AI1103-Assignment-2

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https://github.com/VamsiPreetham−21/AI1103−Assignment−2/blog/main/Assignment2.py

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GATE 2012(EC), Q37 :
A fair coin is tossed till a head appeared for the first time. The probability that the number of tosses required is odd,

Proof. Let X_n define a markov chain where n$\in(0, 1, 2, ...)$. O,E,F,S be four respective states denoting

Symbol	State
O	Odd try
E	Even try
F	Failure
S	Success

TABLE 0: States of the notations used in the Markov Chain

Here O,E states are transient and F,S states are absorbing. Let P denote the state transition matrix for the above markov chain.

$$P = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{0.0.1}$$

Lemma 0.1. The standard form of the matrix is :

$$P = \begin{bmatrix} I & O \\ R & Q \end{bmatrix} \tag{0.0.2}$$

where I,O are Identity and Zero matrices and R,Q are other sub-matrices respectively.
After converting the transition matrix into the form of standard matrix we get

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix} \tag{0.0.3}$$

Lemma 0.2. The limiting matrix for absorbing markov chain is,

$$P = \begin{bmatrix} I & O \\ FR & O \end{bmatrix} \tag{0.0.4}$$

here F=$(I - Q)^{-1}$ is called the fundamental matrix of P.
Solving this leaves us with

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.333 & 0.667 & 0 & 0 \\ 0.667 & 0.333 & 0 & 0 \end{bmatrix} \tag{0.0.5}$$

Here an element P_{ij} represents the probability of state j starting from state i. That means the probability of the first head to appear on odd numberth trail starting with a odd numberth trail is

$$\Pr(A) = p_{14} = 0.667 \tag{0.0.6}$$

□

Fig. 0: Theoretical and simulated results