

AI1103-Assignment-1

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<https://github.com/VamsiPreetham-21/AI1103-Assignment---1/blob/main/Assignment1.py>

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Question - 5.5

If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries are chosen independently each value being assumed with probability $1/2$)

Solution :

Total number of entries in the matrix are 4. Let the entries be $(a_{11}, a_{12}, a_{21}, a_{22})$ where the suffixes represent their row and column number respectively. Matrix

$$\mathbf{M} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \quad (0.0.1)$$

Let $X \in \{0,1\}$ be a random variable denoting the possible value for each entry.

$$\Pr(X = 0) = \frac{1}{2} \quad (0.0.2)$$

$$\Pr(X = 1) = \frac{1}{2} \quad (0.0.3)$$

For the determinant of the matrix to be positive $a_{11}a_{22} - a_{12}a_{21} > 0$. Then the entries a_{11}, a_{22} must be one and at least one among a_{12}, a_{21} should be zero.

$$= \Pr(X = 1) \Pr(X = 1) (1 - (\Pr(X = 1) \Pr(X = 1))) \quad (0.0.4)$$

$$= \frac{1}{4} \times \left(1 - \frac{1}{4}\right) \quad (0.0.5)$$

$$= \frac{3}{16} \quad (0.0.6)$$