

Class 11

Chapter 10 - STRAIGHT LINES

The following problem is question 11 from exercise 10.4

1. Find the equation of the lines through the point (3, 2) which make an angle of 45° with the line $x - 2y = 3$.

Solution:

The given line parameters are

$$\mathbf{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, c = -5 \quad (1)$$

$$\mathbf{P} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2)$$

yielding

$$\mathbf{m}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (3)$$

$$\mathbf{m}_2 = \begin{pmatrix} 1 \\ m \end{pmatrix} \quad (4)$$

where m is defined to be the slope of the line. If the angle between the lines be θ ,

$$\cos \theta = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (5)$$

$$\text{given } \theta = 45^\circ \quad (6)$$

$$\Rightarrow \cos 45^\circ = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (7)$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\left\| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ m \end{pmatrix} \right\|} \quad (8)$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{2 + m}{\sqrt{2^2 + 1} \sqrt{m^2 + 1}} \quad (9)$$

$$\Rightarrow \frac{1}{2} = \frac{m^2 + 4m + 4}{5m^2 + 5} \quad (10)$$

$$\text{or, } 3m^2 - 8m - 3 = 0 \quad (11)$$

yielding

$$m = -\frac{1}{3}, 3 \quad (12)$$

when $m=3$,

The equation of line passing through \mathbf{P} is then obtained as

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{P}) = 0 \quad (13)$$

$$\text{where, } \mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \quad (14)$$

$$\mathbf{n} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad (15)$$

$$\Rightarrow \begin{pmatrix} 3 & -1 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} = 0 \quad (16)$$

$$= 7 \quad (17)$$

$$\Rightarrow \begin{pmatrix} 3 & -1 \end{pmatrix} \mathbf{x} = 7 \quad (18)$$

And, when $m = -\frac{1}{3}$,

The equation of the line passing through \mathbf{P} and having a slope of $-\frac{1}{3}$ is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{P}) = 0 \quad (19)$$

$$\mathbf{n} = \begin{pmatrix} -\frac{1}{3} \\ -1 \end{pmatrix} \quad (20)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (21)$$

$$\Rightarrow \begin{pmatrix} 1 & 3 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} = 0 \quad (22)$$

$$= 9 \quad (23)$$

$$\Rightarrow \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = 9 \quad (24)$$

Therefore, the equations of the lines are

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \mathbf{x} = 7 \quad \text{and} \quad \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = 9. \quad (25)$$

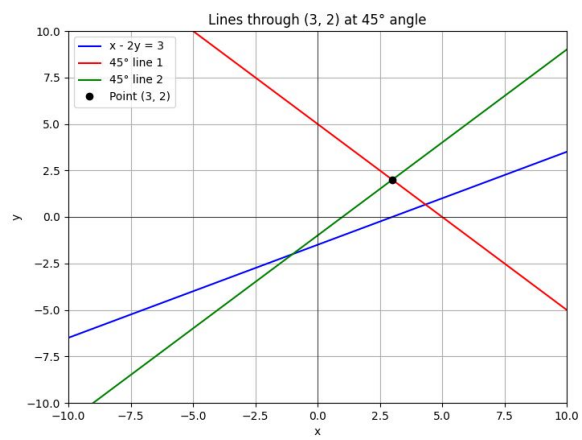


Figure 1: STRAIGHT LINES