EE338 : Digital Signal Processing

Filter Design Assignment

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180070020

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Filter Number : 33

Group No .2

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Contents

1 Student Details:

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2 Filter 1: Butterworth Band Pass Filter

- Filter Number Assigned = m = 33
- Both Pass band and Stop band tolerances are 0.15
- Pass band: Monotonic (as $1 \leq m \leq 80$)
- Stop band: Monotonic
- Butterworth Approximation
- Band-Pass filter.
- The Transition band is 4kHz on either side of the pass band
- The input signal is Band limitted to 160kHz and the sampling rate is 330kHz.

2.1 Unnormalized Specifications

m = 33
q(m) = 3
r(m) = 33 - 30 = 3
BL(m) = $25 + 1.7 \times 3 + 6.1 \times 3 = 48.4$ kHz
BH(m) = BL(m) + 20 = 68.4 kHz

Hence the filter specifications for the Bandpass Filter are:

- Pass band is 48.4 kHz to 68.4 kHz
- Transition band is 4kHz on either side of the Pass band
- Stop band is 0 to 44.4 kHz and 72.4 kHz to 165 kHz
- Tolerances for both bands are 0.15
- Both Pass band and Stop band are monotonic

2.2 Normalized specifications

Given sampling rate = 330 kHz. Using

$$\omega = \frac{2\pi * \Omega}{\Omega_s}$$

where ω is the normalized frequency, Ω is the Un-normalized frequency and Ω_s is the sampling frequency.

- Pass band is $\frac{22}{75}\pi$ to $\frac{114}{275}\pi \sim (0.29\pi$ to $0.415\pi)$
- Transition band is $\frac{4}{165}\pi \sim (0.024\pi)$ on either side of the Pass band
- Stop band is 0 to $\frac{74}{275}\pi$ and $\frac{362}{825}\pi$ to π $\sim$(0 to 0.27π and 0.44π to π)
- Tolerances for both bands are 0.15
- Both Pass band and Stop band are monotonic

2.3 Analog Band-pass Filter Specifications

Using Bilinear Transformation,

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

where Ω is the analog domain frequency and ω is the discrete domain frequency

Domain	Zero	Ω_{S1}	Ω_{P1}	Ω_{P2}	Ω_{S2}	Infinity
ω	0	$\frac{74}{275}\pi$	$\frac{22}{75}\pi$	$\frac{114}{275}\pi$	$\frac{362}{825}\pi$	π
Ω	0	0.45	0.496	0.762	0.824	∞

Hence the filter specifications for the corresponding analog domain Bandpass Filter are:

- Pass band is 0.496 (Ω_{P1}) to 0.762 (Ω_{P2})
- Stop band is 0 to 0.45(Ω_{S1}) and 0.824 (Ω_{S2}) to ∞
- Tolerances for both bands are 0.15
- Both Pass band and Stop band are monotonic (as $1 \leq$ m ≤ 80)

2.4 Frequency Transformation in to a Low Pass Filter

Using the Band-Pass Transformation

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = 0.615$$

$$B = \Omega_{P2} - \Omega_{P1} = 0.2656$$

Domain	Zero	Ω_{S1}	Ω_{P1}	Ω_0	Ω_{P2}	Ω_{S2}	Infinity
Ω	0^+	0.45	0.496	0.615	0.762	0.824	∞
Ω_L	-∞	-1.4727	-1	0	1	1.3741	∞
Domain	-Infinity	Ω_{LS1}	Ω_{LP1}	Ω_0	Ω_{LP2}	Ω_{LS2}	Infinity

2.5 Analog Low Pass Filter Specification

Hence the Frequency Transformed Low-Pass Filter Specifications are:

- Pass band edge is at 1 (Ω_{LP})
- Stop band edge = min (-Ω_{LS1}, Ω_{LS2}) = 1.3741 (Ω_{LS})
- Tolerances = 0.15 for both Stop band and Pass band
- Both Pass band and Stop band are monotonic

2.6 Analog Low Pass Transfer function

Using, Butterworth Approximation, As Tolerances of both Pass Band and Stop Band are 0.15.

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$$
$$D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$$

$$H^2_{analog,LPF}(j\Omega) = \frac{1}{1 + (\frac{\Omega}{\Omega_c})^{2N}}$$

Using the inequality for Order N of the Butterworth approximation,

$$N_{min} = \left\lceil \frac{log\left(\sqrt{\frac{D2}{D1}}\right)}{log\left(\frac{\Omega_{LS}}{\Omega_{LP}}\right)} \right\rceil$$

$$N_{min} = 8$$

The Cut-off Frequency should follow the constraint:

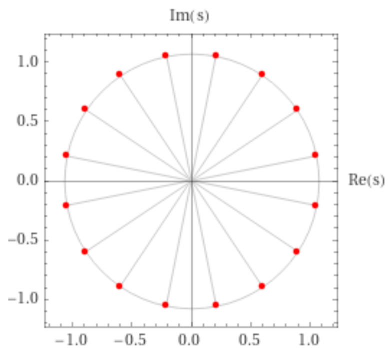
$$\frac{\Omega_{LP}}{(D1)^{\frac{1}{2N}}} \leq \Omega_C \leq \frac{\Omega_{LS}}{(D2)^{\frac{1}{2N}}}$$

$$1.0616 \leq \Omega_C \leq 1.0856$$

Thus we can choose Ω_C as 1.07.
Now, Poles and transfer function can be obtained by solving the equation:

$$1 + \left(\frac{s}{j\Omega_c}\right)^{2N} = 1 + \left(\frac{s}{j1.07}\right)^{16} = 0$$

Using Wolfram to plot these poles,



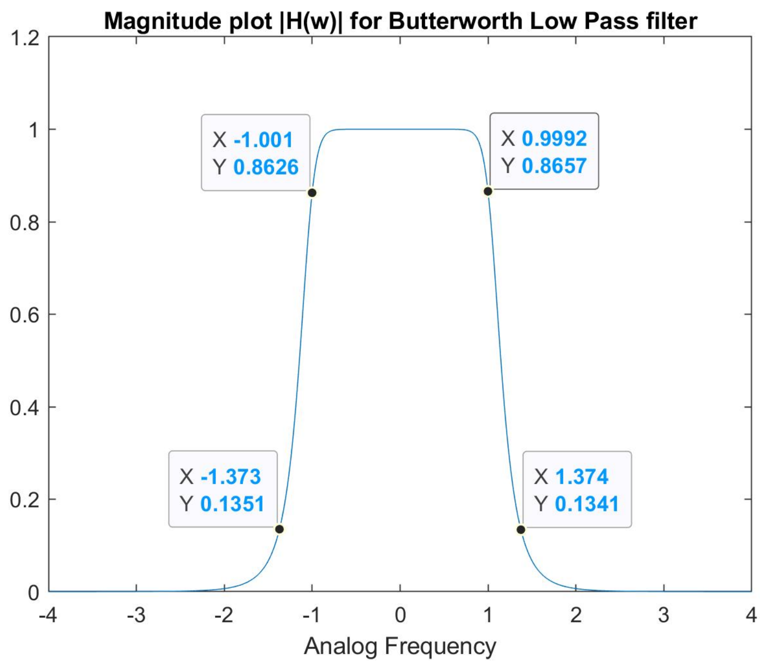
Note we need to only select those poles where the Re(s) < 0 i.e only the left half plane as we want realizable specifications thus we want stability and causality

```
p1=-0.20875+1.0494i
p2=-0.59446+0.88967i
p3=-0.88967+0.59446i
p4=-1.0494+0.20875i
p5=-1.0494-0.20875i
p6=-0.88967-0.59446i
p7=-0.59446-0.88967i
p8=-0.20875-1.0494i
```

Using the above 8 poles, the transfer function of this Analog Low pass filter can be written as:

$$H_{analog,LPF}(s_L) = \frac{\Omega_C^8}{(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)(s_L - p_5)(s_L - p_6)(s_L - p_7)(s_L - p_8)}$$

$$= \frac{1.7182}{s_L^8 + 5.4846 s_L^7 + 15.0406 s_L^6 + 26.7625 s_L^5 + 33.672 s_L^4 + 30.46 s_L^3 + 19.715 s_L^2 + 8.231 s_L + 1.7182}$$



2.7 Analog Band Pass Transfer function

Using the Band pass transformation:

$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$

Substituting the values for B = 0.266 and $\Omega_0 = 0.615$,

$$s_L = \frac{s^2 + 0.378}{0.266s}$$

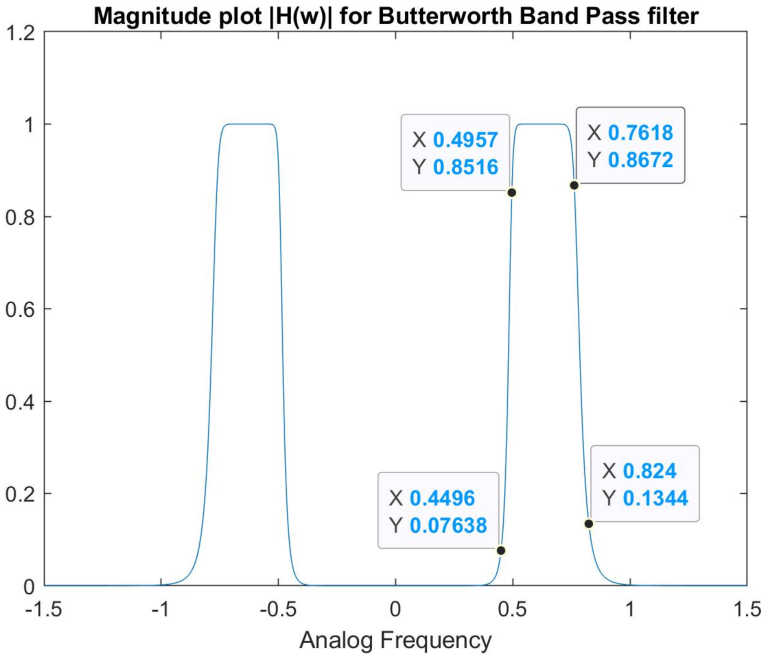
Substituting this back into $H_{analog,LPF}(s_L)$, we get $H_{analog,BPF}(s)$. It can be written in the form N(s)/D(s) where the polynomial N(s) $= 4.2604\times10^{-5}s^8$ and the coefficients of the polynomial D(s) are:

Degree	s^{16}	s^{15}	s^{14}	s^{13}	s^{12}	s^{11}
Coefficient	1	1.4570	4.0876	4.3597	6.5834	5.3676

Degree	s^{10}	s^{9}	s^{8}	s^{7}	s^{6}	s^{5}
Coefficient	5.5702	3.525	2.7317	1.3334	0.7971	0.2906

Degree	s^{4}	s^{3}	s^{2}	s^{1}	1
Coefficient	0.1348	0.0338	0.0120	0.0016	0.00042

Table 1: Coefficients of D(s)



2.8 Discrete time Band Pass Transfer function

Using the Bilinear Transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Substituting this back in $H_{Analog,BPF}(s)$, we get $H_{digital,BPF}(z)$. It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are shown in the below tables.

Coefficients of N(z) for odd powers of z^{-1} are zero.

Degree	1	z^{-2}	z^{-4}	z^{-6}	z^{-8}
Coefficient	1.1426×10^{-6}	-9.141×10^{-6}	3.1994×10^{-5}	-6.3897×10^{-5}	7.9984×10^{-5}

Degree	z^{-10}	z^{-12}	z^{-14}	z^{-16}
Coefficient	-6.3897×10^{-5}	3.1994×10^{-5}	-9.141×10^{-6}	1.1426×10^{-6}

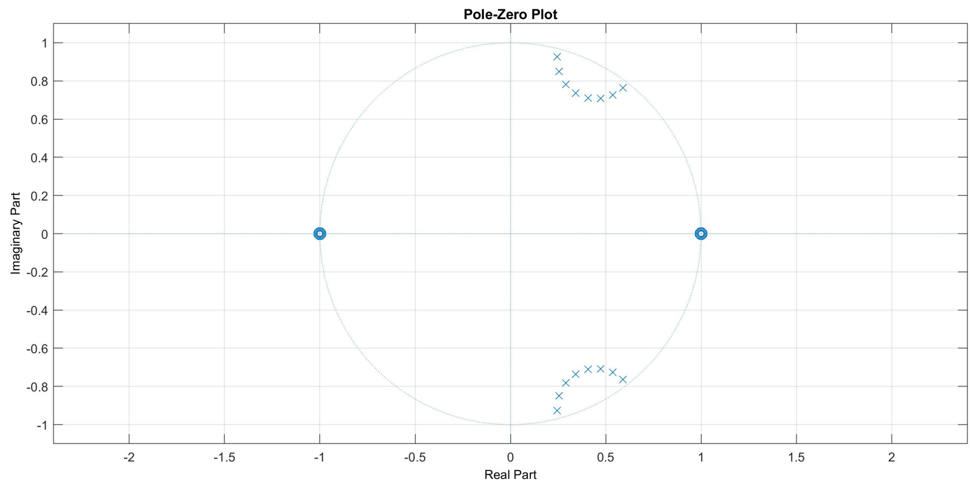
Table 2: Coefficients of N(z)

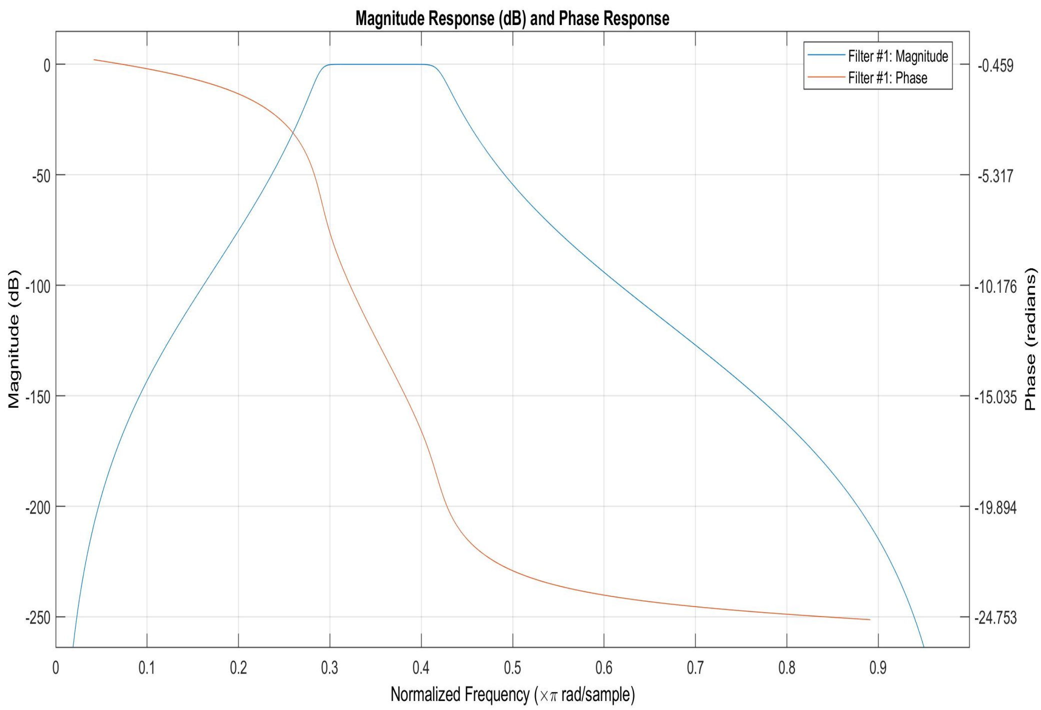
Degree	1	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}
Coefficient	1	-6.2775	23.2001	-59.9462	120.0295	-193.4666

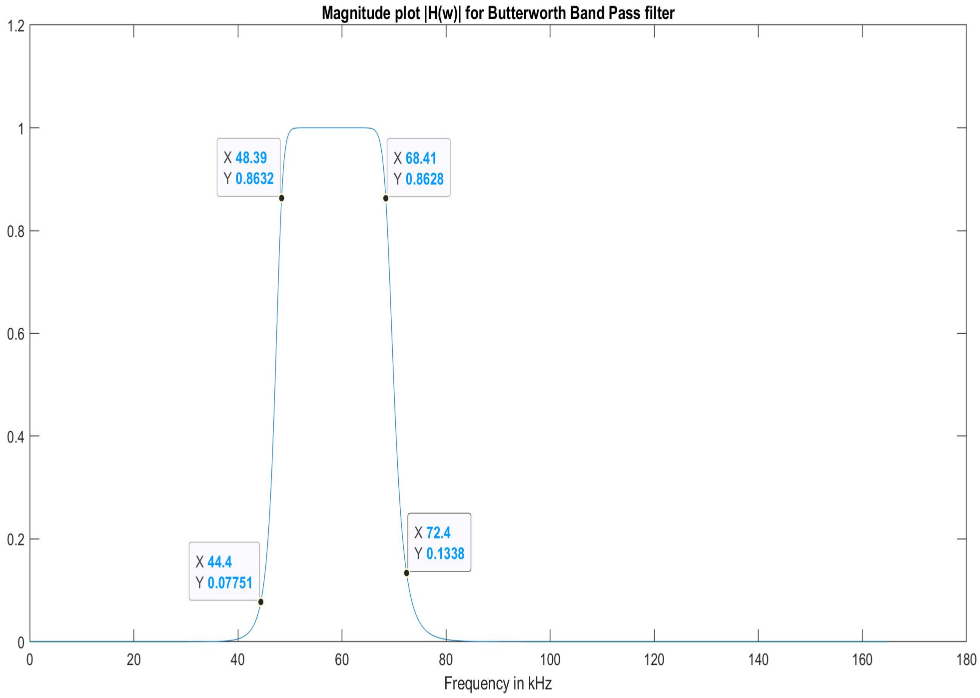
Degree	z^{-6}	z^{-7}	z^{-8}	z^{-9}	z^{-10}
Coefficient	-290.3236	276.4138	-223.3277	153.027	-88.019

Degree	z^{-11}	z^{-12}	z^{-13}	z^{-14}	z^{-15}	z^{-16}
Coefficient	258.6412	41.9788	-16.1093	4.7895	-0.9951	0.122

Table 3: Coefficients of D(z)







2.9 FIR Filter Transfer Function using Kaiser window

Tolerance in both stop band and pass band is given to be 0.15. Therefore, $\delta = 0.15$, using this we get value of A to be

$$A = -20 * log(0.15) = 16.4782dB$$

Since A<21, we get the value of the shape parameter of the Kaiser window $\alpha = 0$. Hence we essentially get a rectangular window, Now the transition bandwidth $\Delta\omega_T = \frac{4}{165}\pi \sim 0.0242\pi$. Using the empirical formula for the length of the window,

$$2Nmin + 1 \geq 1 + \frac{A - 8}{2.285\Delta\omega_T}$$

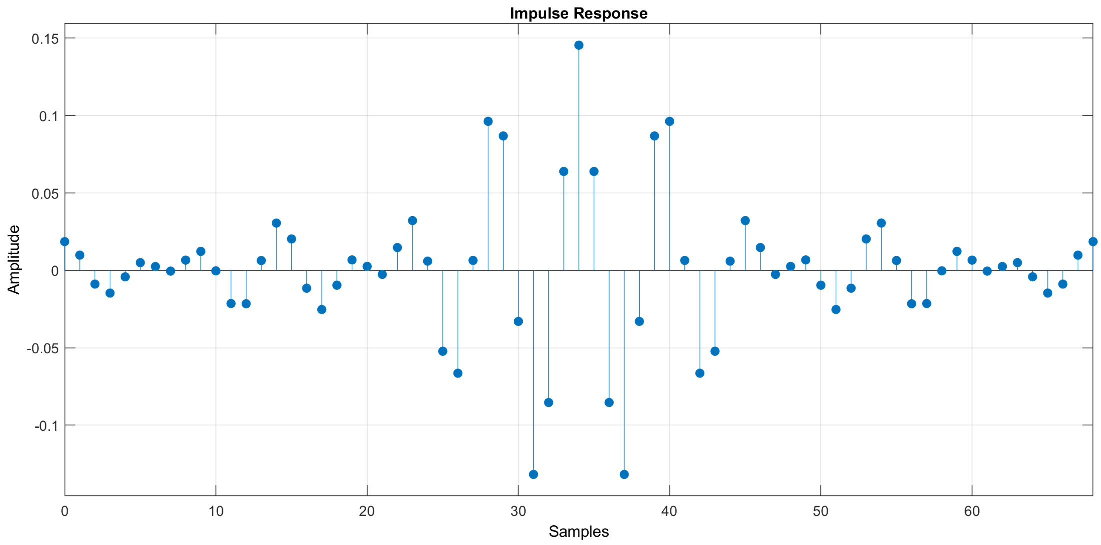
This gives us Nmin = 25, which implies the minimum length of the Kaiser window = 51. But this length does not satisfy all the conditions, the least window length satisfying all the conditions found by trial and error on MATLAB is n = 69.

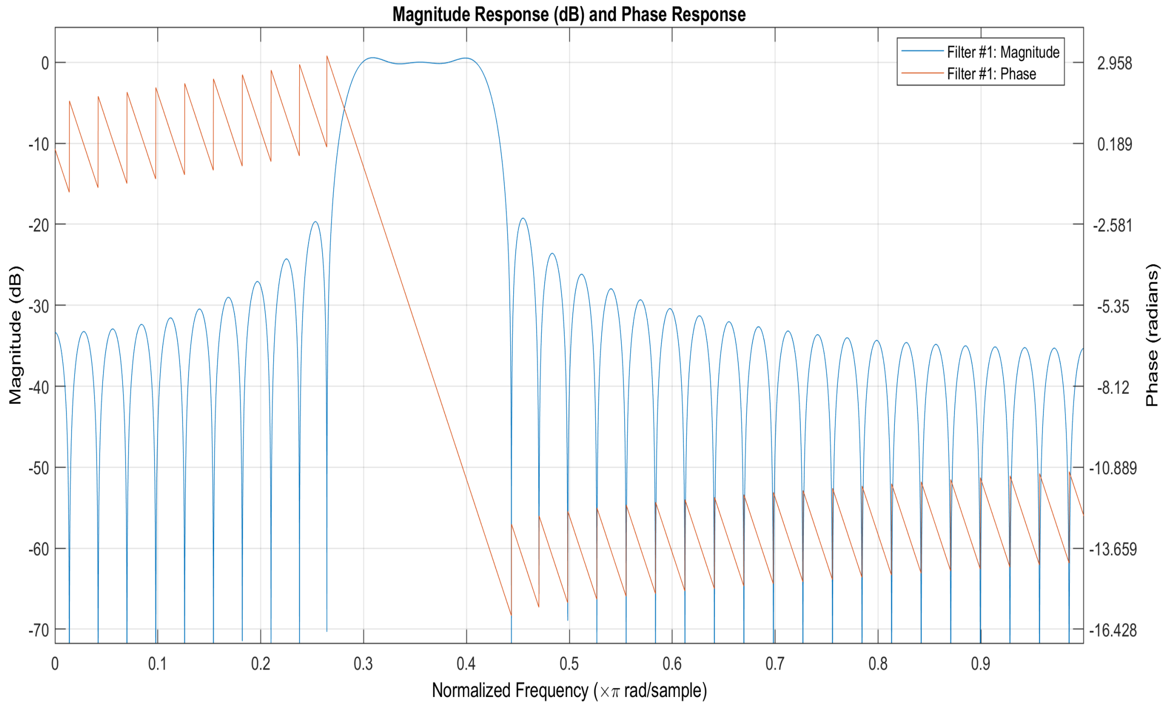
We know the an Ideal Band pass filter can be obtained by subtracting two Ideal Low pass filters, So first we obtain the samples of the Ideal Band Pass filter by subtracting samples of two Low Pass filters of same length as the Kaiser window. Now, we obtain the time domain representation of the final FIR filter by multiplying the Ideal impulse response samples with the Kaiser window.

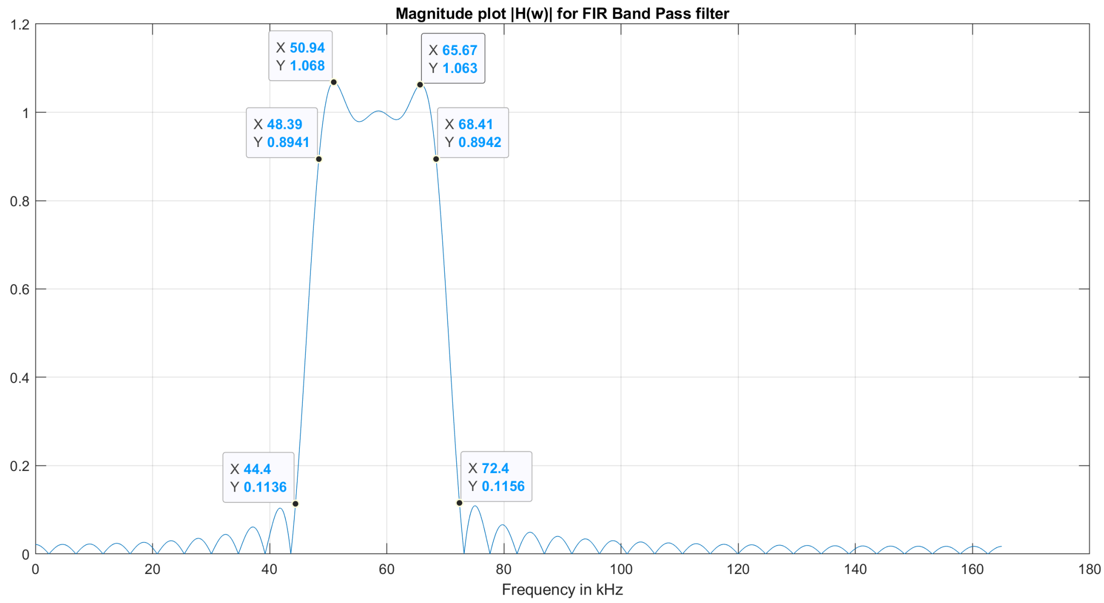
```
FIR_BandPass =

  Columns 1 through 12

    0.0185    0.0098   -0.0089   -0.0147   -0.0042    0.0050    0.0025   -0.0005    0.0066    0.0122   -0.0003   -0.0214

  Columns 13 through 24

   -0.0216    0.0063    0.0305    0.0203   -0.0115   -0.0253   -0.0096    0.0067    0.0026   -0.0026    0.0147    0.0321

  Columns 25 through 36

    0.0059   -0.0522   -0.0664    0.0064    0.0962    0.0868   -0.0330   -0.1318   -0.0853    0.0639    0.1455    0.0639

  Columns 37 through 48

   -0.0853   -0.1318   -0.0330    0.0868    0.0962    0.0064   -0.0664   -0.0522    0.0059    0.0321    0.0147   -0.0026

  Columns 49 through 60

    0.0026    0.0067   -0.0096   -0.0253   -0.0115    0.0203    0.0305    0.0063   -0.0216   -0.0214   -0.0003    0.0122

  Columns 61 through 69

    0.0066   -0.0005    0.0025    0.0050   -0.0042   -0.0147   -0.0089    0.0098    0.0185
```

Figure 1: Time Domain sequence values of the filter

These time domain sequence values are also the coefficients of the Z-transform from 1 to Z^{-68}.







2.10 Comparison between FIR and IIR realizations

- As discussed in the class, we can see that the FIR filter has linear / Pseudo linear phase in the Pass band where as IIR filter has non-linear phase.
- The number of delay lines required for IIR filter is 8+16 = 24, where as for FIR filter we need 68 delay lines. Hence it is very evident that for the same Filter specifications the FIR filters need a lot more hardware.

2.11 Review report

Reviewed Shubham Kar's report

- Specifications were correctly chosen.
- All the frequency response specifications are met. We can see this clearly as he marked the critical points on the plots.
- All the mandatory parts are done. Although there were some typo's which were corrected.

3 Filter 2: Chebyshev Band Stop Filter

- Filter Number Assigned = m = 33
- Both Pass band and Stop band tolerances are 0.15
- Pass Band is Equiripple (as $1 \leq$ m ≤ 80)
- Stop band is Monotonic
- Band-stop Filter
- Chebyshev Approximation
- The Transition band is 4kHz on either side of the stop band
- The input signal is Band limitted to 120kHz and the sampling rate is 260kHz.

3.1 Unnormalized Specifications

m = 33
q(m) = 3
r(m) = 33 - 30 = 3
BL(m) = 25 + 1.9 × 3 + 4.1 × 3 = 43 kHz
BH(m) = BL(m) + 20 = 63 kHz

Hence the filter specifications for the Band-stop Filter are:

- Stop band is 43 kHz to 63 kHz
- Transition band is 4kHz on either side of the Stop band
- Pass band is 0 to 39 kHz and 67 kHz to 130 kHz
- Tolerances for both bands are 0.15
- Pass band: Equiripple
- Stop band: Monotonic

3.2 Normalized specifications

Given sampling rate = 260 kHz. Using

$$\omega = \frac{2\pi * \Omega}{\Omega_s}$$

where ω is the normalized frequency, Ω is the Un-normalized frequency and Ω_s is the sampling frequency.

- Stop band is $\frac{43}{130}\pi$ to $\frac{63}{130}\pi \sim (0.33\pi$ to $0.48\pi)$
- Transition band is $\frac{4}{130}\pi \sim (0.03\pi)$ on either side of the Stop band

- Pass band is 0 to $\frac{3}{10}\pi$ and $\frac{67}{130}\pi$ to $\pi \sim$ (0 to 0.3π and 0.52π to π)
- Tolerances for both bands are 0.15
- Pass band: Equiripple
- Stop band: Monotonic

3.3 Analog Band-Stop Filter Specifications

Using Bilinear Transformation,

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

where Ω is the analog domain frequency and ω is the discrete domain frequency

Domain	Zero	Ω_{P1}	Ω_{S1}	Ω_{S2}	Ω_{P2}	Infinity
ω	0	$\frac{3}{10}\pi$	$\frac{43}{130}\pi$	$\frac{63}{130}\pi$	$\frac{67}{130}\pi$	π
Ω	0	0.5095	0.572	0.9528	1.0495	∞

Hence the filter specifications for the corresponding analog domain Bandstop Filter are:

- Stop band is 0.572 (Ω_{S1}) to 0.9528 (Ω_{S2})
- Pass band is 0 to 0.5095(Ω_{P1}) and 1.0495 (Ω_{P2}) to ∞
- Tolerances for both bands are 0.15
- Pass band: Equiripple
- Stop band: Monotonic

3.4 Frequency Transformation in to a Low Pass Filter

Using the Band-Stop Transformation

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = 0.7313$$

$$B = \Omega_{P2} - \Omega_{P1} = 0.54$$

Domain	Zero	Ω_{P1}	Ω_{S1}	Ω_0^-	Ω_0^+	Ω_{S2}	Ω_{P2}	Infinity
Ω	0^+	0.5095	0.572	0.7313	0.7313	0.9528	1.0495	$+\infty$
Ω_L	0^+	1	1.4879	$+\infty$	$-\infty$	-1.3792	-1	0^-
Domain	Zero	Ω_{LP1}	Ω_{LS1}	+Infinity	-Infinity	Ω_{LS2}	Ω_{LP2}	Zero

3.5 Analog Low Pass Filter Specification

Hence the Frequency Transformed Low-Pass Filter Specifications are:

- Pass band edge is at 1 (Ω_{LP})
- Stop band edge = min (-Ω_{LS2}, Ω_{LS1}) = 1.3792 (Ω_{LS})
- Tolerances = 0.15 for both Stop band and Pass band
- Pass band: Equiripple
- Stop band: Monotonic

3.6 Analog Low Pass Transfer function

Using, Chebyshev Approximation, As Tolerances of both Pass Band and Stop Band are 0.15.

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta_2^2} - 1 = 43.4444$$

$$H^2_{analog,LPF}(j\Omega) = \frac{1}{1+\epsilon^2 C_N^2(\frac{\Omega}{\Omega_{LP}})}$$

Choosing the parameter ϵ of Chebyshev filter to be $\sqrt{D_1}$, the inequality for Order N is

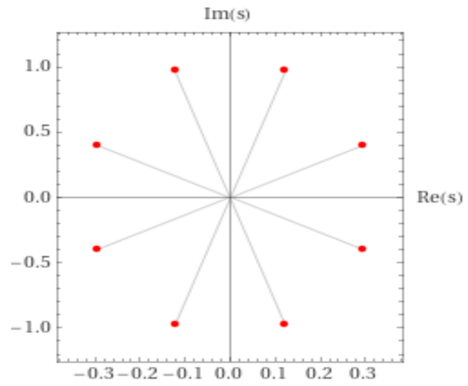
$$N_{min} = \left\lceil \frac{cosh^{-1}\left(\sqrt{\frac{D2}{D1}}\right)}{cosh^{-1}\left(\frac{\Omega_{LS}}{\Omega_{LP}}\right)} \right\rceil$$

$$N_{min} = 4$$

Hence, the poles of the transfer function can be obtained by,

$$1 + D_1 cosh^2\left(N_{min} cosh^{-1}\left(\frac{s}{j\Omega_{LP}}\right)\right) = 1 + 0.3841 cosh^2\left(4 cosh^{-1}\left(\frac{s}{j}\right)\right) = 0$$

Using Wolfram to plot the poles,



Note that to keep the stability and causality, we need to choose only the poles that lie on the left half plane i.e, Re(s) < 0.

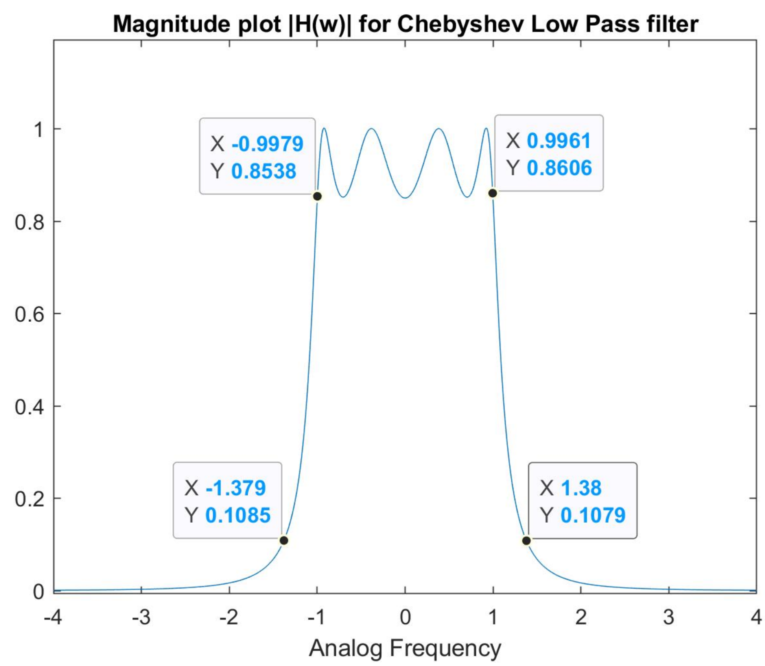
```
p1=-0.12216+0.96981i
p2=-0.29492+0.40171i
p3=-0.29492-0.40171i
p4=-0.12216-0.96981i
```

Using these four poles and the fact that N is even the transfer function Low pass analog filter,

$$H_{analog,LPF}(s_L) = \frac{(-1)^4 p_1 p_2 p_3 p_4}{\sqrt{1+D_1}(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)}$$

Note that since it is even order, we take the DC order to be $\frac{1}{\sqrt{1+\epsilon^2}}$

$$H_{analog,LPF}(s_L) = \frac{0.2011}{s_L^4 + 0.8342 s_L^3 + 1.3451 s_L^2 + 0.6226 s_L + 0.2366}$$



3.7 Analog Band Stop Transfer function

Using the Band stop transformation:

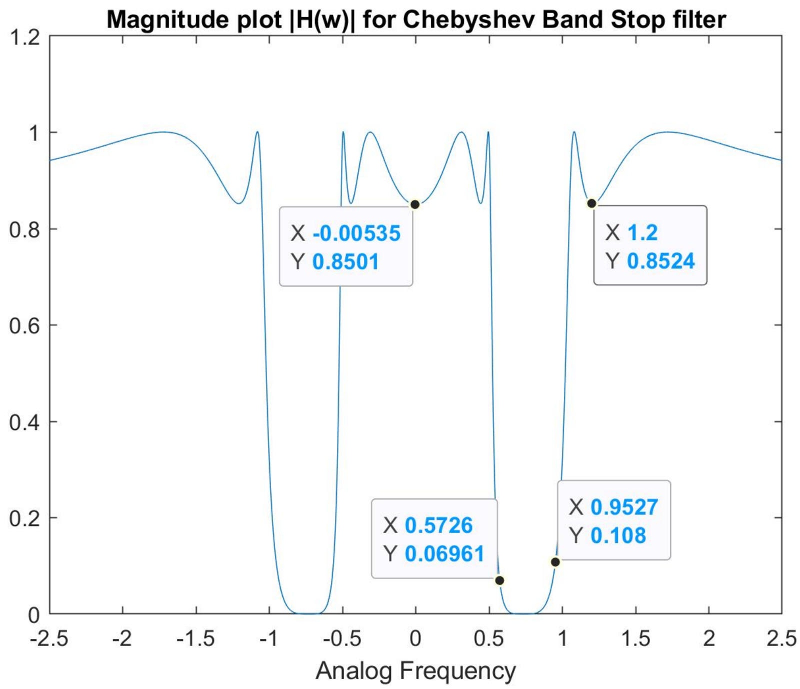
$$s_L = \frac{Bs}{s^2 + \Omega_0^2}$$

Substituting the values B = 0.54 and $\Omega_0 = 0.7313$,

$$s_L = \frac{0.54s}{s^2 + 0.535}$$

Substituting this back into $H_{analog,LPF}$, we get $H_{analog,BSF}$ as

$$\frac{0.85s^8 + 1.8182s^6 + 1.4585s^4 + 0.52s^2 + 0.0695}{s^8 + 1.4211s^7 + 3.797s^6 + 2.835s^5 + 3.8485s^4 + 1.5161s^3 + 1.0859s^2 + 0.2173s + 0.0818}$$



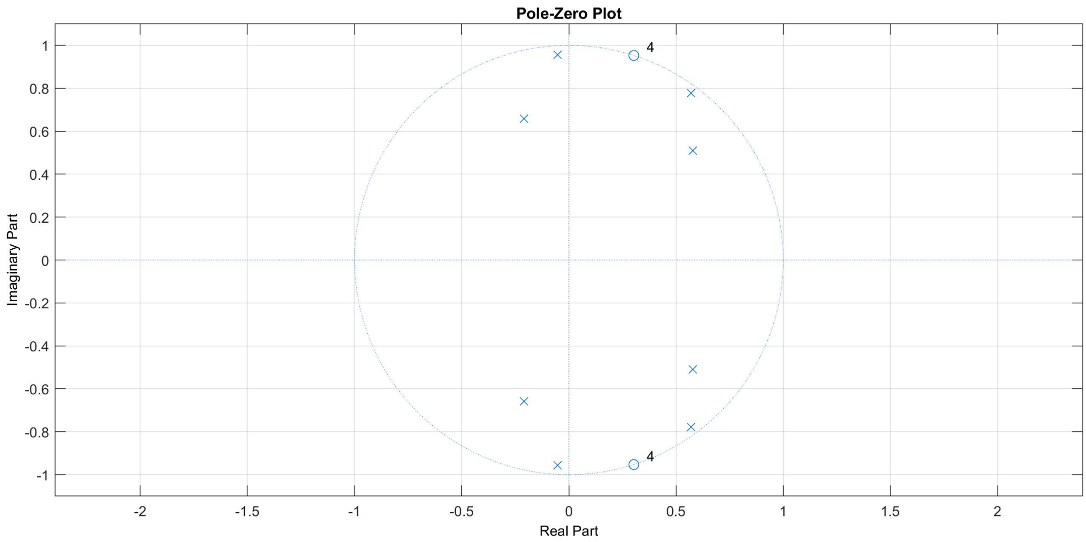
3.8 Discrete time Band Stop Transfer function

Using the Bilinear Transformation:

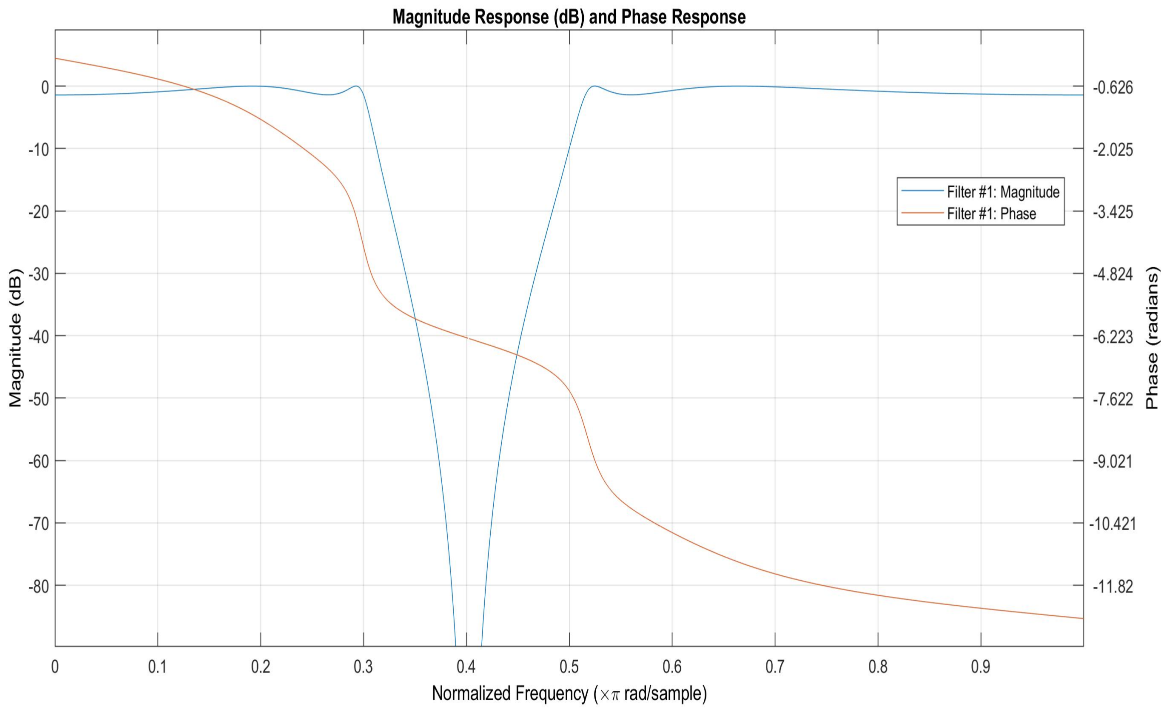
$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Substituting this back into $H_{Analog,BSF}(s)$, we get $H_{digital,BSF}$ as

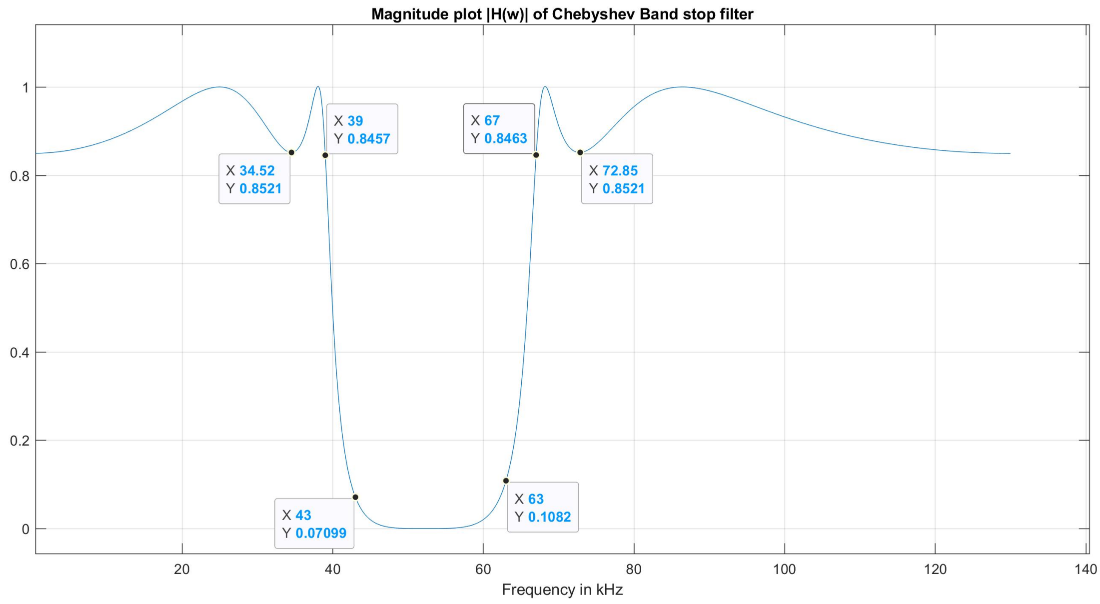
$$\frac{0.298 - 0.724z^{-1} + 1.852z^{-2} - 2.437z^{-3} + 3.147z^{-4} - 2.437z^{-5} + 1.852z^{-6} - 0.724z^{-7} + 0.298z^{-8}}{1 - 1.775z^{-1} + 3.0794z^{-2} - 3.133z^{-3} + 3.163z^{-4} - 2.002z^{-5} + 1.278z^{-6} - 0.527z^{-7} + 0.242z^{-8}}$$



Pole-Zero Plot
Imaginary Part
Real Part
4
4
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2



Magnitude Response (dB) and Phase Response
Magnitude (dB)
Phase (radians)
Normalized Frequency (×π rad/sample)
Filter #1: Magnitude
Filter #1: Phase
0
-10
-20
-30
-40
-50
-60
-70
-80
-0.626
-2.025
-3.425
-4.824
-6.223
-7.622
-9.021
-10.421
-11.82
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9



3.9 FIR Filter Transfer Function using Kaiser window

Tolerance in both stop band and pass band is given to be 0.15. Therefore, $\delta = 0.15$, using this we get value of A to be

$$A = -20 * log(0.15) = 16.4782dB$$

Since A<21, we get the value of the shape parameter of the Kaiser window $\alpha = 0$. Hence we essentially get a rectangular window, Now the transition bandwidth $\Delta\omega_T = \frac{4}{130}\pi \sim 0.03\pi$. Using the empirical formula for the length of the window,

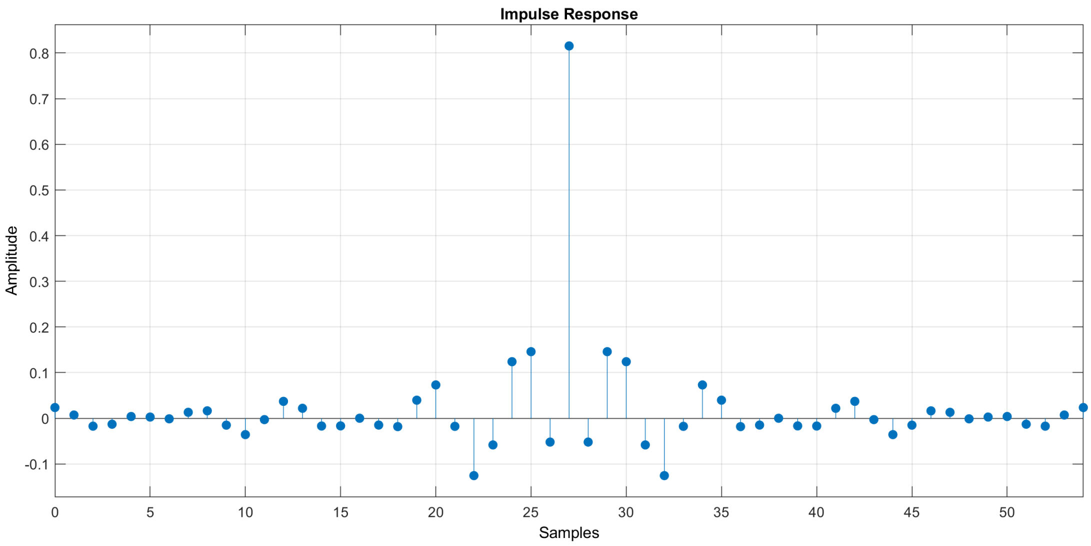
$$2Nmin + 1 \geq 1 + \frac{A - 8}{2.285\Delta\omega_T}$$

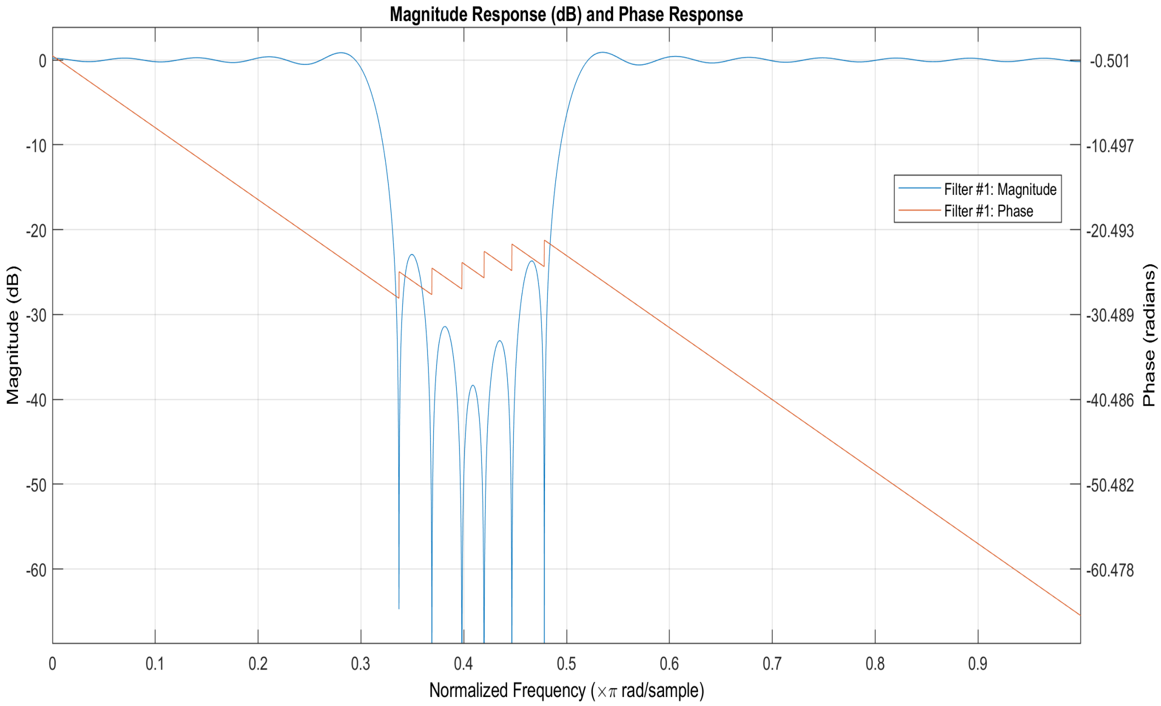
This gives us Nmin = 20, which implies the minimum length of the Kaiser window = 41. But this length does not satisfy all the conditions, the least window length satisfying all the conditions found by trial and error on MATLAB is n = 55.
We know the an Ideal Band Stop filter can be written as linear combination of three Ideal Low Pass Filters, So first we obtain the samples of the Ideal Band Stop filter by subtracting samples of two Low Pass filters and adding to another Low pass filter of same length as the Kaiser window. Now, we obtain the time domain representation of the final FIR filter by multiplying the Ideal impulse response samples with the Kaiser window.

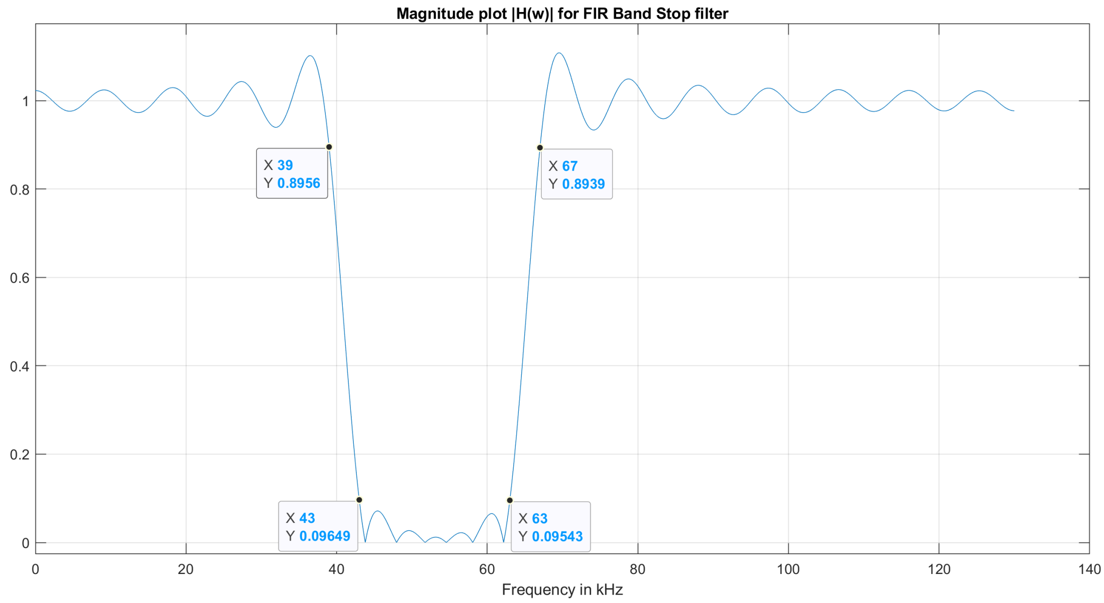
```
FIR_BandStop =

  Columns 1 through 11

    0.0236    0.0072   -0.0172   -0.0130    0.0039    0.0028   -0.0011    0.0131    0.0163   -0.0150   -0.0357

  Columns 12 through 22

   -0.0029    0.0371    0.0219   -0.0169   -0.0166    0.0001   -0.0148   -0.0182    0.0397    0.0731   -0.0176

  Columns 23 through 33

   -0.1255   -0.0583    0.1240    0.1459   -0.0521    0.8154   -0.0521    0.1459    0.1240   -0.0583   -0.1255

  Columns 34 through 44

   -0.0176    0.0731    0.0397   -0.0182   -0.0148    0.0001   -0.0166   -0.0169    0.0219    0.0371   -0.0029

  Columns 45 through 55

   -0.0357   -0.0150    0.0163    0.0131   -0.0011    0.0028    0.0039   -0.0130   -0.0172    0.0072    0.0236
```

Figure 2: Time Domain sequence values of the filter

These time domain sequence values are also the coefficients of the Z-transform from 1 to Z^{-54}.







3.10 Comparison between FIR and IIR realizations

- As discussed in the class, we can see that the FIR filter has linear / Pseudo linear phase in the Pass band where as IIR filter has non-linear phase.
- The number of delay lines required for IIR filter is 8+8 = 16, where as for FIR filter we need 54 delay lines. Hence it is very evident that for the same Filter specifications the FIR filters need a lot more hardware.

3.11 Review report

Reviewed Shubham Kar's report

- Specifications we correctly chosen but the formula written BL(m) was incorrect (but the final value was correct) and was now modified.
- All the frequency response specifications are met. We can see this clearly as he marked the critical points on the plots.
- All the mandatory parts are done. Although there were some typo's which were corrected.

Part2: Elliptic Filters

4 Filter 1: Elliptic Band Pass Filter

Since all the specification except the Stop band and Pass Band Nature are same, the same analysis will be done until specifying the Low pass filter Transfer function as we have done for Filter 1.

- Filter Number Assigned = m = 33
- Both Pass band and Stop band tolerances are 0.15
- Pass band: Equiripple
- Stop band: Equiripple
- Elliptic / Jacobi Approximation
- Band-Pass filter.
- The Transition band is 4kHz on either side of the pass band
- The input signal is Band limitted to 160kHz and the sampling rate is 330kHz.

4.1 Unnormalized Specifications

m = 33
q(m) = 3
r(m) = 33 - 30 = 3
BL(m) = 25 + 1.7 $\times$ 3 + 6.1 $\times$ 3 = 48.4 kHz
BH(m) = BL(m) + 20 = 68.4 kHz

Hence the filter specifications for the Bandpass Filter are:

- Pass band is 48.4 kHz to 68.4 kHz
- Transition band is 4kHz on either side of the Pass band
- Stop band is 0 to 44.4 kHz and 72.4 kHz to 165 kHz
- Tolerances for both bands are 0.15
- Both Pass band and Stop band are Equiripple

4.2 Normalized specifications

Given sampling rate = 330 kHz. Using

$$\omega = \frac{2\pi * \Omega}{\Omega_s}$$

where ω is the normalized frequency, Ω is the Un-normalized frequency and Ω_s is the sampling frequency.

- Pass band is $\frac{22}{75}\pi$ to $\frac{114}{275}\pi \sim (0.29\pi$ to $0.415\pi)$
- Transition band is $\frac{4}{165}\pi \sim (0.024\pi)$ on either side of the Pass band
- Stop band is 0 to $\frac{74}{275}\pi$ and $\frac{362}{825}\pi$ to π $\sim$(0 to 0.27π and 0.44π to π)
- Tolerances for both bands are 0.15
- Both Pass band and Stop band are Equiripple

4.3 Analog Band-pass Filter Specifications

Using Bilinear Transformation,

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

where Ω is the analog domain frequency and ω is the discrete domain frequency

Domain	Zero	Ω_{S1}	Ω_{P1}	Ω_{P2}	Ω_{S2}	Infinity
ω	0	$\frac{74}{275}\pi$	$\frac{22}{75}\pi$	$\frac{114}{275}\pi$	$\frac{362}{825}\pi$	π
Ω	0	0.45	0.496	0.762	0.824	∞

Hence the filter specifications for the corresponding analog domain Bandpass Filter are:

- Pass band is 0.496 (Ω_{P1}) to 0.762 (Ω_{P2})
- Stop band is 0 to $0.45(\Omega_{S1})$ and 0.824 (Ω_{S2}) to ∞
- Tolerances for both bands are 0.15
- Both Pass band and Stop band are Equiripple

4.4 Frequency Transformation in to a Low Pass Filter

Using the Band-Pass Transformation

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = 0.615$$

$$B = \Omega_{P2} - \Omega_{P1} = 0.2656$$

Domain	Zero	Ω_{S1}	Ω_{P1}	Ω_0	Ω_{P2}	Ω_{S2}	Infinity
Ω	0^+	0.45	0.496	0.615	0.762	0.824	∞
Ω_L	-∞	-1.4727	-1	0	1	1.3741	∞
Domain	-Infinity	Ω_{LS1}	Ω_{LP1}	Ω_0	Ω_{LP2}	Ω_{LS2}	Infinity

4.5 Analog Low Pass Filter Specifications

- Pass band edge is at 1 (Ω_{LP})
- Stop band edge = min (-Ω_{LS1}, Ω_{LS2}) = 1.3741 (Ω_{LS})
- Tolerances = δ_1 = δ_2 = 0.15 for both Stop band and Pass band
- Both Pass band and Stop band are Equiripple

4.6 Analog Low Pass Filter Transfer function

The magnitude-squared frequency response of an elliptic filter is defined as

$$H^2_{analog,LPF}(j\Omega) = \frac{1}{1 + \epsilon^2 R_n^2(\Omega, \Omega_{LS}, \delta_1, \delta_2)}$$

Where R_n is the nth-order elliptic rational function also known as a Chebyshev rational function. The order n, ϵ and will depend on the required specifications. The minimum order required can be calculated using the following steps,

$$\delta_1 = \delta_2 = 0.15$$

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta_2^2} - 1 = 43.4444$$

$$\epsilon = \sqrt{D1} = 0.61975$$

$$k = \frac{\Omega_{LP}}{\Omega_{LS}} = \frac{1}{1.3741} = 0.72775$$

$$k' = \sqrt{1-k^2} = 0.6858$$

$$k_1 = \sqrt{\frac{D1}{D2}} = 0.094$$

$$k_1' = \sqrt{1-k_1^2} = 0.99557$$

$$Nmin \geq \frac{KK_1'}{K'K_1}$$

Where K, K', K_1, K_1' are the values of complete elliptic integrals evaluated at k, k', k_1, k_1'. We get, K = 1.88 , K' = 1.8298, K_1 = 1.5743, K_1' = 3.7566. Using these values, we get the order of the elliptic rational function as 3. Once we get N_{min}, we need to recompute k and Ω_{LS} for all the conditions to be satisfied.

$$Nmin = \lceil 2.451 \rceil = 3$$

Where, Elliptic integral is defined as,

$$u(\phi, k) = \int_0^\phi \frac{dy}{\sqrt{1 - k^2 sin^2(y)}}$$

and complete elliptic integral is equal to u($\pi/2$, k). i.e,

$$U(k) = \int_0^{\pi/2} \frac{dy}{\sqrt{1 - k^2 sin^2(y)}}$$

Using the above elliptic integral equation we can also write ϕ in terms of u,k. i.e, $\phi(u$,k). This is the inverse of elliptic integral function. Using this we can define Jacobi elliptic sine function and other elliptic functions are defined as,

$$sn(u, k) = sin(\phi(u, k))$$
$$cn(u, k) = cos(\phi(u, k))$$
$$dn(u, k) = \sqrt{1 - k^2 sn^2(u, k)}$$
$$cd(u, k) = \frac{cn(u, k)}{dn(u, k)}$$

The Chebyshev rational function can be represented as,

$$R_n(\Omega) = sn(N_{min} \times sn^{-1}(\Omega, k), k_1)$$
$$R_n(\Omega) = sn(\phi, k_1), \Omega = sn(\phi, k)$$

The zero locations of the Transfer function can be given by,

$$\Omega = \frac{\pm 1}{kcd(iK/N_{min}, k)}, i.e.,$$
$$s = \frac{\pm j}{kcd(iK/N_{min}, k)}$$

Where i = 0, 2, 4, ..., N-1 for Odd N, and i = 1, 3, 5, ... N-1 for Even N.
The pole locations of the transfer function can be given by,

$$1 + \epsilon^2 R_n(s/j)^2 = 0$$

Using the periodicity of sn(u,k), we get

$$sn(N_{min}\phi, +2K_1 i, k_1) = \pm j\frac{1}{\epsilon}$$
$$\phi = (-2Ki + sn^{-1}(\frac{j}{\epsilon}, k_1))/N_{min}$$
$$\Omega = sn(\phi, k)$$

Define,

$$jV_0 = sn^{-1}(\frac{j}{\epsilon}, k_1)/N_{min}$$

Using these equations, location of poles can be found using

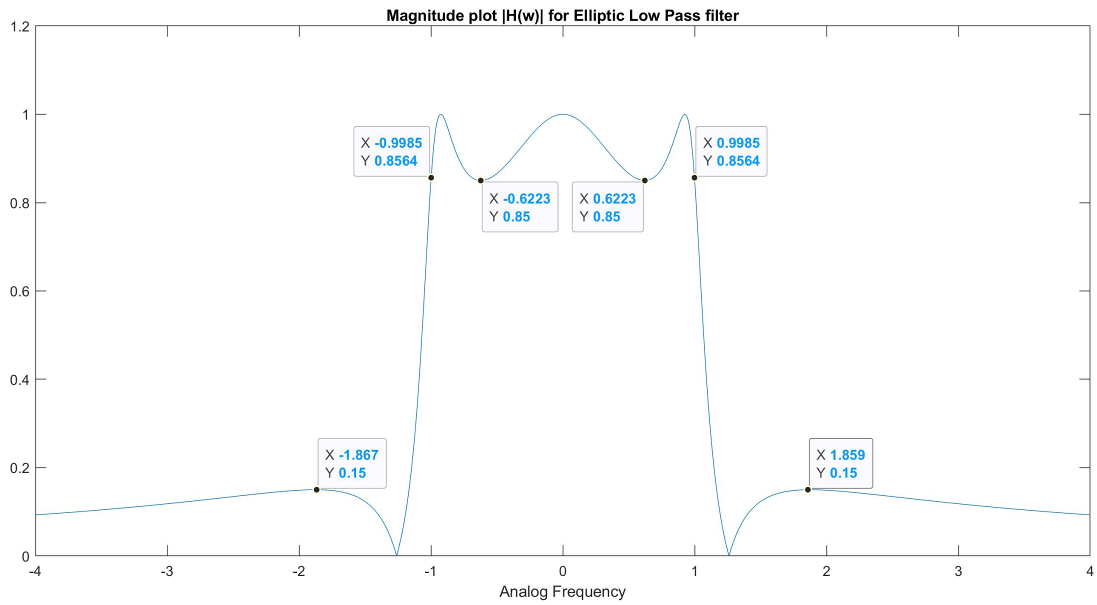
$$s = jsn(Ki/N + jV_0, k) = jcd(Ki/N + (1 - jV_0), k)$$

Note that here j represents complex number and i = 0, 2, 4, ..., N-1 for Odd N, and i = 1, 3, 5, ... N-1 for Even N. From the above equations, the poles and zeroes are found to be:

```
p1=-0.11533-0.9936i
p2=-0.11533+0.9936i
p3=-0.6232
z1=-1.2604i
z2=1.2604i
```

Using these poles and zeroes the transfer function Low pass analog filter is given by,

$$H_{analog,LPF}(s_L) = \frac{0.3925s_L^2 + 0.6235}{s_L^3 + 0.8538s_L^2 + 1.1443s_L + 0.6235}$$



4.7 Analog Band Pass Transfer function

Using the Band pass transformation:

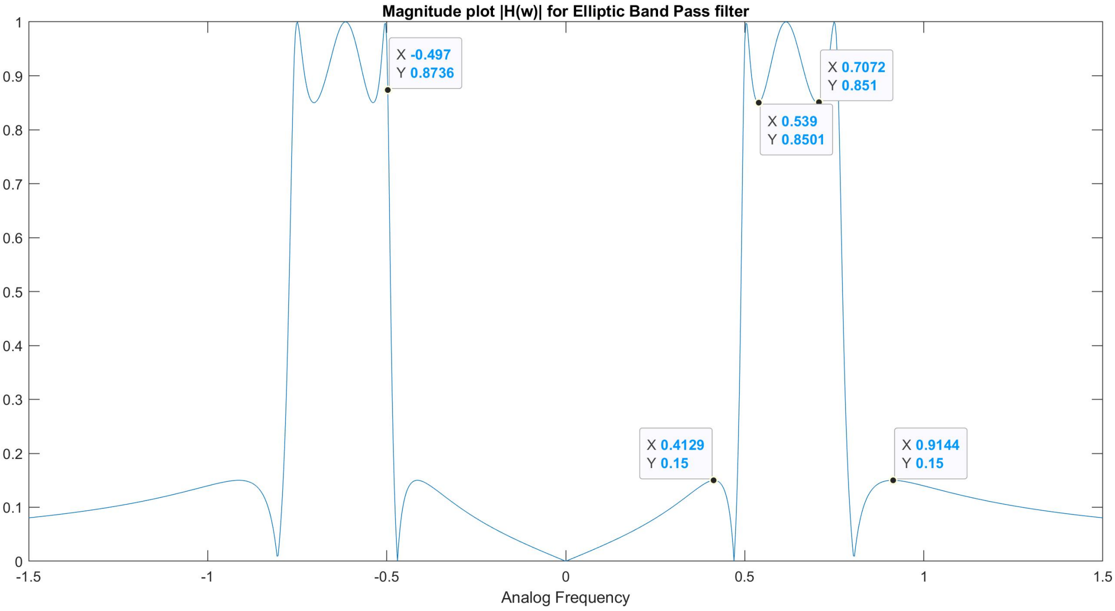
$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$

Substituting the values B = 0.266 and $\Omega_0 = 0.615$,

$$s_L = \frac{s^2 + 0.378}{0.266s}$$

Substituting this back into $H_{analog,LPF}$, we get $H_{analog,BPF}$ as

$$\frac{0.1043s^5 + 0.0906s^3 + 0.0149s}{s^6 + 0.2268s^5 + 1.2156s^4 + 0.1833s^3 + 0.4598s^2 + 0.0325s + 0.0541}$$



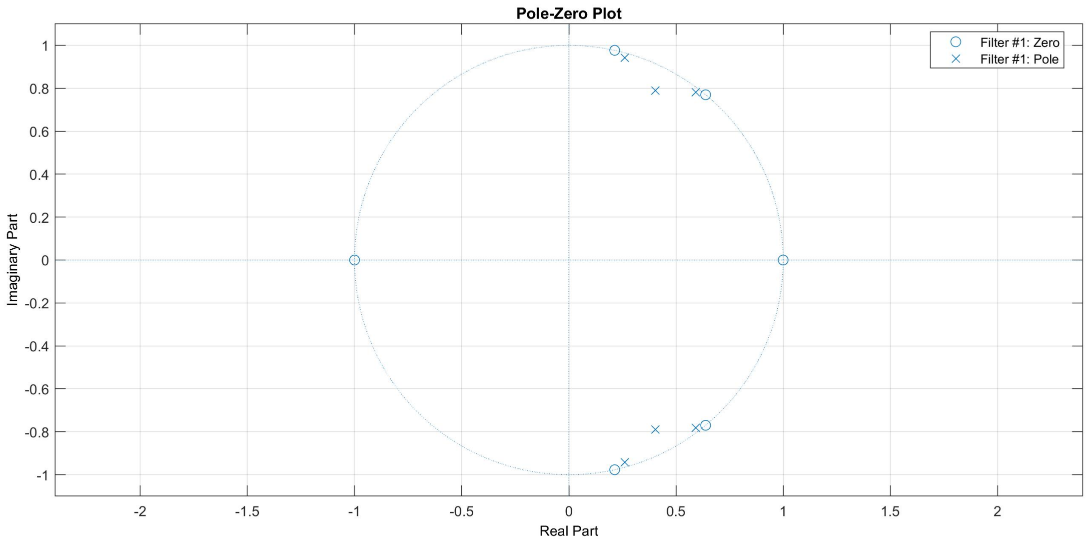
4.8 Discrete time Band Pass Transfer function

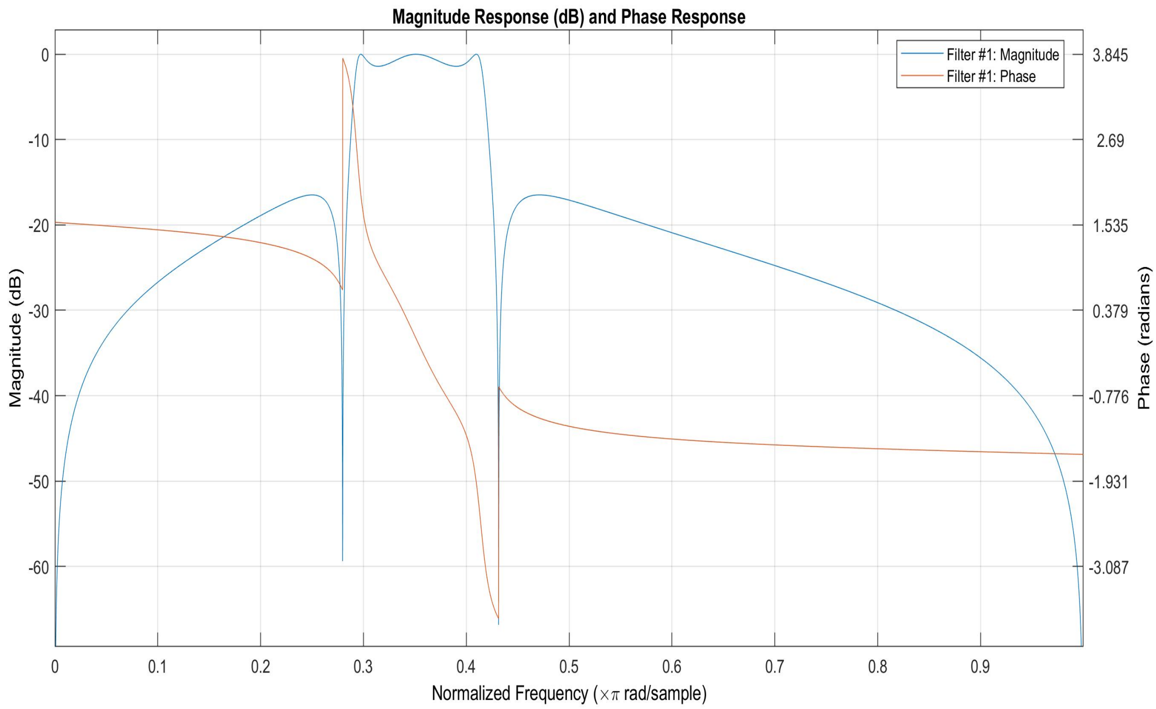
Using the Bilinear Transformation:

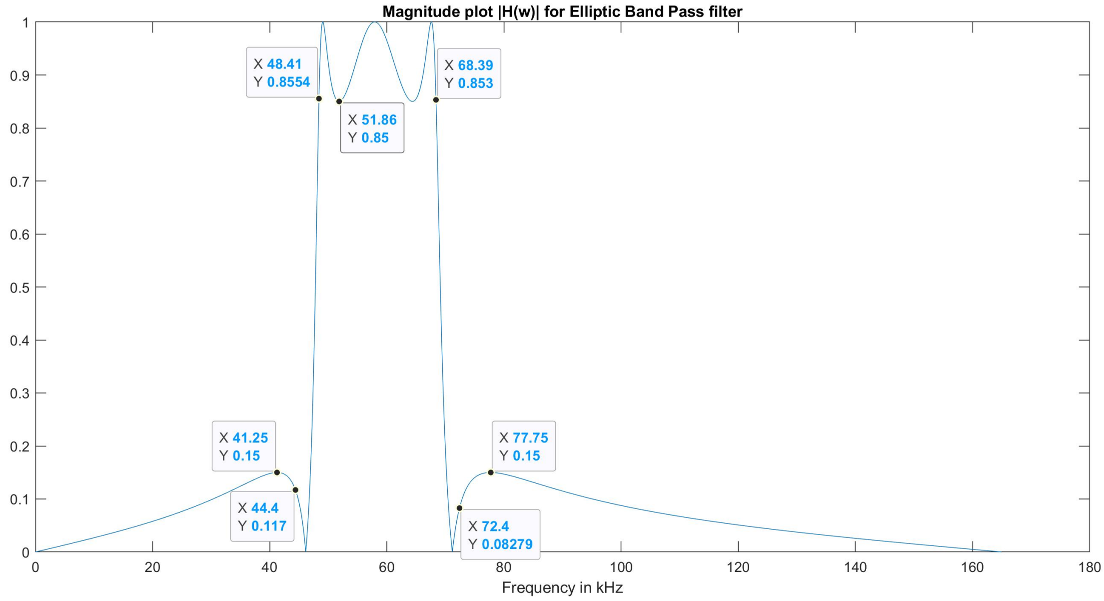
$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Substituting this back into $H_{Analog,BPF}(s)$, we get $H_{digital,BPF}$ as

$$\frac{0.0661 - 0.1127z^{-1} + 0.1022z^{-2} - 0.1022z^{-4} + 0.1127z^{-5} - 0.0661z^{-6}}{1 - 2.5107z^{-1} + 4.6918z^{-2} - 5.0106z^{-3} + 4.2212z^{-4} - 2.0205z^{-5} + 0.721z^{-6}}$$


Pole-Zero Plot
Filter #1: Zero
Filter #1: Pole
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Magnitude Response (dB) and Phase Response
Filter #1: Magnitude
Filter #1: Phase
Magnitude (dB)
Phase (radians)
Normalized Frequency (×π rad/sample)
0
-10
-20
-30
-40
-50
-60
3.845
2.69
1.535
0.379
-0.776
-1.931
-3.087
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9



4.9 Review report

5 Filter 2: Elliptic Band Stop Filter

- Filter Number Assigned = m = 33
- Both Pass band and Stop band tolerances are 0.15
- Pass Band is Equiripple
- Stop band is Equiripple
- Elliptic / Jacobi Approximation
- Band-stop Filter
- The Transition band is 4kHz on either side of the stop band
- The input signal is Band limitted to 120kHz and the sampling rate is 260kHz.

5.1 Unnormalized Specifications

m = 33
q(m) = 3
r(m) = 33 - 30 = 3
BL(m) = 25 + 1.9 × 3 + 4.1 × 3 = 43 kHz
BH(m) = BL(m) + 20 = 63 kHz

Hence the filter specifications for the Band-stop Filter are:

- Stop band is 43 kHz to 63 kHz
- Transition band is 4kHz on either side of the Stop band
- Pass band is 0 to 39 kHz and 67 kHz to 130 kHz
- Tolerances for both bands are 0.15
- Both Stop band and Pass band are Equiripple

5.2 Normalized specifications

Given sampling rate = 260 kHz. Using

$$\omega = \frac{2\pi * \Omega}{\Omega_s}$$

where ω is the normalized frequency, Ω is the Un-normalized frequency and Ω_s is the sampling frequency.

- Stop band is $\frac{43}{130}\pi$ to $\frac{63}{130}\pi \sim (0.33\pi$ to $0.48\pi)$
- Transition band is $\frac{4}{130}\pi \sim (0.03\pi)$ on either side of the Stop band
- Pass band is 0 to $\frac{3}{10}\pi$ and $\frac{67}{130}\pi$ to $\pi \sim (0$ to 0.3π and 0.52π to $\pi)$
- Tolerances for both bands are 0.15
- Both Stop band and Pass band are Equiripple

5.3 Analog Band-stop Filter Specifications

Using Bilinear Transformation,

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

where Ω is the analog domain frequency and ω is the discrete domain frequency

Domain	Zero	Ω_{P1}	Ω_{S1}	Ω_{S2}	Ω_{P2}	Infinity
ω	0	$\frac{3}{10}\pi$	$\frac{43}{130}\pi$	$\frac{63}{130}\pi$	$\frac{67}{130}\pi$	π
Ω	0	0.5095	0.572	0.9528	1.0495	∞

Hence the filter specifications for the corresponding analog domain Bandstop Filter are:

- Stop band is 0.572 (Ω_{S1}) to 0.9528 (Ω_{S2})
- Pass band is 0 to 0.5095(Ω_{P1}) and 1.0495 (Ω_{P2}) to ∞
- Tolerances for both bands are 0.15
- Both Stop band and Pass band are Equiripple

5.4 Frequency Transformation in to a Low Pass Filter

Using the Band-Stop Transformation

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = 0.7313$$

$$B = \Omega_{P2} - \Omega_{P1} = 0.54$$

Domain	Zero	Ω_{P1}	Ω_{S1}	Ω_0^-	Ω_0^+	Ω_{S2}	Ω_{P2}	Infinity
Ω	0^+	0.5095	0.572	0.7313	0.7313	0.9528	1.0495	$+\infty$
Ω_L	0^+	1	1.4879	$+\infty$	-∞	-1.3792	-1	0^-
Domain	Zero	Ω_{LP1}	Ω_{LS1}	+Infinity	-Infinity	Ω_{LS2}	Ω_{LP2}	Zero

5.5 Analog Low Pass Filter Specification

Hence the Frequency Transformed Low-Pass Filter Specifications are:

- Pass band edge is at 1 (Ω_{LP})
- Stop band edge = min (-Ω_{LS2}, Ω_{LS1}) = 1.3792 (Ω_{LS})
- Tolerances = 0.15 for both Stop band and Pass band
- Both Stop band and Pass band are Equiripple

5.6 Analog Low Pass Filter Transfer function

The magnitude-squared frequency response of an elliptic filter is defined as

$$H^2_{analog,LPF}(j\Omega) = \frac{1}{1+\epsilon^2 R_n^2(\Omega, \Omega_{LS}, \delta_1, \delta_2)}$$

Where R_n is the nth-order elliptic rational function also known as a Chebyshev rational function. The order n, ϵ and will depend on the required specifications. The minimum order required can be calculated using the following steps,

$$\delta_1 = \delta_2 = 0.15$$
$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.3841$$
$$D_2 = \frac{1}{\delta_2^2} - 1 = 43.4444$$
$$\epsilon = \sqrt{D1} = 0.61975$$
$$k = \frac{\Omega_{LP}}{\Omega_{LS}} = \frac{1}{1.3792} = 0.72505$$
$$k' = \sqrt{1-k^2} = 0.6886$$
$$k_1 = \sqrt{\frac{D1}{D2}} = 0.094$$
$$k_1' = \sqrt{1-k_1^2} = 0.99557$$
$$Nmin \geq \frac{KK_1'}{K'K_1}$$

Where K, K', K_1, K_1' are the values of complete elliptic integrals evaluated at k, k', k_1, k_1'. We get, K = 1.8765 , K' = 1.8329, K_1 = 1.5743, K_1' = 3.7566. Using these values, we get the order of the elliptic rational function as 3. Once we get N_{min}, we need to recompute k and Ω_{LS} for all the conditions to be satisfied.

$$Nmin = \lceil 2.443 \rceil = 3$$

Where, Elliptic integral is defined as,

$$u(\phi, k) = \int_0^{\phi} \frac{dy}{\sqrt{1-k^2 sin^2(y)}}$$

and complete elliptic integral is equal to u($\pi/2$, k). i.e,

$$U(k) = \int_0^{\pi/2} \frac{dy}{\sqrt{1 - k^2 sin^2(y)}}$$

Using the above elliptic integral equation we can also write ϕ in terms of u,k. i.e, $\phi(u$,k). This is the inverse of elliptic integral function. Using this we can define Jacobi elliptic sine function and other elliptic functions are defined as,

$$sn(u,k) = sin(\phi(u,k))$$
$$cn(u,k) = cos(\phi(u,k))$$
$$dn(u,k) = \sqrt{1 - k^2 sn^2(u,k)}$$
$$cd(u,k) = \frac{cn(u,k)}{dn(u,k)}$$

The Chebyshev rational function can be represented as,

$$R_n(\Omega) = sn(N_{min} \times sn^{-1}(\Omega, k), k_1)$$
$$R_n(\Omega) = sn(\phi, k_1), \Omega = sn(\phi, k)$$

The zero locations of the Transfer function can be given by,

$$\Omega = \frac{\pm 1}{kcd(iK/N_{min}, k)}, i.e.,$$
$$s = \frac{\pm j}{kcd(iK/N_{min}, k)}$$

Where i = 0, 2, 4, ..., N-1 for Odd N, and i = 1, 3, 5, ... N-1 for Even N.
The pole locations of the transfer function can be given by,

$$1 + \epsilon^2 R_n(s/j)^2 = 0$$

Using the periodicity of sn(u,k), we get

$$sn(N_{min}\phi, +2K_1 i, k_1) = \pm j\frac{1}{\epsilon}$$
$$\phi = (-2Ki + sn^{-1}(\frac{j}{\epsilon}, k_1))/N_{min}$$
$$\Omega = sn(\phi, k)$$

Define,

$$jV_0 = sn^{-1}(\frac{j}{\epsilon}, k_1)/N_{min}$$

Using these equations, location of poles can be found using

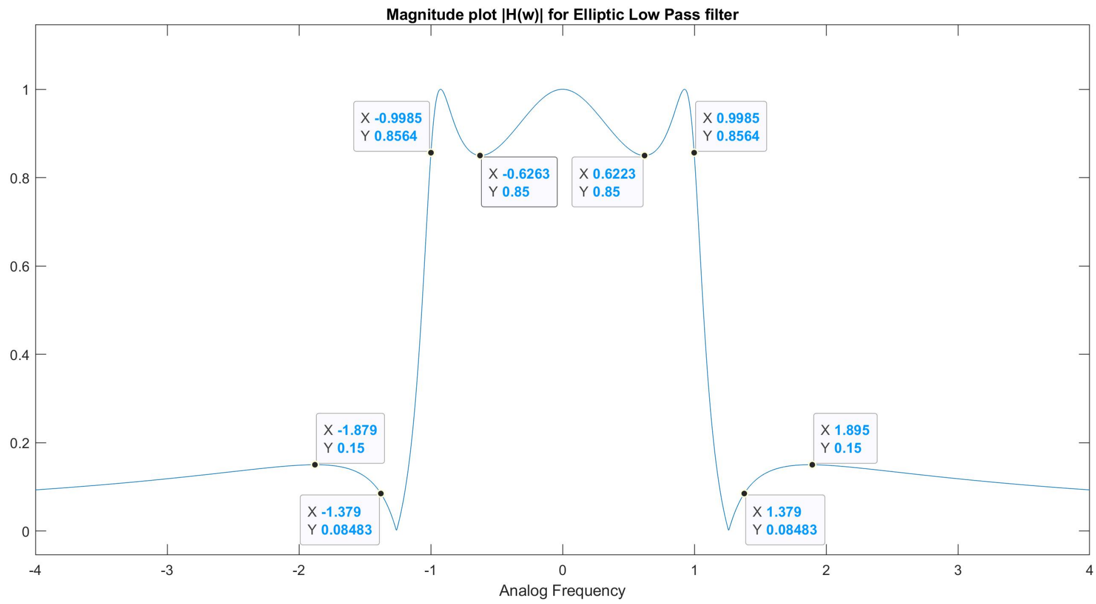
$$s = jsn(Ki/N + jV_0, k) = jcd(Ki/N + (1 - jV_0), k)$$

Note that here j represents complex number and i = 0, 2, 4, ..., N-1 for Odd N, and i = 1, 3, 5, ... N-1 for Even N. From the above equations, the poles and zeroes are found to be:

```
p1=-0.11533-0.9936i
p2=-0.11533+0.9936i
p3=-0.6232
z1=-1.2604i
z2=1.2604i
```

Using these poles and zeroes the transfer function Low pass analog filter is given by,

$$H_{analog,LPF}(s_L) = \frac{0.3925s_L^2 + 0.6235}{s_L^3 + 0.8538s_L^2 + 1.1443s_L + 0.6235}$$



5.7 Analog Band Stop Transfer function

Using the Band Stop Transformation:

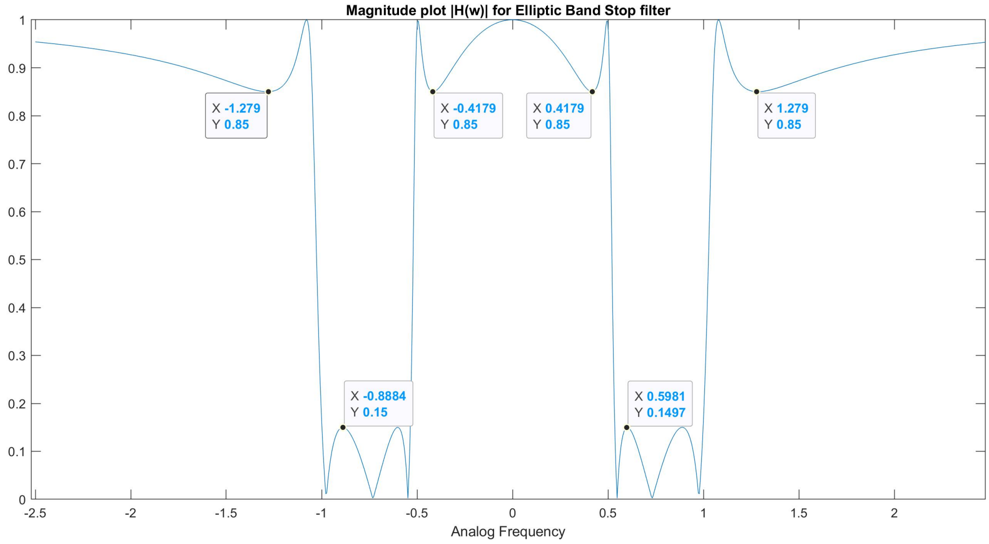
$$s_L = \frac{Bs}{s^2 + \Omega_0^2}$$

Substituting the values B = 0.54 and $\Omega_0 = 0.7313$,

$$s_L = \frac{0.54s}{s^2 + 0.535}$$

Substituting this back into $H_{analog,LPF}$, we get $H_{analog,BSF}$ as

$$\frac{s^6 + 1.7879s^4 + 0.9561s^2 + 0.1529}{s^6 + 0.9911s^5 + 2.0036s^4 + 1.3126s^3 + 1.0715s^2 + 0.2834s + 0.1529}$$



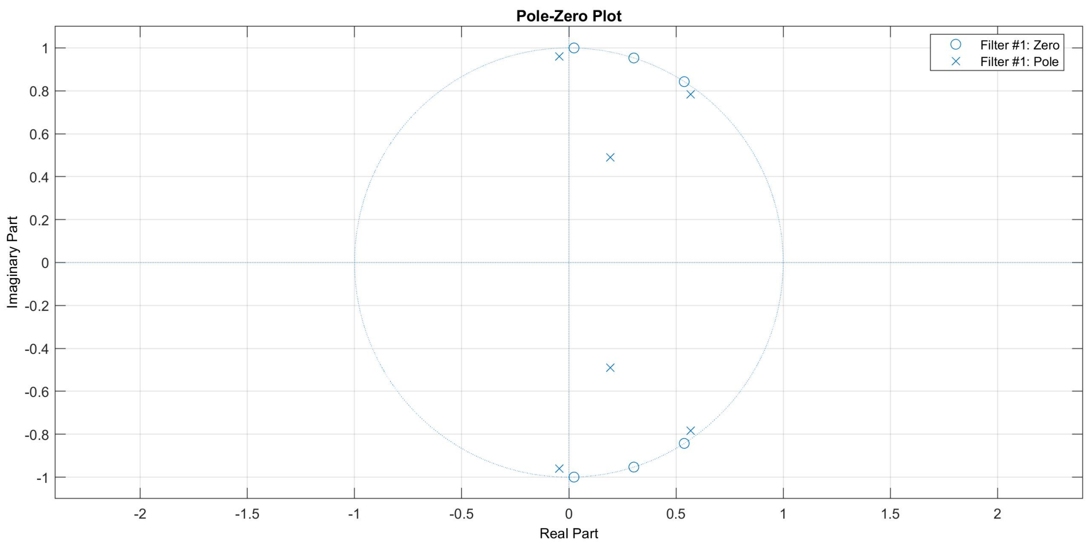
5.8 Discrete time Band Stop Transfer function

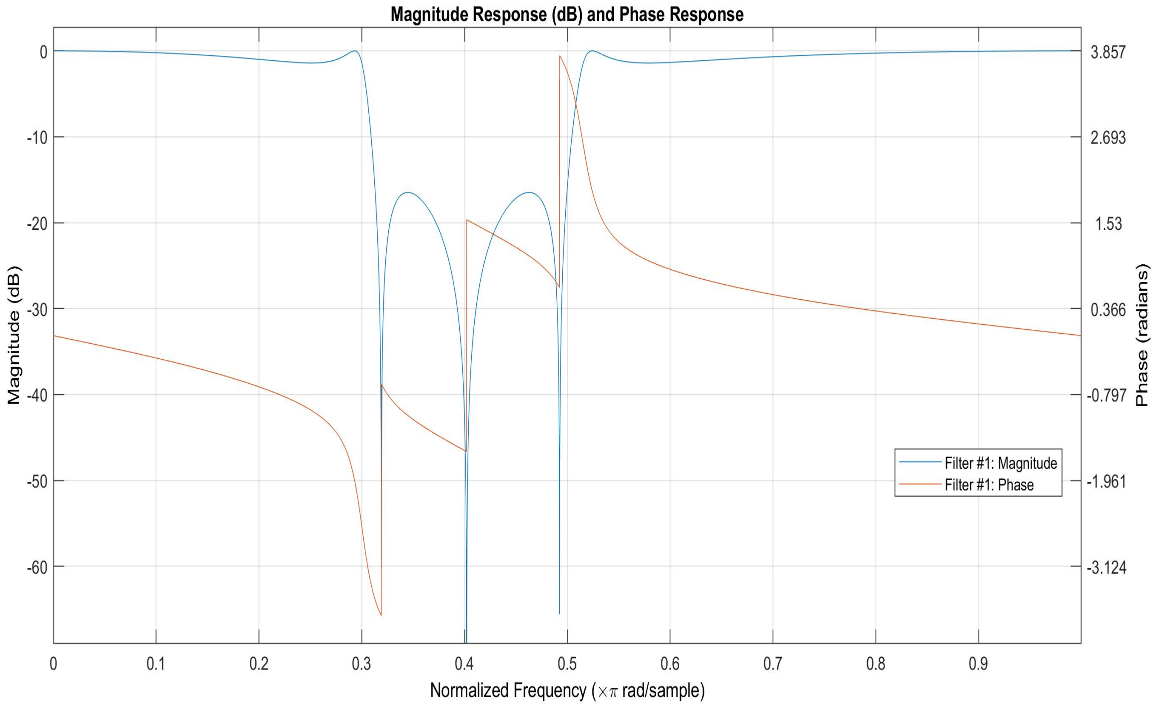
Using the Bilinear Transformation:

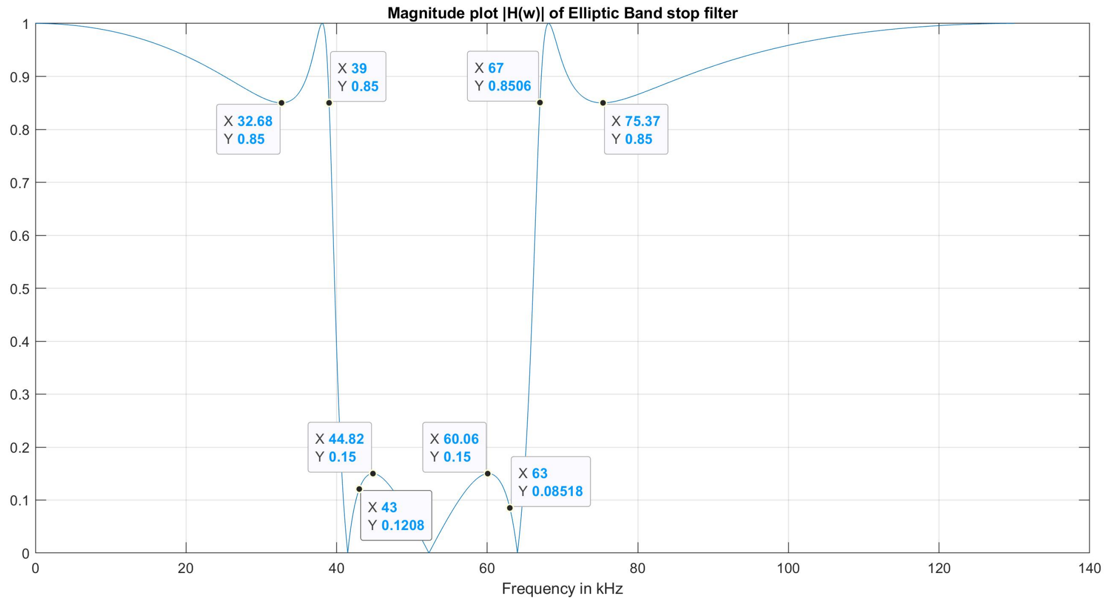
$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Substituting this back into $H_{Analog,BPF}(s)$, we get $H_{digital,BSF}$ as

$$\frac{0.5718 - 0.9899z^{-1} + 0.2135z^{-2} - 1.9977z^{-3} + 2.1350z^{-4} - 0.9899z^{-5} + 0.5718z^{-6}}{1 - 1.4347z^{-1} + 2.4436z^{-2} - 1.9388z^{-3} + 1.7291z^{-4} - 0.604z^{-5} + 0.2408z^{-6}}$$







5.9 Review report

6 Conclusion

7 Appendix