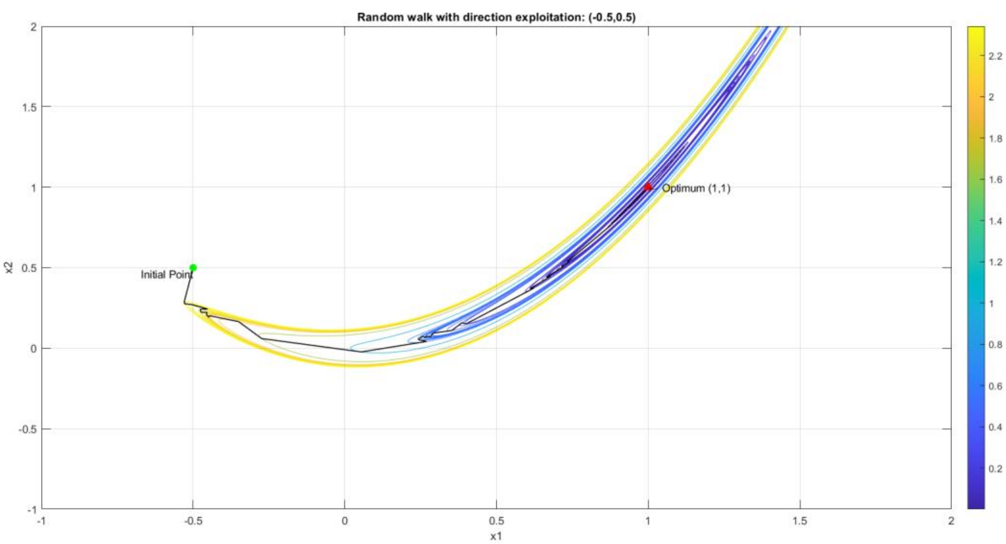
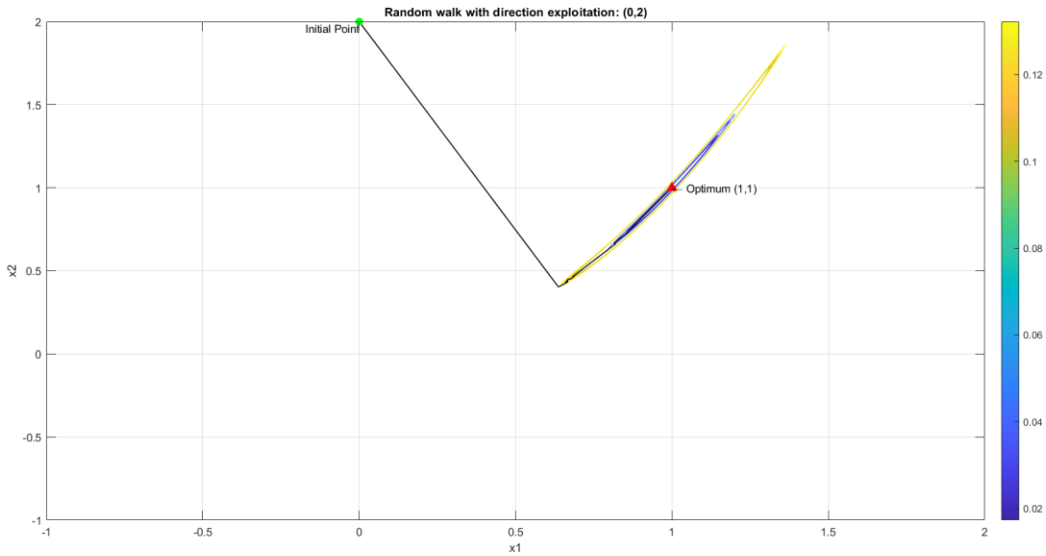
ME7223: Assignment 3

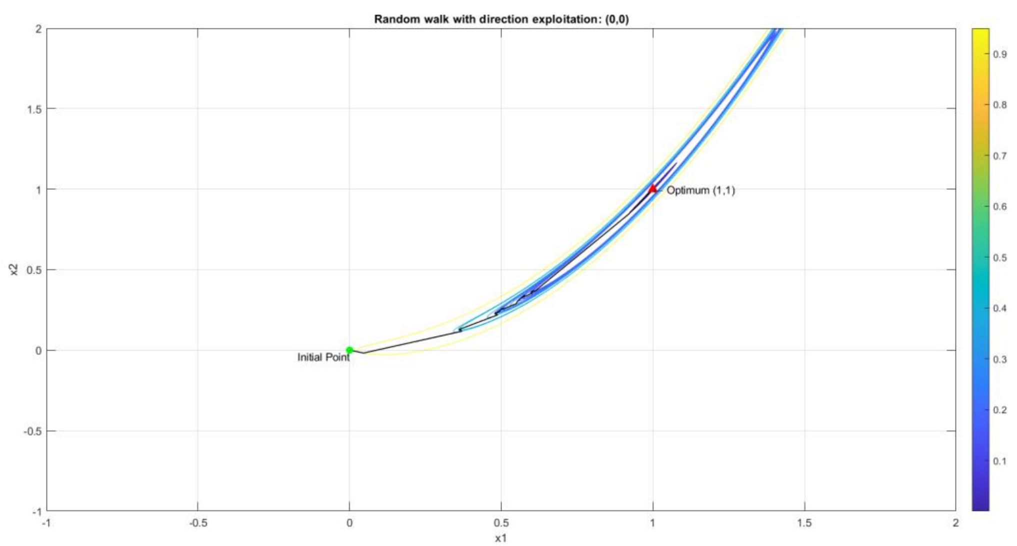
Question 1:

I have used three starting points: (-0.5,0.5), (0,0), (0,2). The contour plots for the same are presented below

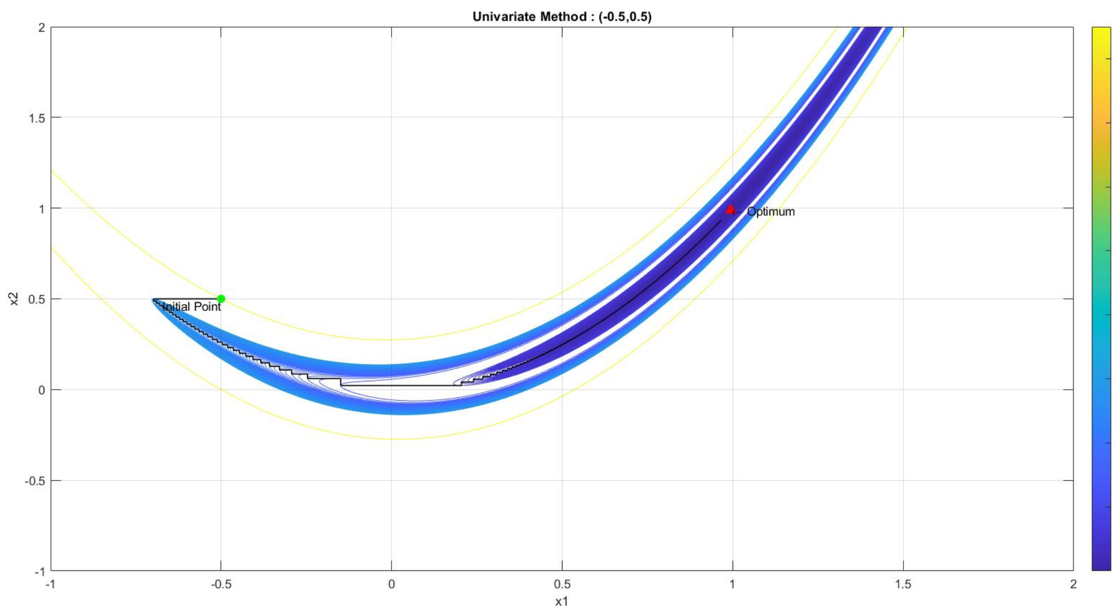
(a) **Random Walk with Direction Exploitation**

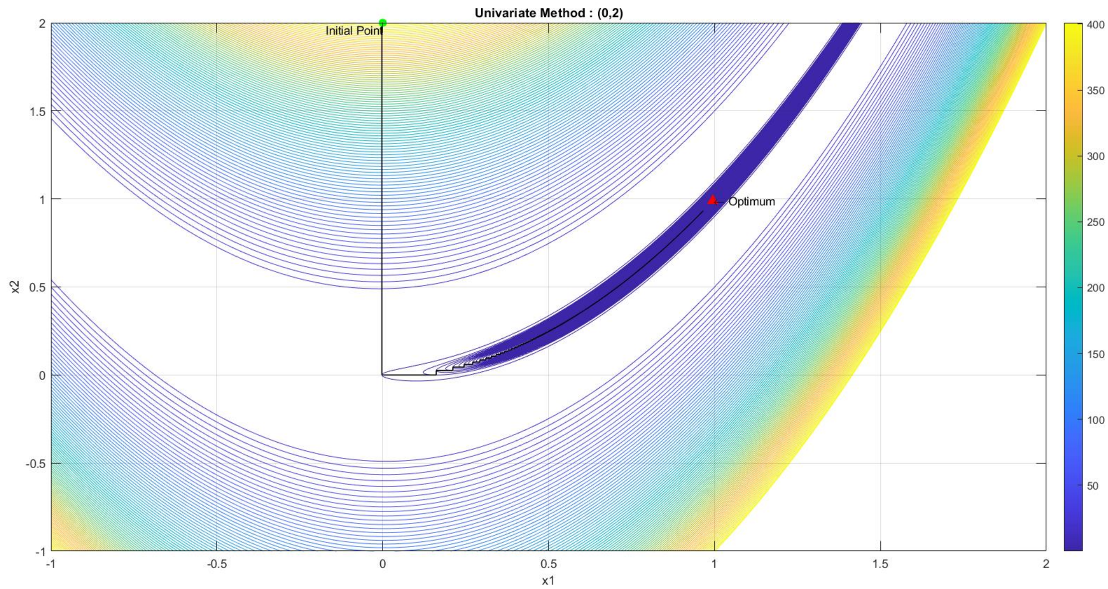


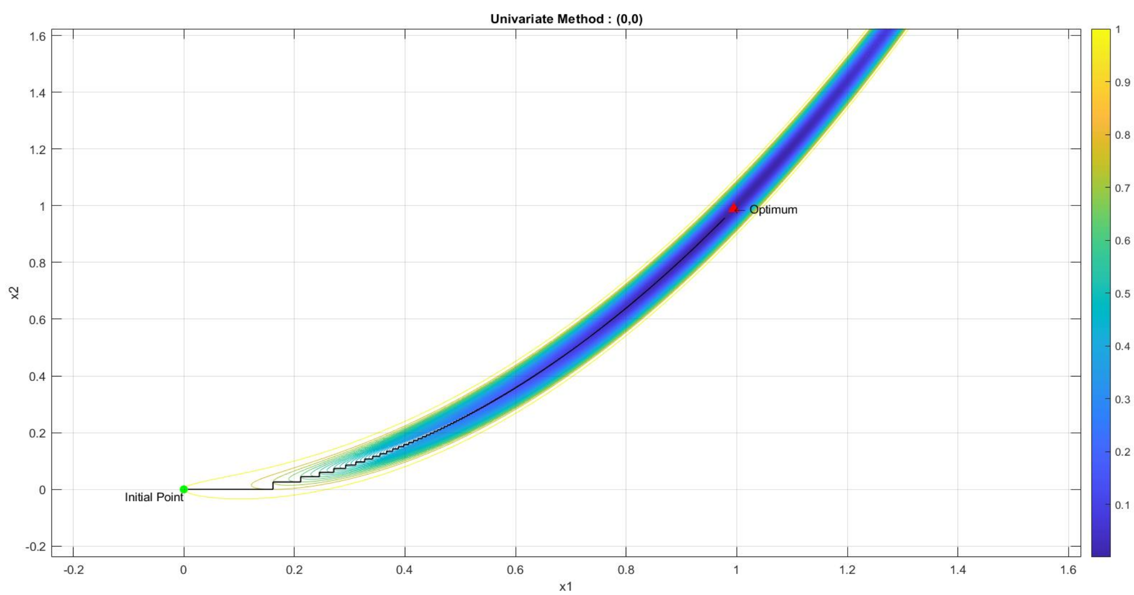

Random walk with direction exploitation: (0,2)
Initial Point
Optimum (1,1)
x1
x2
-1
-0.5
0
0.5
1
1.5
2
0.02
0.04
0.06
0.08
0.1
0.12


Random walk with direction exploitation: (0,0)
Initial Point
Optimum (1,1)
x1
x2
-1
-0.5
0
0.5
1
1.5
2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

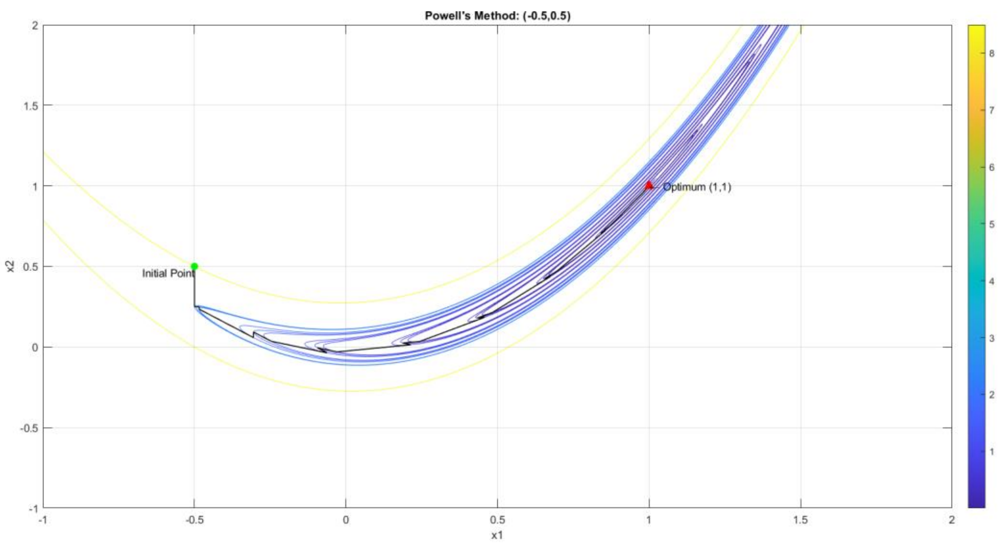
(b) **Univariate Method**

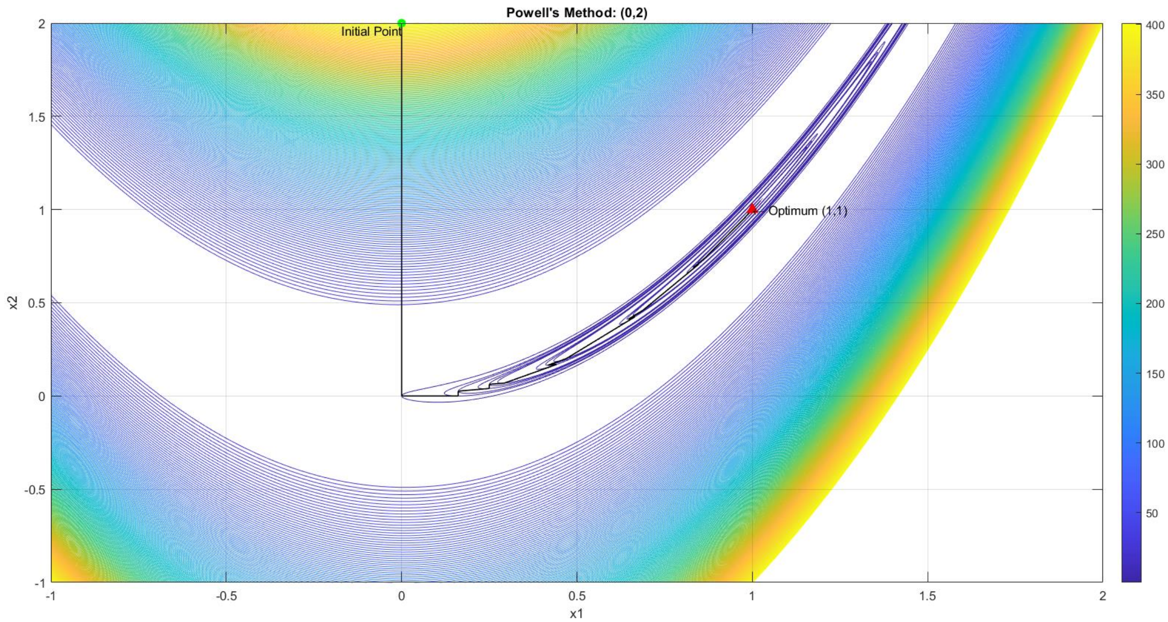


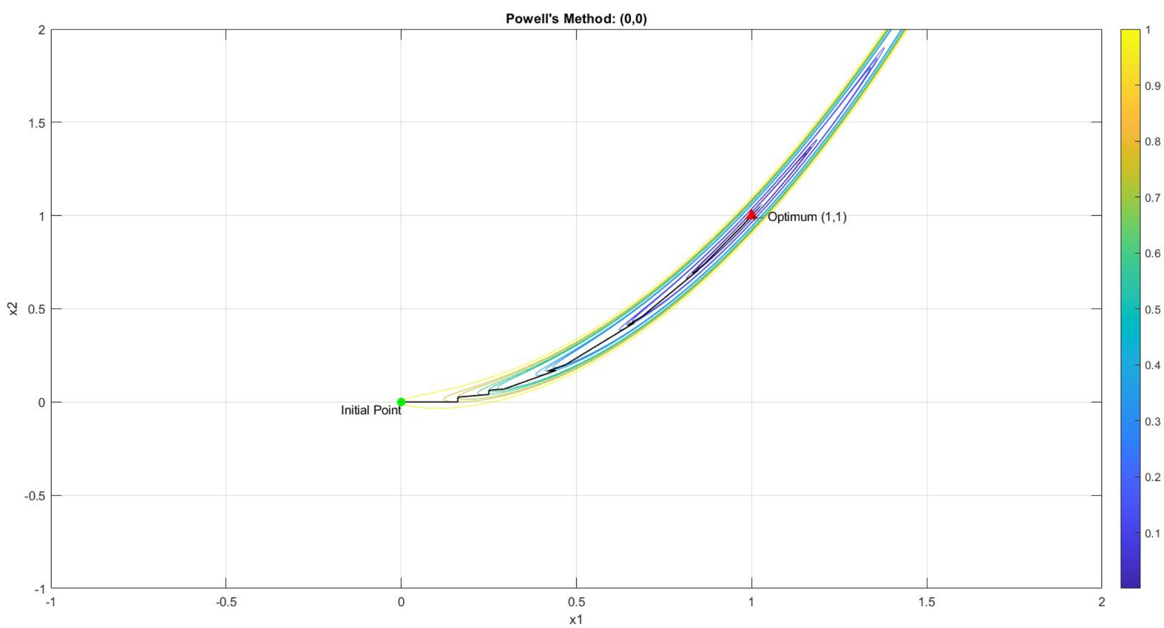

Univariate Method : (0,2)
Initial Point
Optimum
x1
x2


Univariate Method : (0,0)
Initial Point
Optimum
x1
x2

(c) **Conjugate Directions Method**




Powell's Method: (0,2)
Initial Point
Optimum (1,1)
x1
x2


Powell's Method: (0,0)
Initial Point
Optimum (1,1)
x1
x2

(a) Random Walk with direction exploitation:

Random Walk		
Initial Guess	Iterations	Optima
(0,0)	635	(1,1)
(0,2)	516	(1,1)
(-0.5,0.5)	241	(1,1)

Since random walk method utilizes random direction at each step, the number of iterations to converge to optima is not fixed. It's an inefficient method.

(b) Univariate Method:

NOTE: The initial search direction for the below tabulated values is: **[1;0]**

Univariate Method			
Initial Guess	Iterations	Final Optima	Actual Optima
(0,0)	3000	(0.9946,0.9893)	(1,1)
(0,2)	3000	(0.9946,0.9893)	(1,1)
(-0.5,0.5)	3000	(0.9942,0.9885)	(1,1)
(0,1)	1	(1,1)	(1,1)

Univariate Method (iterations upto 10000)			
Initial Guess	Iterations	Final Optima	Actual Optima
(-0.5,0.5)	3295	(0.9960,0.9921)	(1,1)
(0,0)	3237	(0.9960,0.9921)	(1,1)
(0,2)	3239	(0.9960,0.9921)	(1,1)
(0,1)	1	(1,1)	(1,1)

It can be seen that for the given convergence criteria, univariate method takes large number of iterations to converge, hence the number of iterations is capped to 10000, and obtained optima is not so far away from the actual optima.

(c) Conjugate Directions Method (Powell's):

NOTE: The initial search direction for the below tabulated values is: **[0;1]**

Conjugate Directions Method (Powell's)			
Initial Guess	Cycles	Final Optima	Actual Optima
(-0.5,0.5)	18	(1,1)	(1,1)
(0,0)	14	(1,1)	(1,1)
(0,2)	14	(1,1)	(1,1)
(0,1)	14	(1,1)	(1,1)

Question 2:

(a) Compare the number of steps taken by Univariate method and Powell's Method:

In order to compare the two methods, we have to take the first starting direction as same.

Previously I have taken the directions as follows:

(a) Univariate Method: S= [1;0]

(b) Powell's Method: S= [0;1]

Now taking the same Starting Direction as: **S= [1;0]**

Univariate Method			
Initial Guess	Iterations	Final Optima	Actual Optima
(0,0)	3000	(0.9946,0.9893)	(1,1)
(0,2)	3000	(0.9946,0.9893)	(1,1)
(-0.5,0.5)	3000	(0.9942,0.9885)	(1,1)
(0,1)	1	(1,1)	(1,1)

Univariate Method (iterations upto 10000)			
Initial Guess	Iterations	Final Optima	Actual Optima
(-0.5,0.5)	3295	(0.9960,0.9921)	(1,1)
(0,0)	3237	(0.9960,0.9921)	(1,1)
(0,2)	3239	(0.9960,0.9921)	(1,1)
(0,1)	1	(1,1)	(1,1)

Conjugate Directions Method (Powell's)			
Initial Guess	Cycles	Final Optima	Actual Optima
(-0.5,0.5)	18	(1,1)	(1,1)
(0,0)	12	(1,1)	(1,1)
(0,2)	14	(1,1)	(1,1)
(0,1)	1	(1,1)	(1,1)

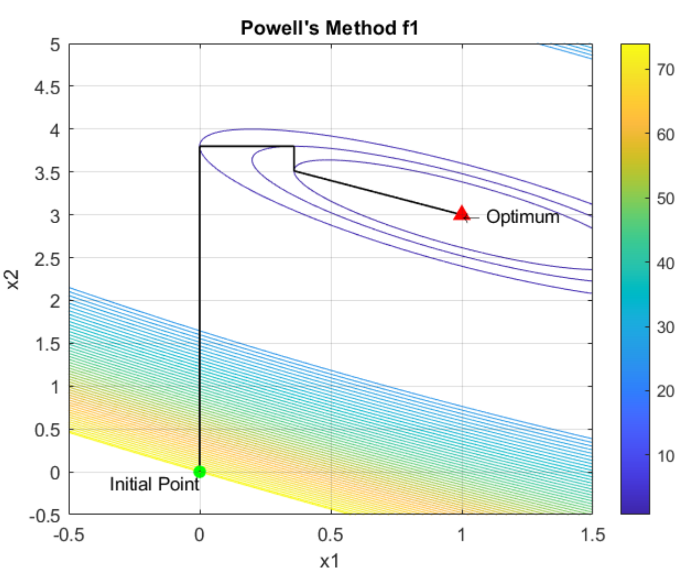
Observations:

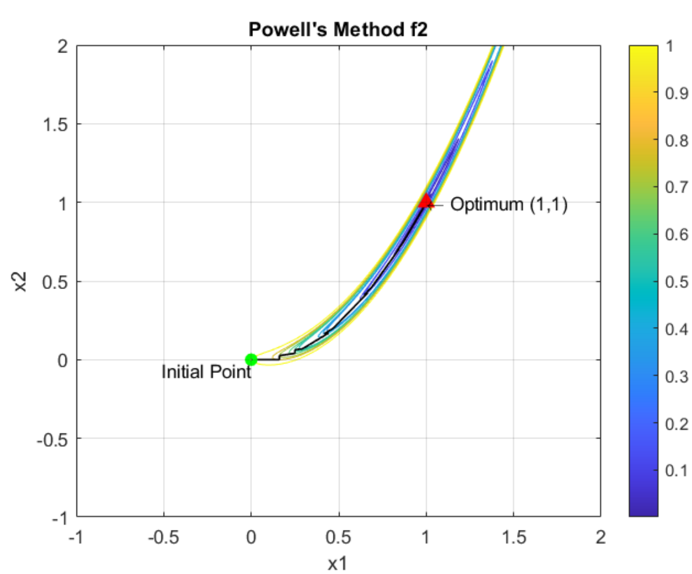
- It can be seen that both Powell's Method and Univariate method reaches the optimum in 1 step when the initial guess point is (0,1) when the starting direction is [1;0].
- When the starting direction is changed for the same starting point, we can see that:

  - The method has taken 1 cycle to reach optima when the direction is [1;0] and starting guess is (0,1) but it took 14 cycles to reach optima when the direction is [0;1]
  - Likewise, there is a decrease in the number of cycles when the starting point is (0,0)
- This shows that certain directions favour certain starting points by a good margin, thus making the selection of starting point and search direction critical.

(b) Comment on the differences in which the Powell's Method progresses for two objective functions

- Since the first objective function is quadratic, Powell's method converges in 2 steps as shown in the figure above for all the initial guesses. Barring a special case of Starting point being (1, n) where n can be any real number, for the first objective function, the method terminates in 1 step.
- Since the second objective function is non quadratic (power of 4), Powell's method takes more steps to converge.





Question 3:

Exact Optima for the problem: (0.045455, -0.04545)

(a) **Steepest Descent Method**

Steepest Descent							
Iterations n	Xn=(X1,X2)		f(Xn)	|grad (F(Xn))|	Sn-1=(s1,s2)		Optimal Step Length
	Xn1	Xn2			s1	s2	
0	100	0	79900	1707.513397	0	0	0
1	19.8179	30.03695	6798.342	387.2967025	-1599	599	0.050145155
2	8.550203	-0.04159	578.4033	145.2857551	-135.865	-362.684	0.082933222
3	1.727816	2.514142	49.17252	32.95358853	-136.053	50.96661	0.050145155
4	0.769091	-0.04513	4.142314	12.36180676	-11.5602	-30.8594	0.082933222
5	0.1886	0.172332	0.310867	2.803894249	-11.5762	4.336553	0.050145155
6	0.107026	-0.04543	-0.01514	1.051818646	-0.98361	-2.62571	0.082933222
7	0.057634	-0.02692	-0.04287	0.23857259	-0.98498	0.368981	0.050145155
8	0.050693	-0.04545	-0.04524	0.089495208	-0.08369	-0.22341	0.082933222
9	0.046491	-0.04388	-0.04544	0.020299225	-0.08381	0.031395	0.050145155
10	0.0459	-0.04545	-0.04545	0.007614803	-0.00712	-0.01901	0.082933222
11	0.045543	-0.04532	-0.04545	0.001727183	-0.00713	0.002671	0.050145155
12	0.045492	-0.04545	-0.04545	0.000647914	-0.00061	-0.00162	0.082933222
13	0.045462	-0.04544	-0.04545	0.000146959	-0.00061	0.000227	0.050145155
14	0.045458	-0.04545	-0.04545	5.51E-05	-5.16E-05	-0.00014	0.082933222
15	0.045455	-0.04545	-0.04545	1.25E-05	-5.16E-05	1.93E-05	0.050145155

(b) **Conjugate Directions Method**

Conjugate Directions							
Iterations n	Xn=(X1,X2)		f(Xn)	|grad (F(Xn))|	Sn-1=(s1,s2)		Optimal Step Length
	Xn1	Xn2			s1	s2	
0	100	0	79900	1707.513397	0	0	0
1	19.8179	30.03695	6798.342	387.2967025	-1599	599	0.050145155
2	0.045455	-0.04545	-0.04545	2.03E-14	-218.128	-331.867	0.090645936
3	0.045455	-0.04545	-0.04545	7.62E-15	7.13E-15	1.90E-14	0.082933222

(c) **Newton's Method**

			Newton's method				
Iterations n	Xn=(X1,X2)		f(Xn)	|grad (F(Xn))|	Sn-1=(s1,s2)		Optimal Step Length
	Xn1	Xn2			s1	s2	
0	100	0	79900	1707.513397	0	0	0
1	0.045455	-0.04545	-0.04545	2.80E-13	99.95455	0.045455	1
2	0.045455	-0.04545	-0.04545	0	1.55E-14	-1.29E-15	1.00E+00

(d) **Marquardt Method**

α=10000, c1=0.5 c2=1.5

			Marquardt Method					
Iterations n	Xn=(X1,X2)		f(Xn)	|grad (F(Xn))|	Sn-1=(s1,s2)		Optimal Step Length	α
	Xn1	Xn2			s1	s2		
0	100	0	79900	1707.513397	0	0	0	10000
1	19.7879	30.00687	6783.367	386.9095536	0.159609	-0.05971	502.5543844	10000
2	8.514147	-0.04666	573.7658	144.726624	0.027113	0.072278	415.8060938	5000
3	1.710429	2.491612	48.28041	32.6635766	0.05381	-0.02007	126.4397547	2500
4	0.752923	-0.04722	3.966165	12.10910088	0.009133	0.024216	104.8392162	1250
5	0.182025	0.16349	0.281805	2.690095294	0.017615	-0.0065	32.41014622	625
6	0.101387	-0.04609	-0.02021	0.962967574	0.002977	0.007737	27.08889005	312.5
7	0.055521	-0.02982	-0.04363	0.201259814	0.005154	-0.00183	8.899171813	156.25
8	0.049066	-0.0456	-0.04535	0.063370609	0.000847	0.002069	7.624077063	78.125
9	0.045965	-0.04463	-0.04545	0.010680837	0.00103	-0.00032	3.010240473	39.0625
10	0.045579	-0.04547	-0.04545	0.002282821	0.000143	0.000311	2.705461365	19.53125
11	0.045464	-0.04544	-0.04545	0.000214504	7.61E-05	-1.92E-05	1.518009163	9.765625
12	0.045456	-0.04545	-0.04545	2.03E-05	5.86E-06	1.16E-05	1.436772363	4.882813
13	0.045455	-0.04545	-0.04545	6.83E-07	9.21E-07	-1.96E-07	1.132346759	2.441406

Observations:

1) It can be seen that Marquardt method behaves similar to the Conjugate Directions method for the first four iterations. This behaviour can be attributed to the large value of α taken at the beginning of the algorithm

2) The number of iterations taken by each method to converge are tabulated as follows:

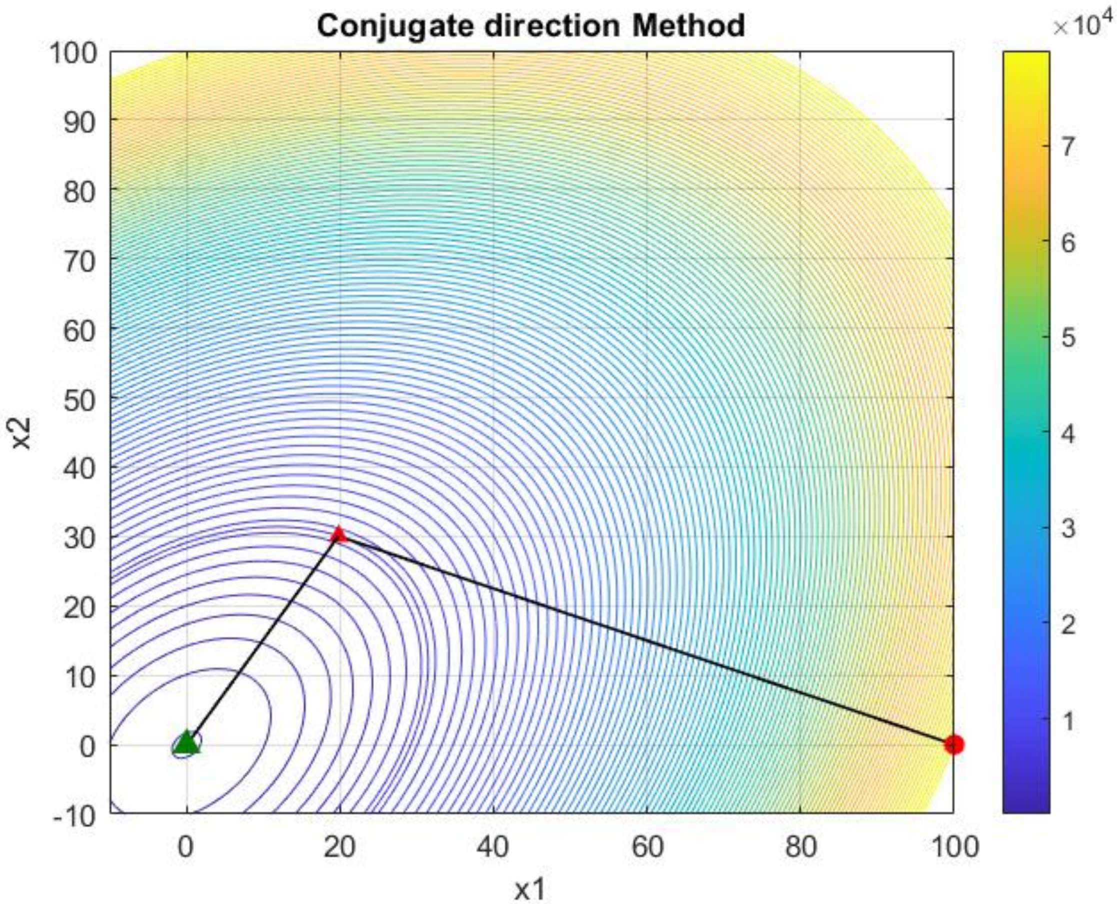
Method	Number of Iterations
Steepest Descent	15
Conjugate Directions	3
Newton's	2
Marquardt's	13

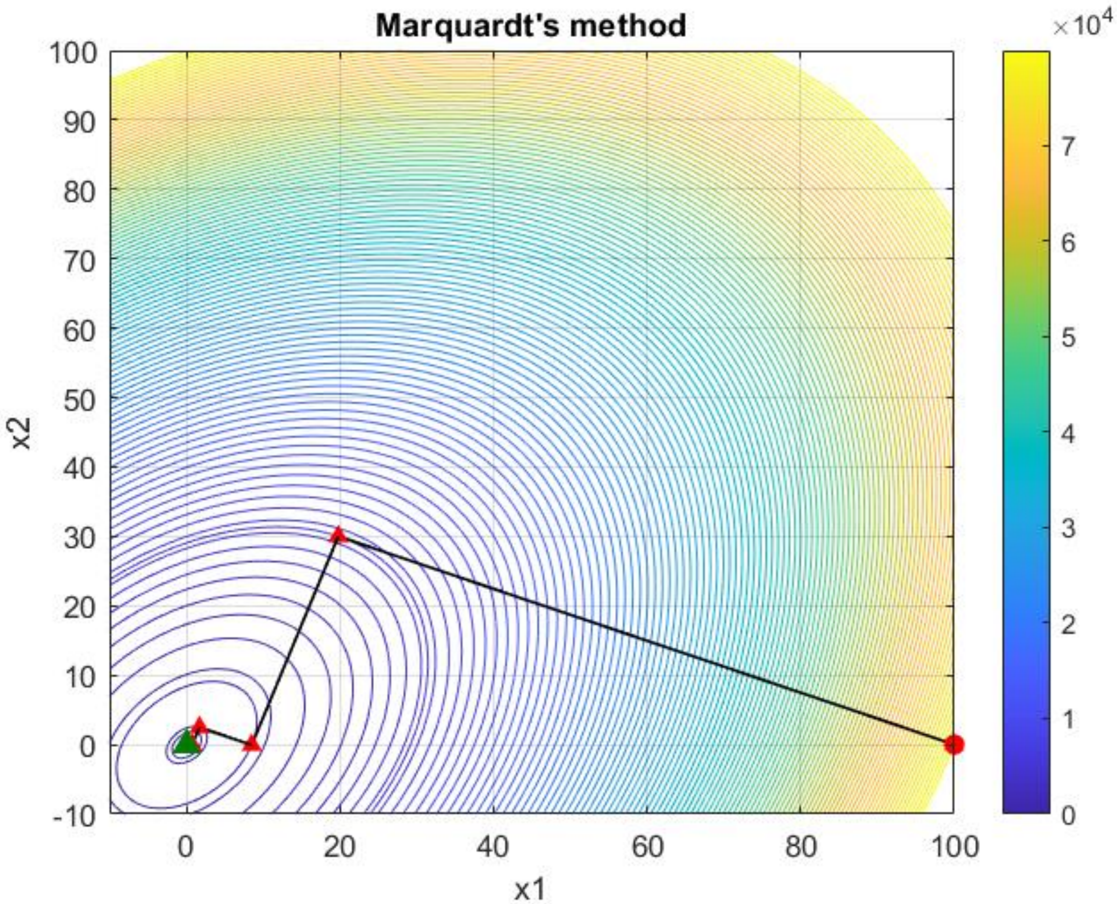
3) Since the given function is a quadratic function, it is expected that Conjugate Directions Method and Newton's Method converge faster than other methods.
   a. Newton's Method converges in 1 step, the other iteration is used to check for convergence, it can be seen that there is no change in optima for 2nd iteration.
   b. Conjugate Directions method takes 2 steps to converge, the 3rd iteration is used to check for convergence.

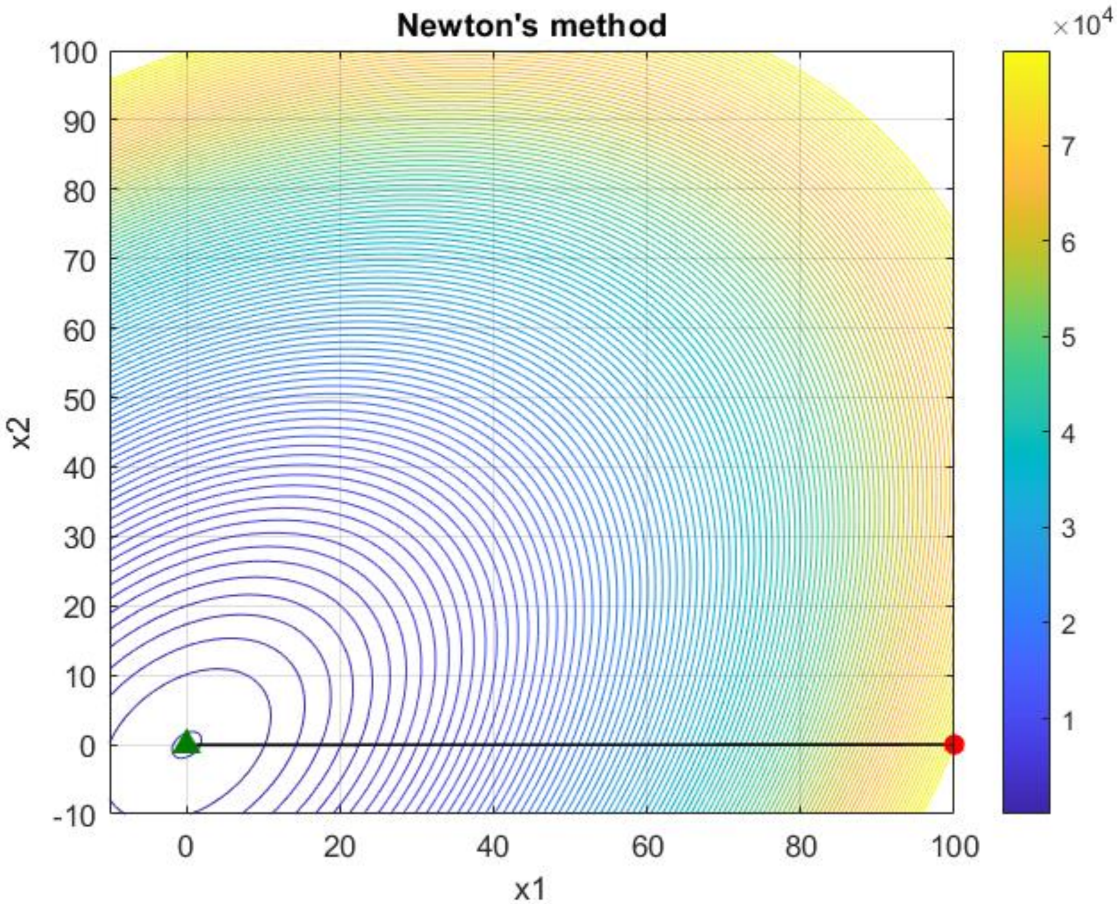
Observations at the end of 4th iteration:

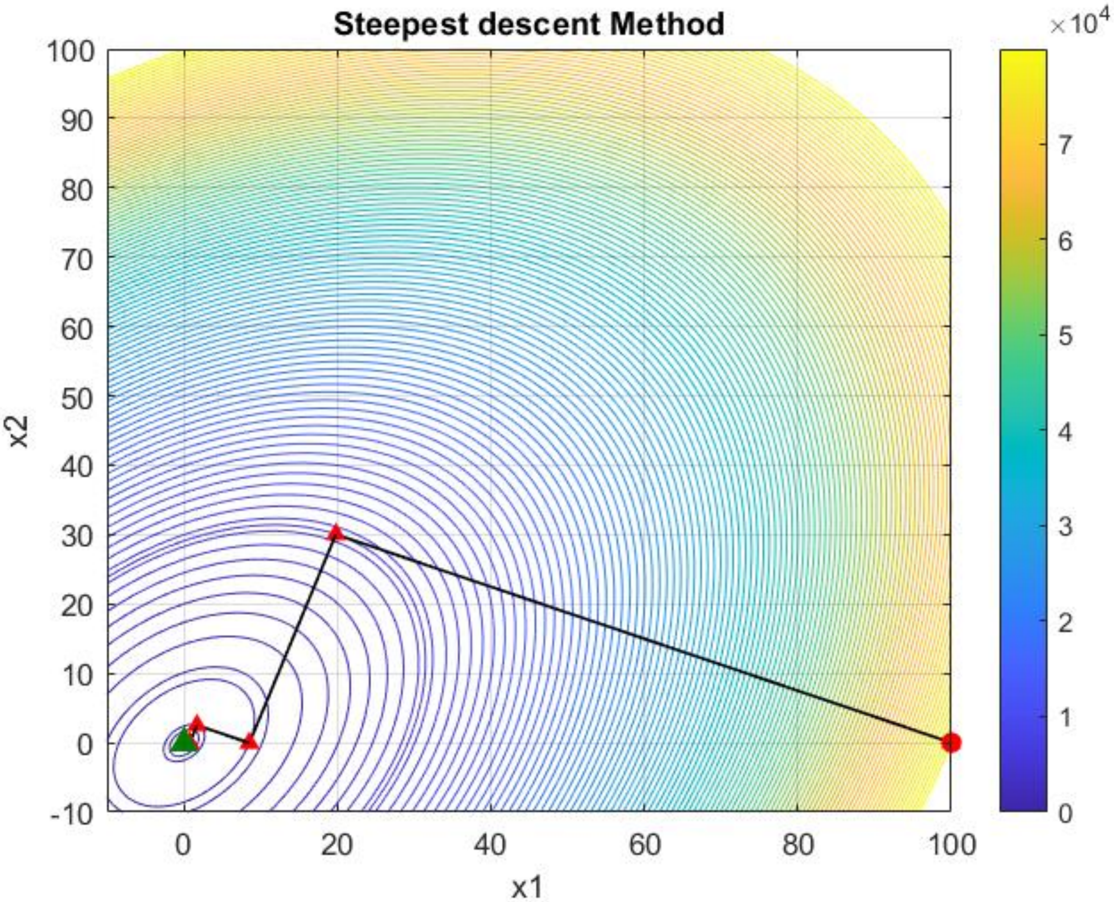
Method	Iteration	Xn=(X1,X2)		f(Xn)	|grad (F(Xn))|
		Xn1	Xn2		
Steepest Descent	4	0.7690909	-0.04513	4.142314	12.36180676
Conjugate Directions	2	0.0454545	-0.04545	-0.04545	2.03E-14
Newton	1	0.0454545	-0.04545	-0.04545	0
Marquardt	4	0.7529233	-0.04722	3.966165	12.10910088

1) Conjugate directions method and Newton's method have produced the exact optimum point.
2) Marquardt method is slightly better than Steepest Descent method, which is evident from the gradient of the objective function.
3) The function value of Marquardt method is lesser than Steepest Descent method at the fourth iteration.
4) It can be seen that the point (Xn), the Marquardt method reaches is almost same as Steepest Descent method due to the large value of α at the beginning. Marquardt method behaves like Steepest descent method.
5) As Marquardt method reaches convergence, α can be seen to be decreasing rapidly hence it behaves as newton's method.


Conjugate direction Method
x2
x1
$\times 10^4$
100
90
80
70
60
50
40
30
20
10
0
-10
0
20
40
60
80
100
7
6
5
4
3
2
1


Marquardt's method
x1
x2
$\times 10^4$
100
90
80
70
60
50
40
30
20
10
0
-10
0
20
40
60
80
100
7
6
5
4
3
2
1
0


Newton's method
x2
x1
100
90
80
70
60
50
40
30
20
10
0
-10
0
20
40
60
80
100
×10^4
7
6
5
4
3
2
1


Steepest descent Method
x2
x1
×10^4
100
90
80
70
60
50
40
30
20
10
0
-10
0
20
40
60
80
100
7
6
5
4
3
2
1
0

Question 4

The objective function which is bivariate can be reduced to univariate function using the equality constraint as follows.

Objective Function:

$$f(x, y) = x^2 + y^2$$

Equality constraint:

$$x - y = 3$$

Inequality constraint:

$$x + 2y \geq 6$$

After substituting the equality constraint into the objective function and the inequality constraint, the final objective function and the inequality constraint are as follows:

Final objective function:

$$f(x) = 2x^2 - 6x + 9$$

Subject to:

$$4 - x \leq 0$$