Optimization Methods for Mechanical Design - ME7223

Assignment 1

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September 6,2021

Max Marks 20 **Due Date** 13/09/2021

- Answer all questions.
- Assume any missing data appropriately.
- Append the graphs to the scanned version of the answer sheets.
- Contact the TA (Kaushik) if you have any questions.

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1. (a) Using any software package plot the contours of $f(x, y) = 25x^2 + 12y^2 - 20xy + 120y$ in the range of (x,y) from (-15,-15) to (10,10).

   (b) If there is a feasible region of $x + y \geq 0$, what is the minima? (Note: On the hardcopy of the contour plot draw the feasible region and find where the contour touches the feasible region approximately, submit only the scanned copy of the annotated contour plot as answer)

2. Find the maxima and minima of the following problem manually on a Graph Paper,
   Objective function: $f = y + 0.58x$
   Constraints:

   (a) $y - 2x - 5 \leq 0$
   (b) $y - x - 7.5 \leq 0$
   (c) $y + 5.5x - 66 \leq 0$
   (d) $(x - 7)^2 + (y - 15)^2 - 9 \geq 0$
   (e) $y \geq 0$

3. Express the function

$$f(x_1, x_2, x_3) = -x_1^2 - x_2^2 + 2x_1x_2 - x_3^2 + 6x_1x_3 + 4x_1 - 5x_3 + 2$$

in matrix form as

$$f(\mathbf{X}) = \frac{1}{2}\mathbf{X}^T[A]\mathbf{X} + B^T\mathbf{X} + C$$

and determine whether the matrix A is positive definite, negative definite, or indefinite.

4. The potential energy of a particle moving along the x direction is given by,

$$U(x) = 3x^2 - x^3$$

Plot the potential energy as a function of x. Identify all the possible equilibrium points, and label the stable equilibrium position.

5. Find the second-order Taylor's series approximation of the function

$$f(x_1, x_2, x_3) = x_2^2x_3 + x_1e^{x_3}$$

at the point (1,0,-2).

6. For a triangle ABC, find the maximum value of $\sin(A) + \sin(B) + \sin(C)$. Formulate it as a constrained optimization problem.

7. Minimize

$$f(\mathbf{X}) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$

subject to

$$g_1(\mathbf{X}) = x_1 - x_2 = 0,$$
$$g_2(\mathbf{X}) = x_1 + x_2 + x_3 - 1 = 0$$

by

   (a) direct substitution
   (b) constrained variation
   (c) Lagrange multiplier method

8. (a) Minimise the function

$$f(x, y) = x^2 + y^2$$

   subject to

$$g(x, y) = xy = 1,$$

   using the Lagrange multiplier method. Find the solution point(s) and the corresponding Lagrange multiplier(s).

   (b) Find ∇f and ∇g at the solution point. How are they related? What implication does this have on the contour lines of f and g ?

   (c) How does the relation between the gradients ∇f and ∇g computed at the solution point compare with the Lagrange multiplier ?