the behaviors when the invader has a medium learning rate and the humans have differing learning rates. Amplitudes across these two sets of images are different, but general shape remains.

Overall, analyzing how differing learning rates amongst the three parties in the game showcases how both humans and invaders adapt their strategy in response to the others. While this toy example with three players may not be realistic, it does showcase the patterns of behaviors that we may expect; (1) Humans being collaborative when the invader is being more aggressive. (2) Human's converging to the same behavior if they have the same learning rates. (3) If humans have differing learning rates not only do they not converge but whomever is the faster learner effects the outcome qualitatively. (4) Humans becoming more self-interested when the invader is being passive. To continue to push this analysis further, seeing how this game applies to a overall population must also be done.

3.3 Population Simulation

While the iterative game analyzed how individuals would react in this game structure, what is more applicable to reality is to analyze how does humanity overall react to an invader. To model this, a overall population will be split into 4 portions; Collaborative-Humans, Self-Interested-Humans, Passive-Invaders, and Active-Invaders. Then based on the payoff matrix described in section 2.3, one can derive how these four populations will evolve over time. The change in each of the 4 population types will be described in the below form:

$$\frac{dx_i}{dt} = x_i(f_i - \phi).$$

Where f_i describes the expected fitness of population i against the other 3 population types and ϕ represented the overall average fitness. f_i can be defined for each portion of the population as multiplying the proportion of the other populations one can encounter as well as multiplying by the payoff for that combination of 3 players. More concretely the formulas for each expected fitness are below. ϕ can then be calculated by taking the weighted average of all f_i such that $\phi = p_C * f_C + p_S * f_S + p_P * f_P + p_A * f_A$

$$f_C = p_C * p_P * U_{C-C-P}[C] + p_C * p_A * U_{C-C-A}[C]$$
$$p_S * p_P * U_{C-S-P}[C] + p_S * p_A * U_{C-S-A}[C]$$

$$f_S = p_C * p_P * U_{S-C-P}[S] + p_C * p_A * U_{S-C-A}[S]$$
$$p_S * p_P * U_{S-S-P}[S] + p_S * p_A * U_{S-S-A}[S]$$

$$f_P = p_C * p_C * U_{C-C-P}[P] + p_C * p_S * U_{C-S-P}[P]$$
$$p_S * p_C * U_{S-C-P}[P] + p_S * p_S * U_{S-S-P}[P]$$

$$f_A = p_C * p_C * U_{C-C-A}[A] + p_C * p_S * U_{C-S-A}[A]$$
$$p_S * p_C * U_{S-C-A}[A] + p_S * p_S * U_{S-S-A}[A]$$

Now that the population dynamics have been defined, given an initial starting condition of the four population types, we can use the defined pay-off matrix and see how the overall population evolves over multiple iterations. To do this, each population type is given an initial starting position, typically given as 4 integers. Those 4 integers are then normalized to sum up to 1, and then for 10,000 iterations the change in each population is found by solving for dx_i given a dt = .001. One side, Humans or Invaders can 'win', if they can drive the other side to reach 0 population. Below multiple initial conditions will be looked at and analyzed.

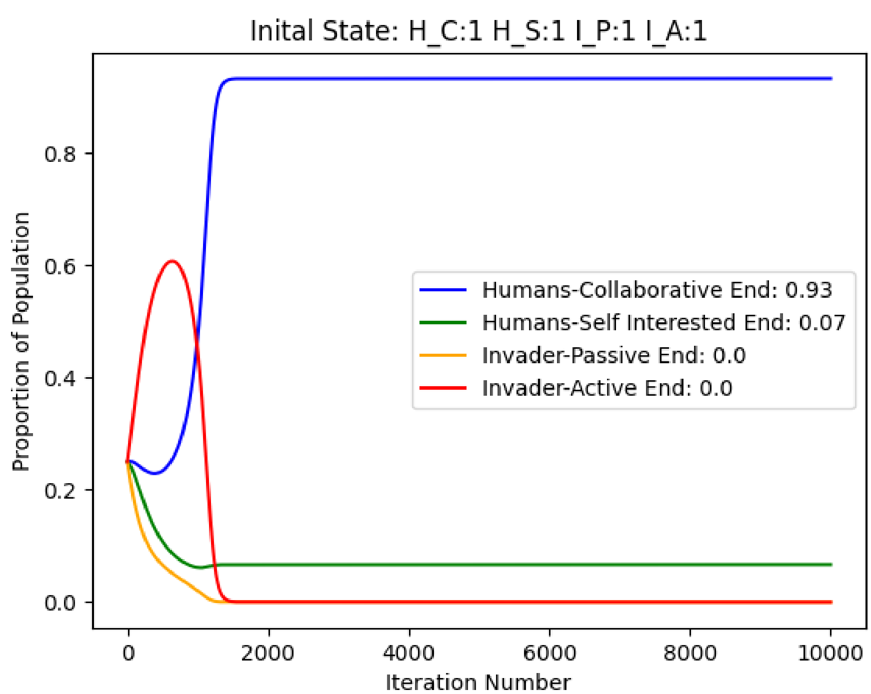


Figure 8: Here, each of the 4 population types exist in equal proportions to each other. Given the current payoff matrix it seems like humans, are more fit on average. While there is an initial surge in aggressive Invader Behavior, Collaborative humans, start to become, on average, more fit. Qualitatively, it seems that once the collaborative humans reach some breaking point, they tend to dominate the population. While the Invaders do completely die out, it is interesting that by the end, there is also some small fraction of self-interested humans.

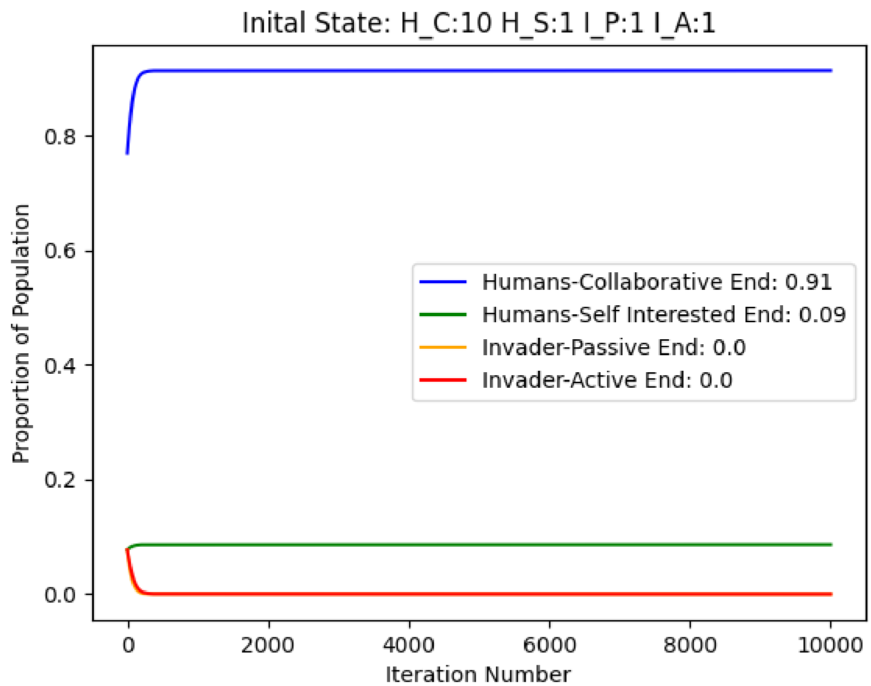


Figure 9: Increasing the populations such that there is 10x more collaborative humans than everyone else, further shows the fitness of this population. The invaders of course continue to die out, however the proportion of self-interested humans does seem to increase when compared to the all equal case.

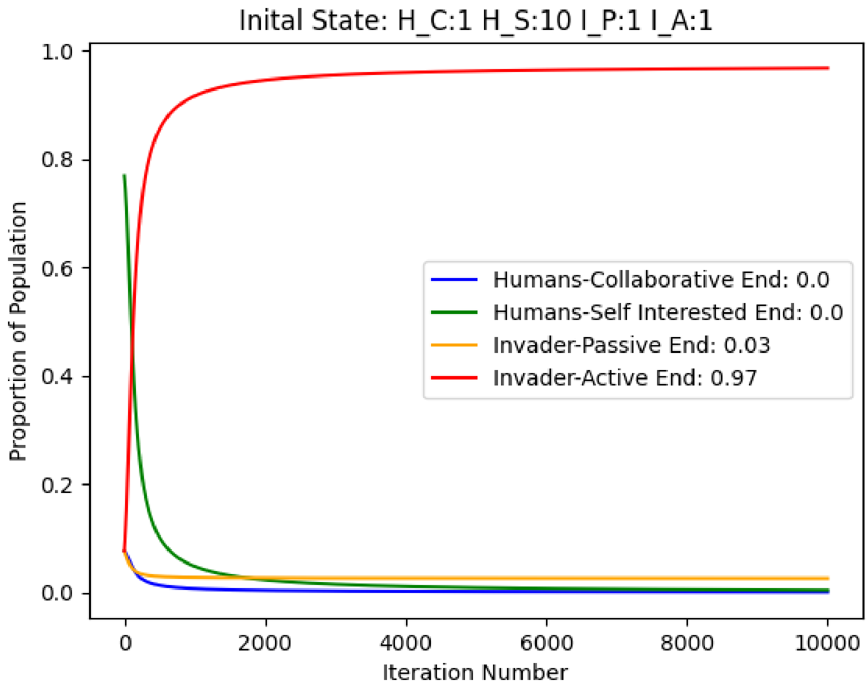


Figure 10: Now taking a look at the case when the population of self-interested humans is 10x the size of the population's of everyone else. In these cases Invaders being Active in their attacks are a more fit population. However, intuitively this may not make sense as if the population of humans is more self-interested, an Invader may not need most of its population to be actively aggressive. This may indicate that the pay-off matrix is not necessarily suited for the scenario that is attempting to be modeled

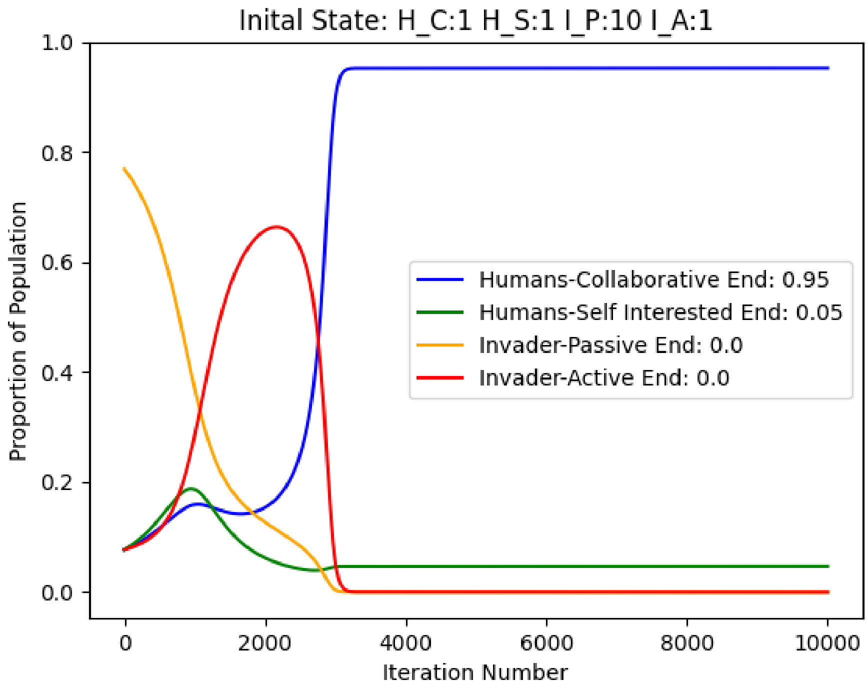


Figure 11: Now taking a look at the case when the population of Passive Invaders is 10x the size of the population's of everyone else. This case is similar to the all-equal starting conditions scenario in the sense that active invaders start to have an initial push, but eventually the collaborative humans start to increase their population fast enough to stop them

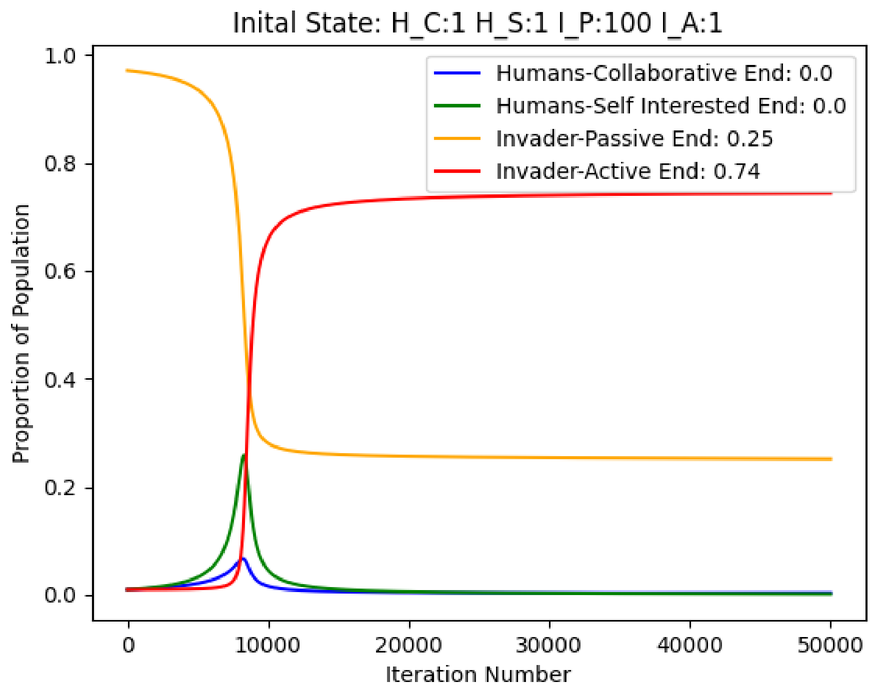


Figure 12: Interestingly enough, starting with 100x more passive invaders than the other populations leads to a scenario in which the invaders last longer than most scenarios. For this scenario 50,000 iterations were used as humanity was still holding on after 10,000 iterations. In the end however, the invaders still won out.

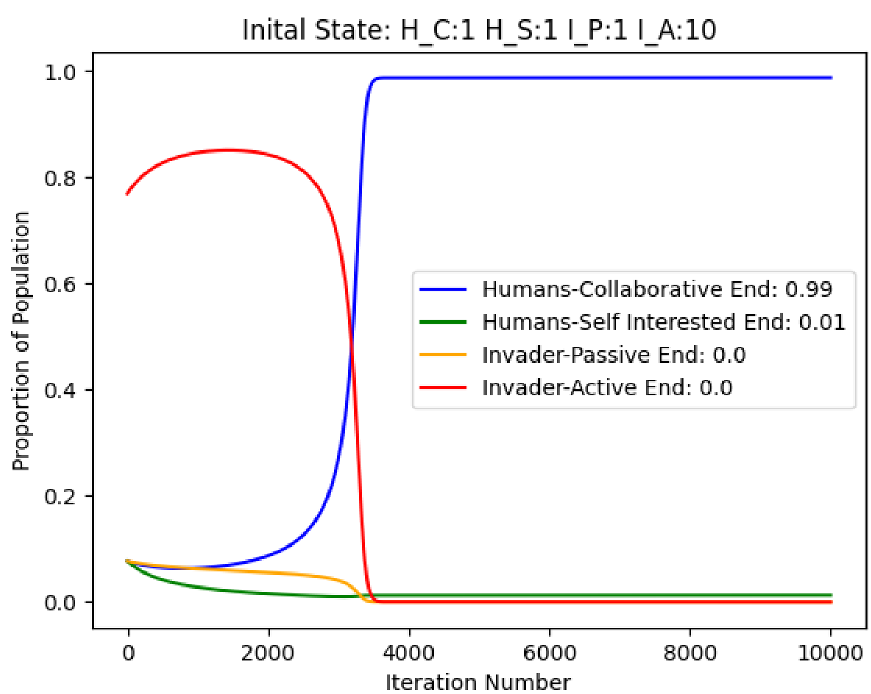


Figure 13: Starting out with 10x more active invaders than the other populations, also interestingly leads to a situation where the humans dominate. From this point it seems like, given the current payoff matrix, the main goal of the invaders should be to destroy the humans before they reach a certain tipping point of collaboration

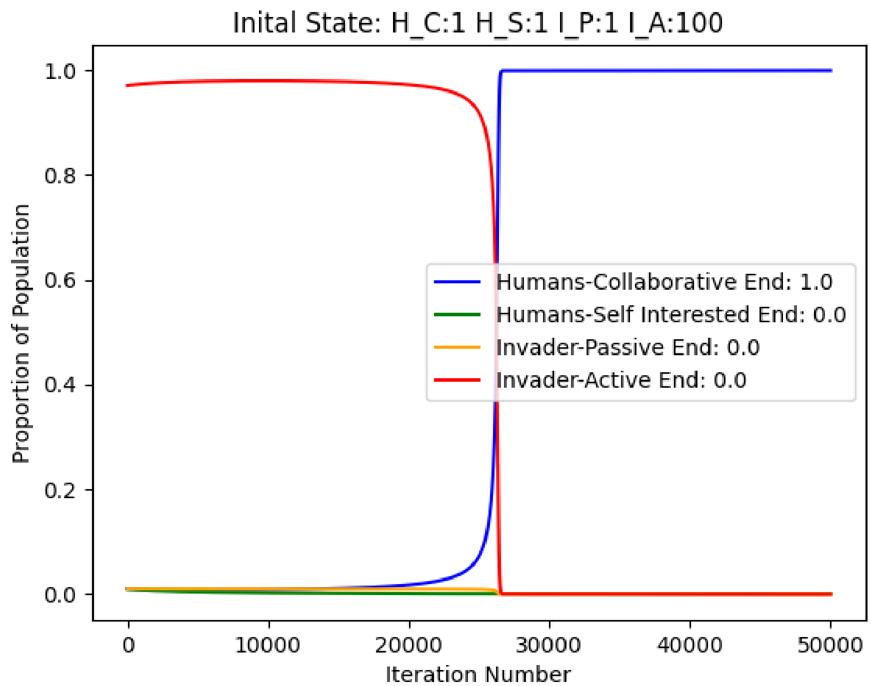


Figure 14: Starting out with 100x more active invaders than the other populations, showcases that after 10,000 iterations there is still some population humans. Interestingly, even with such a small population of collaborative humans at 10,000 iterations, they were able to eventually build up their populations, build strength, and win against the aggressive invaders.

Overall, analyzing this game from a population perspective was an interesting exploration on how these different populations interact with each other on longer time scales. An interesting conclusion that can be seen from even just these few test cases is that human's collaborating is a powering thing. In the cases that the invaders won out, they only won when they took out all of the collaborative humans as quickly as possible.

4 Conclusion and Future Potential Work

This project was intended to analyze how human interaction can impact humanities results when faced against an invader. This was done in two ways; (1) looking at 2 humans and 1 invader and seeing how their strategies against each other change over iterations and (2) looking at overall populations of Humans and Invaders and seeing how those populations change over time.

In the iteration case, cycles of strategies tended to form between the humans and invaders. Mimicking what one might expect from this scenario. When an invader is relatively passive, humans can relax and be more self-interested. However when an invader is more aggressive, humans tended to collaborate more often.

From a population analysis perspective, humans tended to win out in the scenarios that were analyzed. As humans tended to reach a certain population of collaboration, that ended up being the trigger to eliminate the invaders. However, if the invaders could take down the collaborative humans quick enough, they would have defeated humanity.

While there is some semblance of 'realism' to this analysis there is many potential routes that this project could taken on in future iterations. One simple example is to adjust the specific variables used to generate this payoff matrix. The variables chosen here was selected specifically to showcase some interesting results in both the iterative and population simulations, but are there conclusions that can be made from just the formulas listed.

Another route this project could go is adjusting those fundamental formulas themselves. Many assumptions were made when designing those that may not hold to reality. For example, when all humans are self-interested, perhaps it should be in the invaders best-interest to be passive, and let the humans destroy themselves.

Finally, more 'gimmicks' could be added to either simulation style. Perhaps the payoff matrix changes over iterations or the invaders can't be actively aggressive consistently. This types of nuances would be interesting to analyze, but of course would make modeling more complex.

Given this payoff matrix and simulation was a overly simplified representation of what could happen during some invader invasion, it is still amazing to see some early results that reinforce the importance of collaboration amongst humanity towards a common enemy. While this project idea was born from a fictional example, there could be some actual relevance to reality. This type of modeling could be used to look at how does the internal actions of one group impact their results against their competition. Applications could apply to completing companies, sports team dynamics, or even broader politics between multiple countries.

References

Hsi Tse Wang. The construction of the strategy selection behavior of online food delivery platform based on the tripartite evolutionary game model. *Asia Pacific Management Review*, 28(3):316–326, 2023. ISSN 1029-3132. doi: https://doi.org/10.1016/j.apmrv.2022.12.004. URL https://www.sciencedirect.com/science/article/pii/S1029313222000744.