PARABOLA

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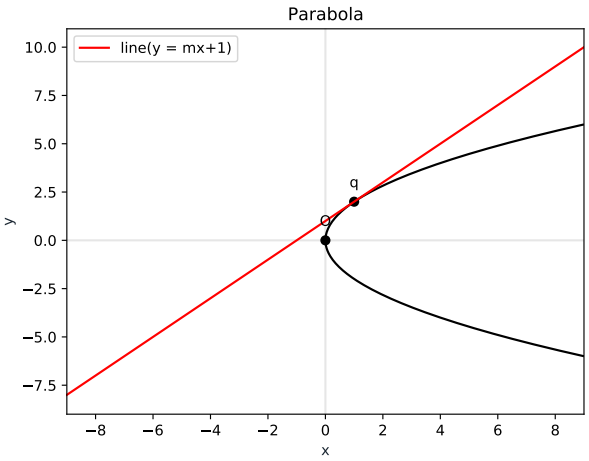
Contents

1 Problem

The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$, if the value of m is

2 Construction



3 Solution

The equation of parabola is:

$$\mathbf{X}^T\mathbf{V}\mathbf{X} + 2\mathbf{u}^T\mathbf{X} + f = 0 \tag{1}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} f = 0$$

Let $\mathbf{q}$ be the point of contact
The condition of point of contact is

$$\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f = 0 \tag{2}$$

$$\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \tag{3}$$

$$\mathbf{m}^T\mathbf{V}\mathbf{q} = -\mathbf{m}^T\mathbf{u} \tag{4}$$

$$\mathbf{n}^T\mathbf{q} = 1 \tag{5}$$

By Solving,

$$\begin{pmatrix} \mathbf{n}^T \\ \mathbf{m}^T\mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} 1 \\ -\mathbf{m}^T\mathbf{u} \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} -m & 1 \\ 0 & m \end{pmatrix} \mathbf{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{7}$$

The augumented matrix is

$$\begin{pmatrix} -m & 1 & 1 \\ 0 & m & 2 \end{pmatrix} \xrightarrow{R_1 \leftarrow mR_1 - R_2} \begin{pmatrix} -m^2 & 0 & m-2 \\ 0 & m & 2 \end{pmatrix} \tag{8}$$

$$\mathbf{q} = \begin{pmatrix} \frac{2-m}{m^2} \\ \frac{2}{m} \end{pmatrix} \tag{9}$$

Substitute in

$$\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f = 0 \tag{10}$$

$$\begin{pmatrix} \frac{2-m}{m^2} & \frac{2}{m} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{2-m}{m^2} \\ \frac{2}{m} \end{pmatrix} + 2 \begin{pmatrix} -2 & 0 \end{pmatrix} \begin{pmatrix} \frac{2-m}{m^2} \\ \frac{2}{m} \end{pmatrix} \tag{11}$$

$$\frac{4}{m^2} = \frac{4}{m^2}(2-m) \tag{12}$$

$$m = 1 \tag{13}$$