AI1103-Assignment-2

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https://github.com/VamsiPreetham−21/AI1103−Assignment−2/blog/main/Assignment2.py

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GATE 2012(EC), Q37 :
A fair coin is tossed till a head appeared for the first time. The probability that the number of tosses required is odd,

Solution:
Let X_n define a markov chain where n$\in(0, 1, 2, ...)$. $1, 2, 3, 4$ be four respective states denoting

Symbol	State
1	Odd try
2	Even try
3	Failure
4	Success

TABLE 0: States of the notations used in the Markov Chain

Here 1,2 states are transient and 3,4 states are absorbing. Let P denote the state transition matrix for the above markov chain.

$$\mathbf{P} = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{0.0.1}$$

Lemma 0.1. The standard form of the matrix is :

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix} \tag{0.0.2}$$

where I,O are Identity and Zero matrices and R,Q are other sub-matrices respectively.

After converting the transition matrix into the form of standard matrix we get

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix} \tag{0.0.3}$$

Lemma 0.2. The limiting matrix for absorbing markov chain is,

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{FR} & \mathbf{O} \end{bmatrix} \tag{0.0.4}$$

here F=$(\mathbf{I} - \mathbf{Q})^{-1}$ is called the fundamental matrix of P.

Solving this leaves us with

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.333 & 0.667 & 0 & 0 \\ 0.667 & 0.333 & 0 & 0 \end{bmatrix} \tag{0.0.5}$$

Here an element $\mathbf{P}_{ij}$ represents the probability of state j starting from state i. Let A be the event of getting the first head to appear on odd numberth trail starting with a odd numberth trail. Then the probability of A is

$$\Pr(A) = p_{14} = 0.667 \tag{0.0.6}$$

Fig. 0: Theoretical and simulated results