Assignment-2

> Answer all the questions. Make any suitable assumptions if needed.

Name: _____

Roll Number: _____

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1. The shear stress induced along the $z-$axis when two cylinders are in contact with each other is given by

$$\frac{\tau_{zy}}{p_{\max}} = -\frac{1}{2}\left[-\frac{1}{\sqrt{1+\left(\frac{z}{b}\right)^2}} + \left\{2 - \frac{1}{1+\left(\frac{z}{b}\right)^2}\right\} \times \sqrt{1+\left(\frac{z}{b}\right)^2} - 2\left(\frac{z}{b}\right)\right] \qquad (1)$$

where $2b$ is the width of the contact area and $p_{\max}$ is the maximum pressure developed at the center of the contact area (see, Figure 1) giveny by

$$b = \left[\frac{2F}{\pi l}\frac{\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2}}{\frac{1}{d_1}+\frac{1}{d_2}}\right]^{1/2} \qquad (2)$$

$$p_{\max} = \frac{2F}{\pi bl} \qquad (3)$$

F is the contact force; E_1, ν_1 and E_2, ν_2 are the Young's modulus and Poisson's ratio of the cylinders 1 and 2, respectively. d_1 and d_2 are the diameters of the two cylinders, and l the axial length of contact. In several applications such as roller bearings, a crack originates at the point of maximum shear stress and propagates to the surface leading to a fatigue failure. Hence, to locate the origin of a crack, it is necessary to find the point at which the shear stress attains its maximum value.
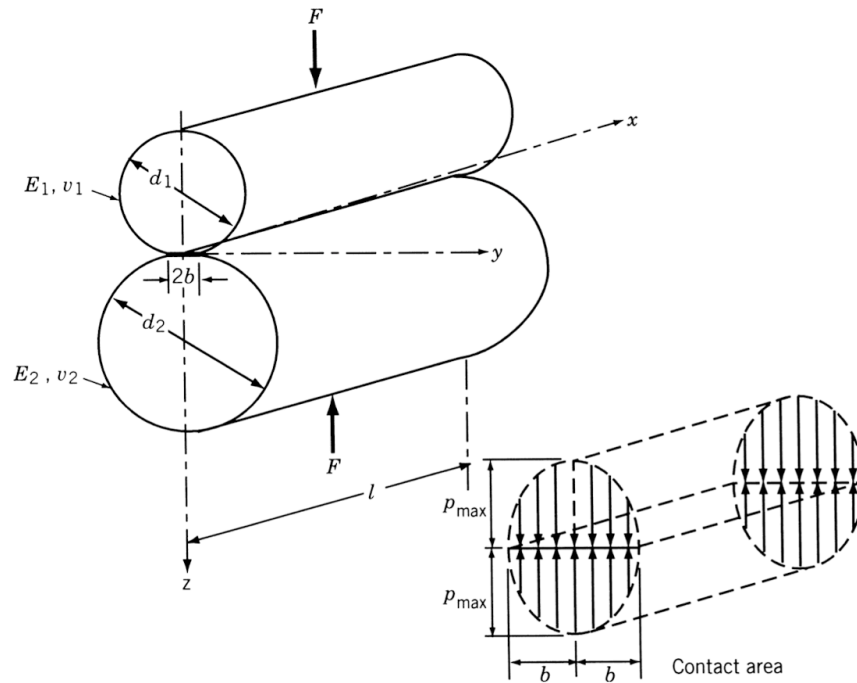
Figure 1: Contact stress between two cylinders

Show that the problem of finding the location of the maximum shear stress for $\nu_1 = \nu_2 = 0.3$ reduces to maximizing the function

$$f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda \tag{4}$$

where $f = \tau_{zy}/p_{\text{max}}$ and $\lambda = z/b$.

(a) Plot the graph of the function $f(\lambda)$ given by Equation (4) in the range (0,3) and identify its maximum. $\boxed{1}$

(b) Find the maximum of the function using the following methods by writing a computer program and also using pen and paper.

    i. Unrestricted search with a fixed step size of 0.1 from the starting point 0.0 $\boxed{1}$

    ii. Unrestricted search with an accelerated step size using an initial step size of 0.1 from the starting point 0.0 $\boxed{1}$

    iii. Exhaustive search method in the interval (0,3) to achieve an accuracy within 5% of the exact value. $\boxed{2}$

    iv. Dichotomous search method in the interval (0,3) to achieve an accuracy within 5% of the exact value. $\boxed{2}$

    v. Interval halving method in the interval (0,3) to achieve an accuracy within 5% of the exact value. $\boxed{2}$

    vi. Fibonacci method with $n = 10$. $\boxed{2}$

    vii. Golden section method with $n = 10$. $\boxed{2}$

(c) Compare the relative efficiencies of all the methods and comment on their behaviour. $\boxed{2}$