Class 11

Chapter 10 - STRAIGHT LINES

The following problem is question 11 from exercise 10.4

1. Find the equation of the lines through the point (3, 2) which make an angle of 45° with the line x − 2y = 3.

Solution:

The given line parameters are

$$\mathbf{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, c = -5 \tag{1}$$

yielding

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2}$$

$$\mathbf{m} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{3}$$

where m is defined to be the slope of the line. If the angle between the lines be θ,

$$\cos\theta = \frac{\begin{pmatrix} m_1 \end{pmatrix}^\top \begin{pmatrix} m_2 \end{pmatrix}}{\left\|\begin{pmatrix} m_1 \end{pmatrix}\right\| \left\|\begin{pmatrix} m_2 \end{pmatrix}\right\|} \tag{4}$$

$$given \quad \theta = 45° \tag{5}$$

$$\implies cos45° = \frac{\begin{pmatrix} m_1 \end{pmatrix}^\top \begin{pmatrix} m_2 \end{pmatrix}}{\left\|\begin{pmatrix} m_1 \end{pmatrix}\right\| \left\|\begin{pmatrix} m_2 \end{pmatrix}\right\|} \tag{6}$$

$$\implies \frac{1}{\sqrt{2}} = \frac{\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\left\|\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right\| \left\|\begin{pmatrix} 1 \\ m \end{pmatrix}\right\|} \tag{7}$$

$$\implies \frac{1}{\sqrt{2}} = \frac{2+m}{\sqrt{2^2+1}\sqrt{m^2+1}} \tag{8}$$

$$\implies \frac{1}{2} = \frac{m^2+4m+4}{5m^2+5} \tag{9}$$

$$\text{or, } 3m^2 - 8m - 3 = 0 \tag{10}$$

yielding

$$m = -\frac{1}{3}, 3 \tag{11}$$

when m=3,
The equation of line passing through $\mathbf{P}$ (3,2) is then obtained as

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{P}) = 0 \tag{12}$$

$$\text{where,}\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \tag{13}$$

$$\mathbf{n} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{14}$$

$$\implies \begin{pmatrix} 3 & -1 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} = 0 \tag{15}$$

$$= 7 \tag{16}$$

$$\implies \begin{pmatrix} 3 & -1 \end{pmatrix} \mathbf{x} = 7 \tag{17}$$

And, when $m = -\frac{1}{3}$,
The equation of the line passing through $\mathbf{P}$ (3,2) and having a slope of $-\frac{1}{3}$is

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{P}) = 0 \tag{18}$$

$$\mathbf{n} = \begin{pmatrix} -\frac{1}{3} \\ -1 \end{pmatrix} \tag{19}$$

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{20}$$

$$\implies \begin{pmatrix} 1 & 3 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} = 0 \tag{21}$$

$$= 9 \tag{22}$$

$$\implies \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = 9 \tag{23}$$

Therefore,the equations of the lines are

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \mathbf{x} = 7 \quad and \quad \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = 9. \tag{24}$$

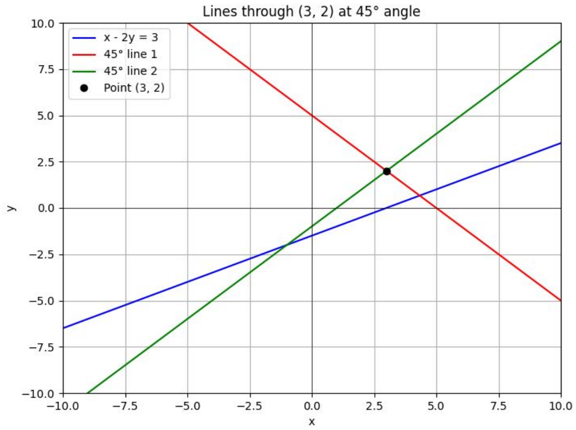

Figure 1: STRAIGHT LINES