CIRCLE

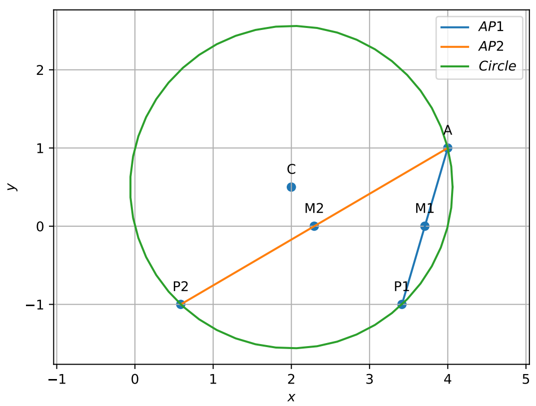
VAMSI SUNKARI
vamsisunkari9849@gmail.com
FWC22040 IITH Future Wireless Communication (FWC) ASSIGN-5

Contents

1 Problem

Let a circle be given by $2x(x-a)+y(2y-a)=0 (a \neq 0, b \neq 0)$. Find the condition on a and b if two chords each bisected by the x-axis, can be drawn to the circle from $\begin{pmatrix} a \\ \frac{b}{2} \end{pmatrix}$

2 Construction



3 Solution

3.1 Considerations

Symbol	Value
a	4
b	2

The given circle is

$$\mathbf{X}^T\mathbf{V}\mathbf{X} + 2\mathbf{u}^T\mathbf{X} + f = 0 \tag{1}$$

$$\mathbf{V} = \mathbf{I}$$
$$\mathbf{u} = \begin{pmatrix} \frac{-a}{2} \\ \frac{-b}{4} \end{pmatrix}$$
$$f = 0$$

3.2 Part 1:

let $\mathbf{P_1}$ and $\mathbf{P_2}$ be the other points of the chord. So, they satisfy the circle equation.

$$\mathbf{P_1}^T\mathbf{P_1} + 2\mathbf{u}^T\mathbf{P_1} = 0 \tag{2}$$

$$\mathbf{P_2}^T\mathbf{P_2} + 2\mathbf{u}^T\mathbf{P_2} = 0 \tag{3}$$

$\frac{\mathbf{A+P_1}}{\mathbf{2}}$ and $\frac{\mathbf{A+P_2}}{\mathbf{2}}$ are the midpoints of the chords and lies on

$$e_2^T\mathbf{x} = 0 \tag{4}$$

so,

$$e_2^T(\frac{\mathbf{A+P_1}}{\mathbf{2}}) = 0 \tag{5}$$

$$e_2^T(\frac{\mathbf{A+P_2}}{\mathbf{2}}) = 0 \tag{6}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x+a \\ y+\frac{b}{2} \end{pmatrix} = 0 \tag{7}$$

$$y = -\frac{b}{2} \tag{8}$$

The other point is $\begin{pmatrix} x \\ -\frac{b}{2} \end{pmatrix}$ which satisfies the parabola equation.

$$\begin{pmatrix} x & -\frac{b}{2} \end{pmatrix} \begin{pmatrix} x \\ -\frac{b}{2} \end{pmatrix} + 2\begin{pmatrix} \frac{-a}{2} & \frac{-b}{4} \end{pmatrix} \begin{pmatrix} x \\ -\frac{b}{2} \end{pmatrix} = 0 \tag{9}$$

$$x^2 - ax + \frac{b^2}{2} = 0 \tag{10}$$

The solution of above equation gives the x coordinates of the points

$$x = \frac{a \pm \sqrt{a^2 - 2b^2}}{2} \tag{11}$$

3.3 Part 2

It is clear that there are two distinct points on the X-axis. $\therefore$ the discriminant of quadratic equation is positive

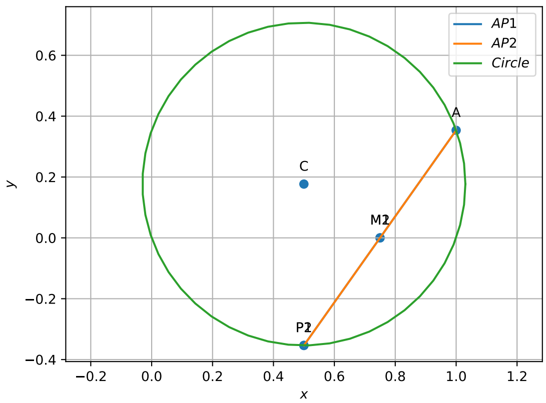
$$\implies \Delta > 0 \tag{12}$$

$$a^2 - 2b^2 > 0 \tag{13}$$

$$a^2 > 2b^2 \tag{14}$$

The condition on a and b if two chords each bisected by the x-axis, can be drawn to the circle from $\begin{pmatrix} a \\ \frac{b}{2} \end{pmatrix}$ is $a^2 > 2b^2$

if $a^2 = 2b^2$ condition is violated then x-axis bisects only 1 chord



if $a^2 < 2b^2$ the conditon is violated roots are imaginary and it is not possible to draw chords