

AI1103-Assignment-2

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Download all python codes from

<https://github.com/VamsiPreetham-21/AI1103-Assignment-2/blob/main/Assignment2.py>

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<https://github.com/VamsiPreetham-21/AI1103-Assignment-2/blob/main/Assignment2.tex>

| Symbol | State |
|--------|----------|
| O | Odd try |
| E | Even try |
| F | Failure |
| S | Success |

TABLE 0: Symbols and States used in the Markov chain

GATE 2012(EC), Q37 :

A fair coin is tossed till a head appeared for the first time. The probability that the number of tosses required is odd,

Solution:

Let X_n define a markov chain where $n \in (0, 1, 2, \dots)$.

O,E,F,S be four respective states denoting

Here O,E states are transient and F,S states are absorbing. Let P denote the state transition matrix for the above markov chain.

$$P = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (0.0.1)$$

The standard form of the matrix is :

$$P = \begin{bmatrix} I & O \\ R & Q \end{bmatrix} \quad (0.0.2)$$

where I,O are Identity and Zero matrices and R,Q are other sub-matrices respectively. After converting the transition matrix into the form of standard matrix we get

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix} \quad (0.0.3)$$

The limiting matrix for absorbing markov chain is,

$$P = \begin{bmatrix} I & O \\ FR & O \end{bmatrix} \quad (0.0.4)$$

here $F = (I - Q)^{-1}$ is called the fundamental matrix of P. Solving this leaves us with

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.333 & 0.667 & 0 & 0 \\ 0.667 & 0.333 & 0 & 0 \end{bmatrix} \quad (0.0.5)$$

Here an element P_{ij} represents the probability of state j starting from state i. That means the probability of the first head to appear on odd numberth trail starting with a odd numberth trail is

$$\Pr(A) = p_{14} = 0.667 \quad (0.0.6)$$

Markov chain diagram

