Optimization Assignment - 2

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Problem Statement **- Amongst all open (from the top) right circular cylindrical boxes of volume $125\pi cm^3$, find the dimensions of the box which has the least surface area.**

Solution

0.1 Considerations

Symbol	Description
r	radius of cylinder
h	height of cylinder

Volume of the cylinder

$$V = \pi r^2 h \tag{1}$$

$$S = 2\pi rh + \pi r^2 \tag{2}$$

$$S = \frac{250\pi}{r} + \pi r^2 \tag{3}$$

Minima using conventional method

$$\frac{dS}{dr} = \frac{-250\pi}{r^2} + 2\pi r \tag{4}$$

$$\frac{-250\pi}{r^2} + 2\pi r = 0 \tag{5}$$

$$r^3 = 125 \tag{6}$$

$$r = 5cm \tag{7}$$

$$V = \pi r^2 h = 125\pi \tag{8}$$

$$h = \frac{125}{r^2} = 5cm \tag{9}$$

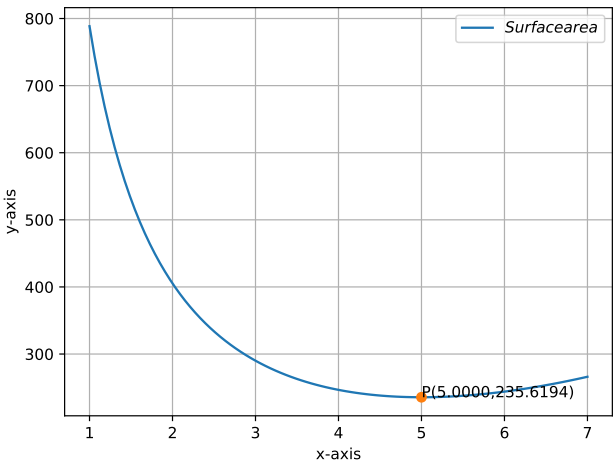

Figure 1: Graph of Surface area

Gradient descent

Using gradient descent method we can find its minima ,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{10}$$

$$\implies x_{n+1} = x_n - \alpha \left(\frac{250\pi}{r^2} + 2\pi r \right) \tag{11}$$

Taking $x_0 = 0.1, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Minima} = 235.61} \tag{12}$$

$$\boxed{\text{Minima Point} = 5.00} \tag{13}$$