Class 11

Chapter 10 - STRAIGHT LINES

The following problem is question 11 from exercise 10.4

1. Find the equation of the lines through the point (3, 2) which make an angle of 45° with the line x – 2y = 3.

Solution:

The given line can be written as

$$y = \frac{1}{2}x - \frac{3}{2} \tag{1}$$

which is of the form

$$y = mx + c \tag{2}$$

Thus,slope of the given line

$$m_2 = \frac{1}{2} \tag{3}$$

It is given that the angle between the required line and line x-2y=3 is 45°.we know that if θ is acute angle between lines l_1 and l_2 with slopes m_1 and m_2 respectively,then

$$tan\theta = \frac{|m_1 - m_2|}{1 + m_1 m_2} \tag{4}$$

$$\implies tan45^\circ = \frac{|m_1 - m_2|}{1 + m_1 m_2} \tag{5}$$

$$\implies 1 = \left|\frac{\frac{1}{2} - m_1}{1 + \frac{m_1}{2}}\right| \tag{6}$$

$$\implies 1 = \left|\frac{\frac{1-2m_1}{2}}{\frac{2+m_1}{2}}\right| \tag{7}$$

$$\implies 1 = \left|\frac{1 - 2m_1}{2 + m_1}\right| \tag{8}$$

$$\implies 1 = \pm\left(\frac{1 - 2m_1}{2 + m_1}\right) \tag{9}$$

$$\implies 1 = \left(\frac{1 - 2m_1}{2 + m_1}\right) \quad and \quad 1 = -\left(\frac{1 - 2m_1}{2 + m_1}\right) \tag{10}$$

$$2 + m_1 = 1 - 2m_1 \quad or \quad 2 + m_1 = -1 + 2m_1 \tag{11}$$

$$\implies m_1 = -\frac{1}{3} \quad or \quad m_1 = 3 \tag{12}$$

Now,when m_1=3 then,
The equation of line passing through (3,2) and having a slope of 3 is:

$$y - 2 = 3(x - 3) \tag{13}$$

$$\implies y - 2 = 3x - 9 \tag{14}$$

$$3x - y = 7 \tag{15}$$

And, when $m_1 = -\frac{1}{3}$ The equation of the line passing through (3,2) and having a slope of$-\frac{1}{3}$is:

$$y - 2 = -\frac{1}{3}(x - 3) \tag{16}$$

$$3y - 6 = -x + 3 \tag{17}$$

$$x + 3y = 9 \tag{18}$$

Therefore,the equations of the lines are

$$3x - y = 7 \quad and \quad x + 3y = 9. \tag{19}$$

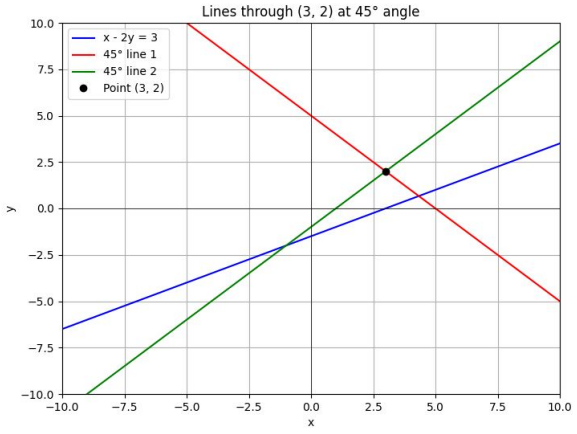

Figure 1: STRAIGHT LINES