

AI1103-Assignment-2

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<https://github.com/VamsiPreetham-21/AI1103-Assignment-2/blob/main/Assignment2.py>

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GATE 2012(EC), Q37 :

A fair coin is tossed till a head appeared for the first time. The probability that the number of tosses required is odd,

Solution:

Let X_n define a markov chain where $n \in (0, 1, 2, \dots)$.
1, 2, 3, 4 be four respective states denoting

Symbol	State
1	Odd try
2	Even try
3	Failure
4	Success

TABLE 0: States of the notations used in the Markov Chain

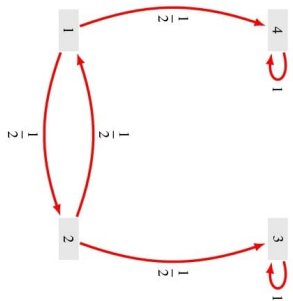


Fig. 0: Theriotical Results

Here 1,2 states are transient and 3,4 states are absorbing. Let \mathbf{P} denote the state transition matrix

for the above markov chain.

$$\mathbf{P} = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (0.0.1)$$

Lemma 0.1. The standard form of the matrix is :

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix} \quad (0.0.2)$$

where \mathbf{I}, \mathbf{O} are Identity and Zero matrices and \mathbf{R}, \mathbf{Q} are other sub-matrices respectively.

After converting the transition matrix into the form of standard matrix we get

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix} \quad (0.0.3)$$

Lemma 0.2. The limiting matrix for absorbing markov chain is,

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{F} & \mathbf{O} \end{bmatrix} \quad (0.0.4)$$

here $\mathbf{F} = (\mathbf{I} - \mathbf{Q})^{-1}$ is called the fundamental matrix of \mathbf{P} .

Solving this leaves us with

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.333 & 0.667 & 0 & 0 \\ 0.667 & 0.333 & 0 & 0 \end{bmatrix} \quad (0.0.5)$$

Here an element \mathbf{P}_{ij} represents the probability of state j starting from state i . Let A be the event of getting the first head to appear on odd numberth trail starting with a odd numberth trail. Then the probability of A is

$$\Pr(A) = p_{14} = 0.667 \quad (0.0.6)$$