AI1103-Assignment-2

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Download all python codes from

https://github.com/VamsiPreetham−21/AI1103−Assignment−2/blog/main/Assignment2.py

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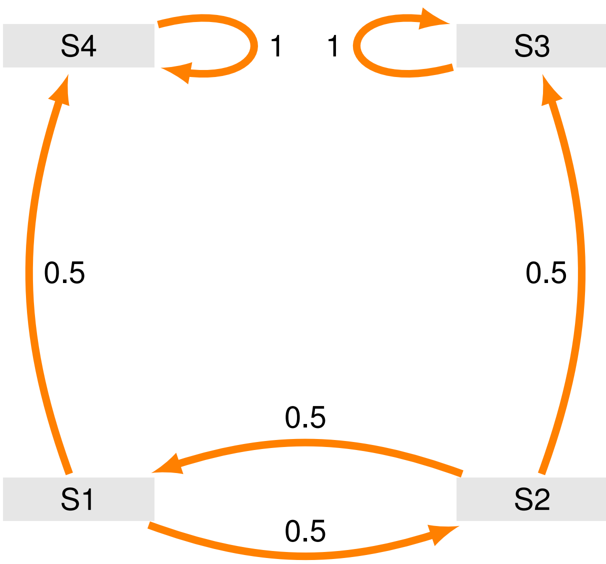
GATE 2012(EC), Q37 :
A fair coin is tossed till a head appeared for the first time. The probability that the number of tosses required is odd,
Solution:
Let X_n define a markov chain where n$\in(0,1,2,...)$. S_1, S_2, S_3, S_4 be four respective states denoting

Odd try	S_1
Even try	S_2
Failure	S_3
Success	S_4



Here 1,2 states are transient and 3,4 states are absorbing. Let P denote the state transition matrix for the above markov chain.

$$P = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The standard form of the matrix is :

$$P = \begin{bmatrix} I & O \\ R & Q \end{bmatrix}$$

where I,O are Identity and Zero matrices and R,Q are other sub-matrices respectively.After converting the transition matrix into the form of standard matrix we get

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix}$$

The limiting matrix for absorbing markov chain is,

$$P = \begin{bmatrix} I & O \\ FR & O \end{bmatrix}$$

here F=$(I-Q)^{-1}$ is called the fundamental matrix of P. Solving this leaves us with

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.333 & 0.667 & 0 & 0 \\ 0.667 & 0.333 & 0 & 0 \end{bmatrix}$$

Here an element P_{ij} represents the probability of state j starting from state i. That means the probability of the first head to appear on odd numberth trail starting with a odd numberth trail is

$$\Pr(A) = p_{14} = 0.667 \tag{0.0.1}$$