Computer Assignment 1

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Indian Institute of Technology, Madras — March 4, 2021

Problem Definition

Solve the 1 Dimensional Heat Diffusion equation using FTCS Scheme.

$$\frac{\partial T}{\partial t} = \alpha * \frac{\partial T^2}{\partial x^2}; \tag{1}$$

Initial conditions : *Temperature of the wire* $= 0^0 C$

Boundary conditions : $T_{x=0}=1^0 C$ and $T_{x=L}=0^0 C$

Space Steps : $\triangle$ x=0.1

Compute the Solution from t=0 to t=10s in the following time steps

Time Steps :

$\triangle$ t=[0.1, 0.01, 0.001]

1 Numerical Formulation

The Finite difference method scheme used to solve the 1 Dimensional Heat Diffusion equation is FTCS Scheme (Forward Difference in Time and Central Difference in Space).

The given parabolic PDE can be discretized as follows using FTCS Scheme:

$$\frac{\partial T}{\partial t} = \alpha * \frac{\partial T^2}{\partial x^2}; \tag{2}$$

$$\frac{T_i^{n+1} - T_i^n}{\triangle t} = \alpha * \frac{T_{i+1}^n - 2 * T_i^n + T_{i-1}^n}{\triangle x^2} \tag{3}$$

$$T_i^{n+1} = \nu * T_{i+1}^n + (1 - 2 * \nu) * T_i^n + \nu * T_{i-1}^n \tag{4}$$

Where :

$$\nu = \frac{\alpha * \triangle t}{\triangle x^2} \tag{5}$$

2 Pseudo code

The Pseudo code for solving the given parabolic equations is :

1. Initialize the following parameters:
   - Length of the Wire (l)
   - Time interval (t)
   - Thermal Diffusivity of the material (a)
   - Time spacing (dt)

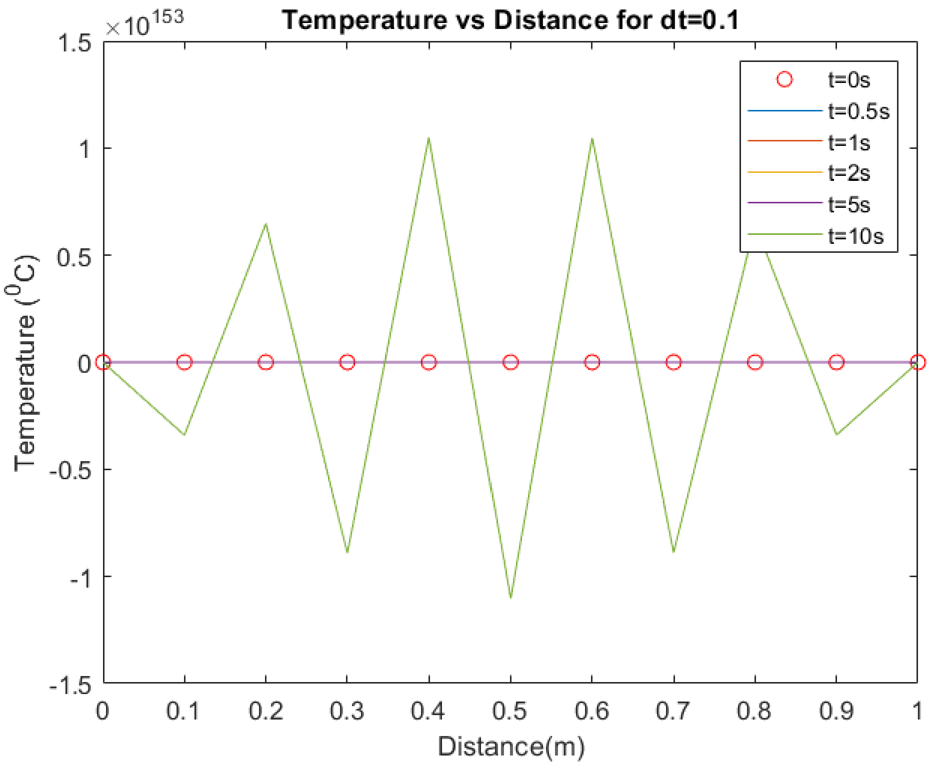
- Grid spacing (dx)

2. Calculate the following for computation :
    - Number of time steps : $ts = \frac{t}{dt}$
    - Number of space steps : $ss = \frac{1}{dx} + 1$
3. Generate the space grid
4. Specify the initial and the boundary conditions
5. Implement the FTCS Scheme using ***Nested For loop***
    (a) Marching in time with time step at every iteration
        i. Marching in space with space step at every iteration
            A. *Perform the Temperature calculation for (n+1)th level using the known information at nth level*
6. Exit the loop when the final time step value corresponding to the final time(10s) is reached
7. Plot the Temperature vs Distance plots for different values of Time spacing

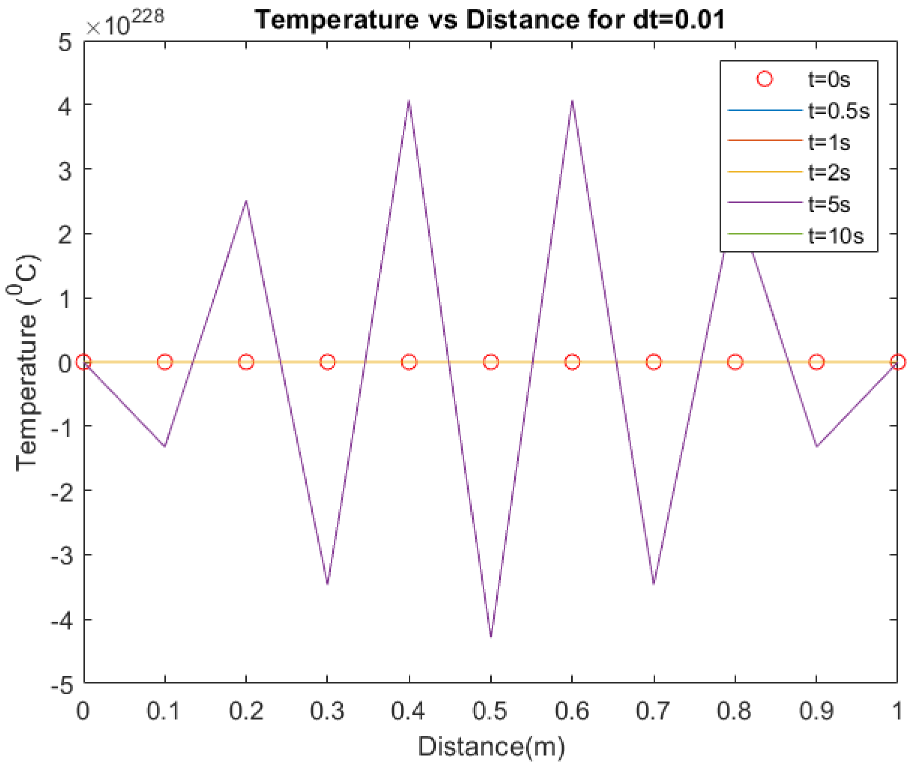
3 Results

The Temperature vs Space graphs for given specific time stamps are as follows :

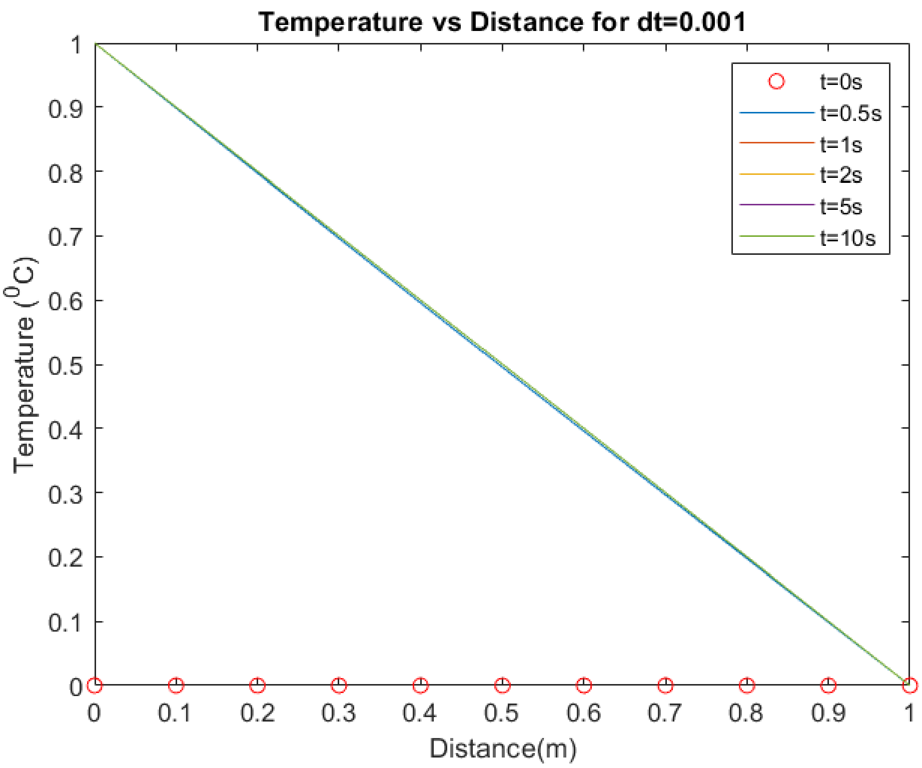
Temperature vs Space plot for Time step : $\triangle t = 0.1$



Temperature vs Space plot for Time step : $\triangle t = 0.01$



Temperature vs Space plot for Time step : $\triangle t = 0.001$



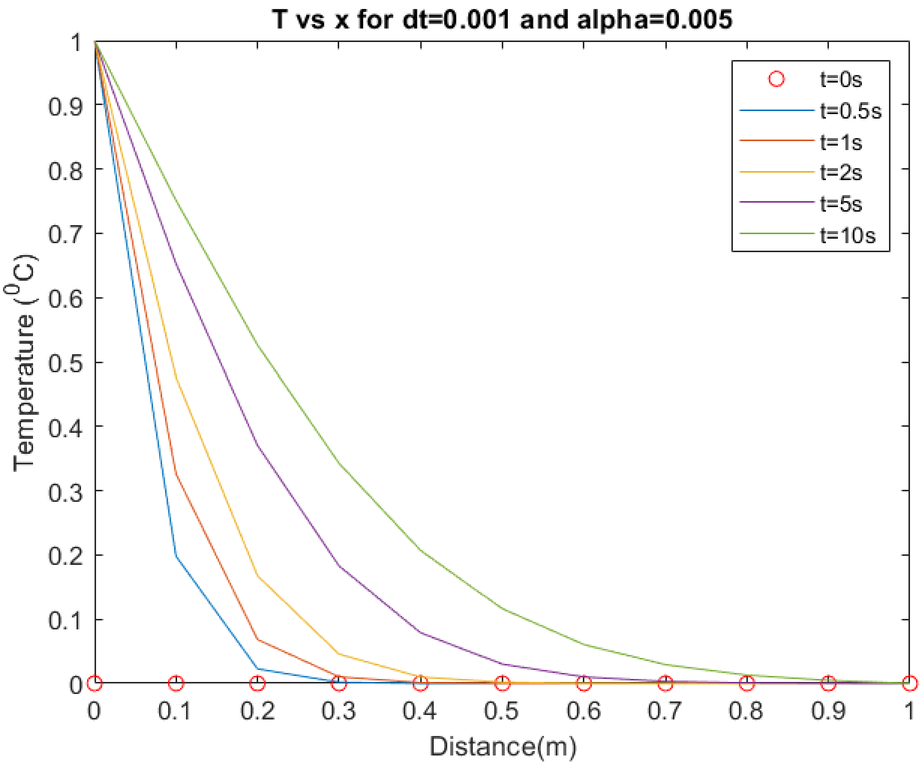
3.1 Inferences

- The FTCS method is not stable for the time step values of 0.1 and 0.001. This instability is evident from the erroneous plots as shown above.
- The FTCS method is stable for the time step value of 0.001.
- For the time step value of 0.001, since the value of thermal diffusivity is very high, the parabolic nature of temperature distribution is not evident.

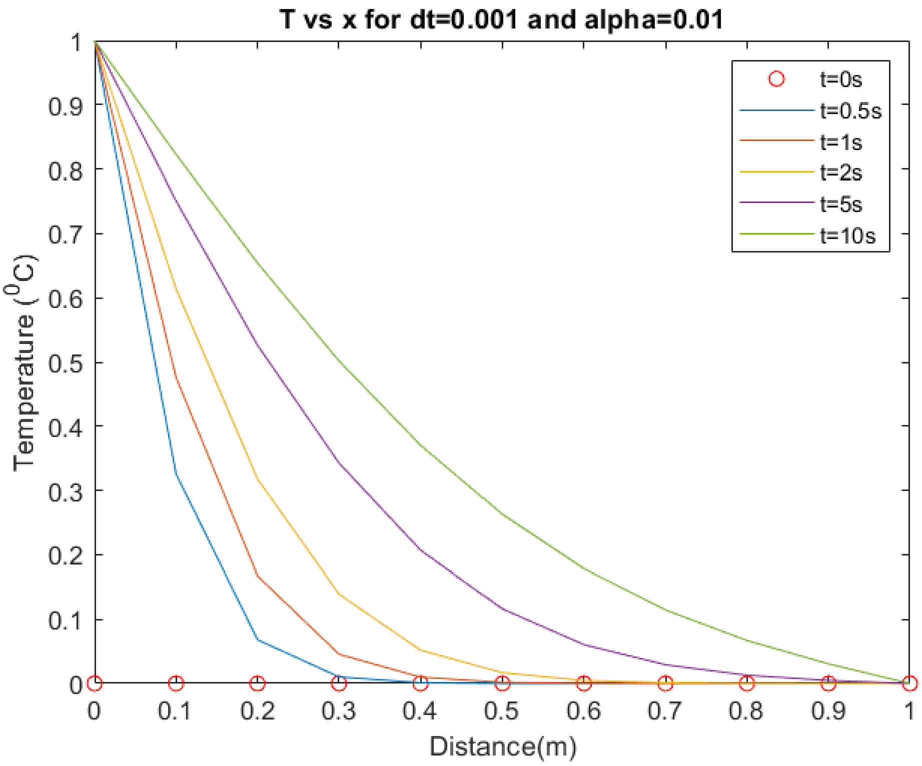
3.2 Temperature distributions with different values of Thermal diffusivity

The following plots for different values of thermal diffusivity present the inherent parabolic nature of the temperature distribution.

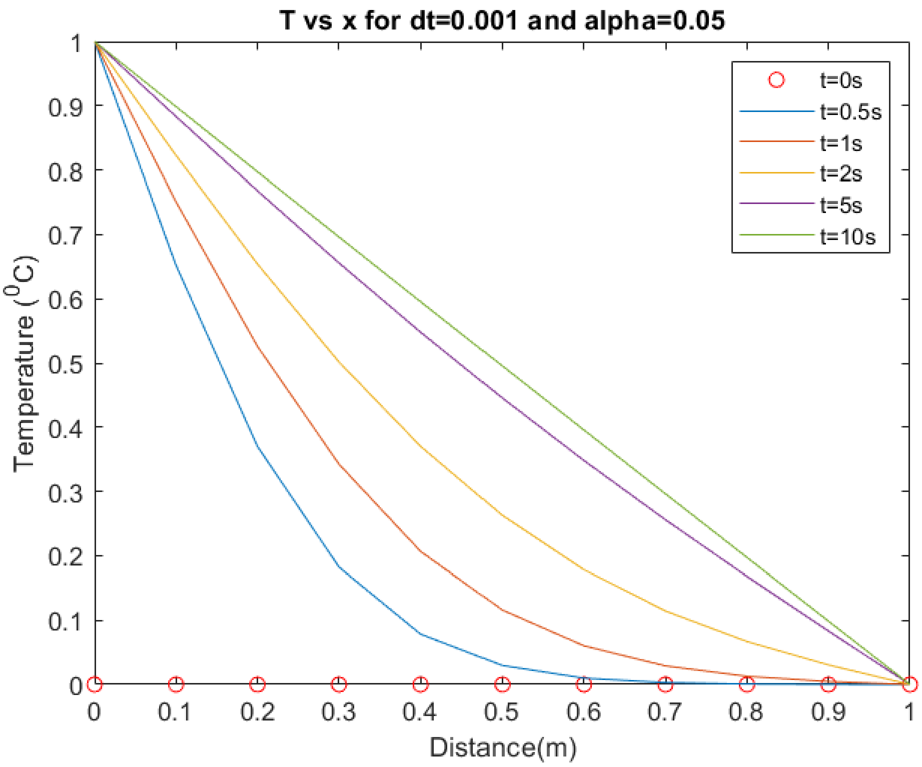
Temperature vs Space plot for Time step : $\triangle t = 0.1$ **and** $\alpha = 0.005$



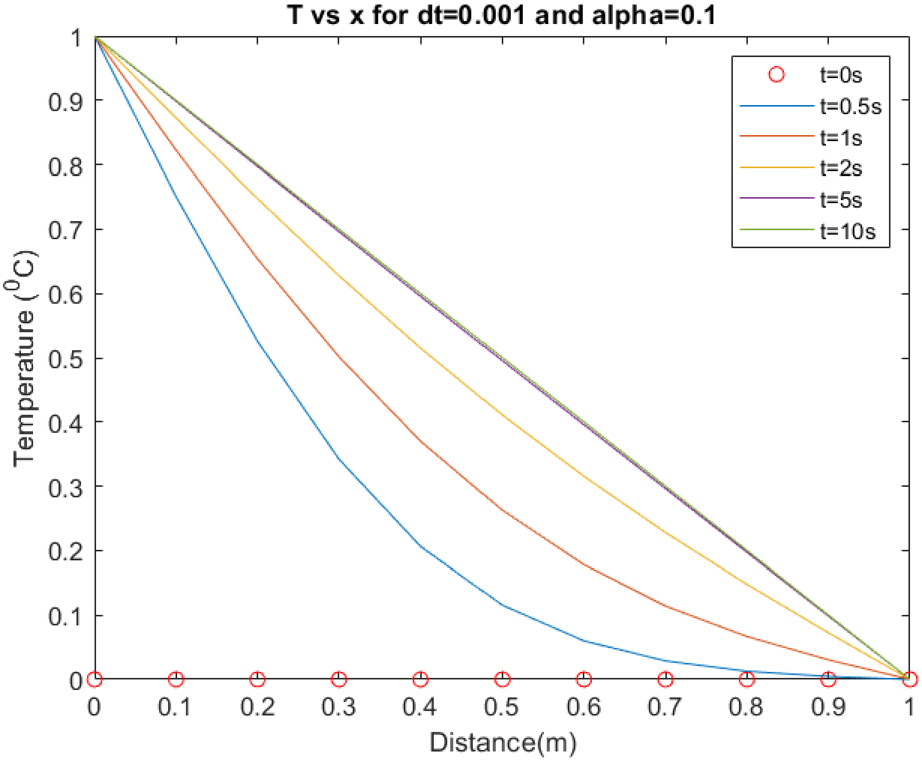
Temperature vs Space plot for Time step : $\triangle t = 0.1$ **and** $\alpha = 0.01$



Temperature vs Space plot for Time step : $\triangle t = 0.1$ **and** $\alpha = 0.05$



Temperature vs Space plot for Time step : $\triangle t = 0.1$ **and** $\alpha = 0.1$



4 Appendix

The source code is written in MATLAB R2018a

```
% Plot temperature vs Distance graph at :
% t = 0 ; 0.5 ; 1 ; 2 ; 5 ; 10 s
% dt= 0.1 ; 0.01 ; 0.001

clc;
clear;

l=1;                                                    % Length of the wire
t=10;                                                   % Final time
a=input('Enter the value of thermal diffusivity:');    % Thermal Diffusivity
dt=input('Enter the value of time spacing(dt):');       % Time spacing
dx=input('Enter the value of grid spacing(dx):');       % Grid spacing


timesteps=t/dt;      % Number of time steps
nx=(l/dx)+1;         % Number of space steps for ftcs scheme
g=a*dt/(dx*dx);      % Constant in FDM form

% Grid Generation and Initial Conditions :

for ss=1:nx
    x(ss)=(ss-1)*dx;
    T(ss,1)=0;
end

% Boundary Conditions :

for k=2:timesteps+1
    T(1,k)=1;
    T(nx,k)=0;
    time(k)=(k-1)*dt;
end

% Scheme Implementation :
for ts=1:timesteps
    for ss=2:nx-1
        T(ss,ts+1)=T(ss,ts)+g*(T(ss-1,ts)+T(ss+1,ts)-2*T(ss,ts));
    end
end

% Plotting the Graphs :
if(dt==0.1)
    figure(1)
    plot(x,T(:,1),'ro',x,T(:,5),x,T(:,10),x,T(:,20),x,T(:,50),x,T(:,100))
    legend("t=0s","t=0.5s","t=1s","t=2s","t=5s","t=10s")
    xlabel("Distance(m)")
    ylabel("Temperature (^0C)")
    title("Temperature vs Distance for dt=0.1")
end

if(dt==0.01)
    figure(2)
    plot(x,T(:,1),'ro',x,T(:,50),x,T(:,100),x,T(:,200),x,T(:,500),x,T(:,1000))
```

```
    legend("t=0s","t=0.5s","t=1s","t=2s","t=5s","t=10s")
    xlabel("Distance(m)")
    ylabel("Temperature (^0C)")
    title("Temperature vs Distance for dt=0.01")
end

if(dt==0.001)
    figure(3)
    plot(x,T(:,1),'ro',x,T(:,500),x,T(:,1000),x,T(:,2000),x,T(:,5000),x,T(:,10000))
    legend("t=0s","t=0.5s","t=1s","t=2s","t=5s","t=10s")
    xlabel("Distance(m)")
    ylabel("Temperature (^0C)")
    title("Temperature vs Distance for dt=0.001")
end
```