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NOTE

Edge Preserving Smoothing

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A new smoothing algorithm is proposed, which looks for the most homogeneous neighborhood area around each point in a picture, and then gives each point the average gray level of the selected neighborhood area. It removes noise in a flat region without blurring sharp edges, nor destroying the details of the boundary of a region. This smoothing also has the ability to sharpen blurred edges.

1. INTRODUCTION

There have been many papers on the subject of smoothing of a digital image [1–3]. A basic difficulty with smoothing in these papers is that, if applied without care, it tends to blur any sharp edges which happen to be present. The smoothing method presented in this paper attempts to resolve the conflict between noise elimination and edge degradation. It looks for the most homogeneous neighborhood around each point in a picture, and then gives each point the average gray level of the selected neighborhood area. Noise in a picture is removed by the repeated usage of this method, while the edges remain sharp.

2. BLURRING EFFECTS OF SMOOTHING

Recently Tsuji *et al.* have proposed a smoothing method which gives the point (x, y) the average gray level of the most homogeneous neighborhood among the five rectangular neighborhoods in Fig. 1 [3]. However, it does not yield a good result if applied to a complex-shaped region, because this method uses rectangular areas as the neighborhoods around (x, y). For example, a wedge-shaped portion of a region is apt to be merged in the surrounding regions, or sometimes it becomes an independent region with a false gray level. Furthermore, an $N \times N$ region, whose size is the same as that of the rectangular neighborhoods, cannot survive after several iterations of the smoothing operation.

Let us consider a simple example. Suppose that we smooth a 3×3 region (Fig. 2) by using five 3×3 rectangular neighborhoods. At the points a, c, e, g, and i there exists a rectangle which is completely included in the 3×3 region, while at b, d, f, and h all the five 3×3 rectangles cross over the boundaries, and contain parts of both the region and the surrounding region. Therefore the new

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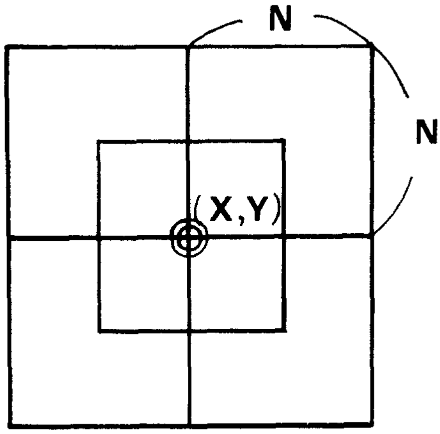


FIG. 1. Five rectangular neighborhood areas around point (X, Y) [3].

gray levels given to these points become smaller (larger) than the original value of the region. If this process is iterated several times, the gray levels of the points in the 3×3 region approach that of the surrounding region, and finally this region is smoothed out.

In order to avoid this effect it is necessary to determine a new shape for the regions in which the local average is taken and also a new rule for iterating the algorithm.

3. EDGE-PRESERVING SMOOTHING

The procedure of our edge preserving smoothing is as follows:

(1) Rotate an elongated bar mask around a point (x, y). (Fig. 3)

(2) Detect the position of the mask for which the variance of the gray level is minimum.

(3) Give the average gray level of the mask at the selected position to the point (x, y).

(4) Apply steps (1) to (3) to all points in the picture.

(5) Iterate the above process until the gray levels of almost all points in the picture do not change.

a	b	c
d	e	f
g	h	i

FIG. 2. A 3×3 rectangular region; smoothing over rectangular neighborhoods smooths out this small region (see text).

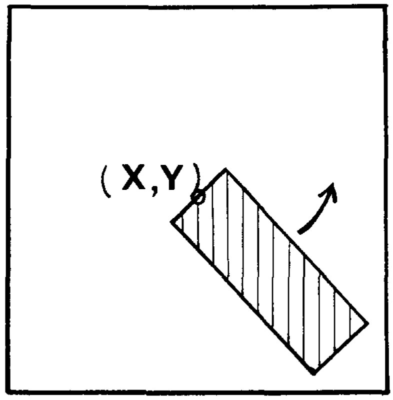


FIG. 3. Mask selection for edge preserving smoothing; rotate an elongated bar mask around (X, Y) and detect the position of the mask for which the variance of the gray level is minimum.

In order to remove the noise without blurring sharp edges, averaging must not be applied to an area which contains an edge because it makes the edge blurred. Thus the most homogeneous neighborhood is to be found around the point to be smoothed. If an area contains a sharp edge, the variance of the gray level in that area becomes large. Therefore we can use the variance as a measure of non-homogeneity of an area.

Suppose that a picture has two regions $R1$ and $R2$ whose means and variances are $(0, \sigma_1^2)$ and (m, σ_2^2), respectively. Let a point (x, y) belong to $R1$. If (x, y) is located in the central part of $R1$, its gray level approaches the average gray level of $R1$ (in this case 0) after several iterations of smoothing. On the other hand, if (x, y) is near the boundary, there exist two kinds of neighborhoods, one of which is completely included in $R1$ while the other includes both parts of $R1$ and $R2$. The variance of the former is about σ_1^2. The variance σ^2 of the latter can be calculated as follows. Let $N1$ and $N2$ denote the numbers of points in $R1$ and

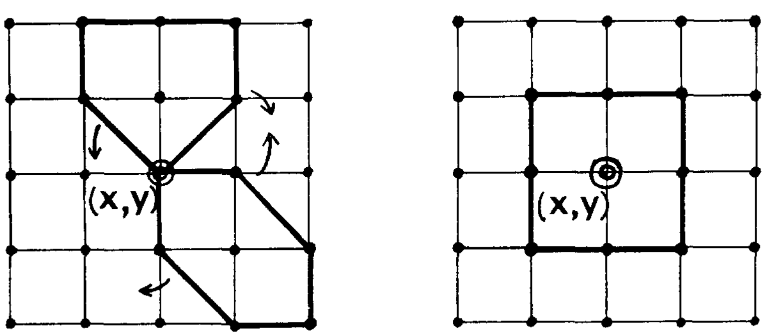


FIG. 4. Discrete realization of the bar masks; four pentagonal and four hexagonal masks have sharp corners at the point (X, Y). A 3×3 rectangular mask is used to smooth a small region (see text).

$R2$, respectively, which are contained in this neighborhood. Then the variance σ^2 is about

$$\sigma^2 \approx \frac{1}{N}\left\{\sum_{i=1}^{N_1}\left(x_i^1 - \frac{N_2}{N}m\right)^2 + \sum_{j=1}^{N_2}\left(x_j^2 - \frac{N_2}{N}m\right)^2\right\}$$

where $N = N1 + N2$, x_i^1 and x_j^2 denote the gray levels of the points belonging to $R1$ and $R2$, respectively, and $\approx$ means nearly equal. Expanding the right-hand side,

$$\sigma^2 \approx \frac{1}{N}\left\{N_1\sigma_1^2 + N_2\sigma_2^2 + \frac{N_1N_2}{N}m^2\right\}$$

where we have used

$$\sum_{i=1}^{N_1} (x_i^1)^2 \approx N_1\sigma_1^2, \qquad \sum_{i=1}^{N_1} x_i^1 \approx 0, \qquad \sum_{j=1}^{N_2} (x_j^2 - m)^2 \approx N_2\sigma_2^2$$

and

$$\sum_{j=1}^{N_2} x_j^2 \approx N_2 m.$$

If $\sigma^2 < \sigma_1^2$, that is, $\sigma_2^2 + (N_1/N)m^2 < \sigma_1^2$, the neighborhood containing parts of both $R1$ and $R2$ is selected. The boundary between $R1$ and $R2$ is then blurred by the averaging operation over this neighborhood. In most pictures, however, it is reasonable to assume $\sigma_1^2 \approx \sigma_2^2$ and $\sigma^2 > \sigma_1^2$, so that the correct neighborhood is selected. Even if $R1$ is very noisy and σ_1^2 is large, σ^2 is sufficiently larger than σ_1^2 providing that the difference of the average gray levels of the two regions, i.e. m, is large.

4. ACTUAL REALIZATION FOR DISCRETE PICTURE DATA

The nine masks in Fig. 4 are the discrete realization of the bar masks of the smallest size for edge-preserving smoothing of a digital picture. Using pentagonal and hexagonal corners at the point (x, y), we can avoid the degradation of sharp edges, and can find the homogeneous neighborhood (i.e., the neighborhood with no sharp edge in it) even if the point (x, y) is located at a sharp angle of a complex-shaped region. Thus we can smooth a region without blurring sharp edges nor destroying the shape of the boundary.

The 3×3 rectangular mask in Fig. 4 is used to smooth a small region. Suppose that we apply this smoothing to the 3×3 rectangular region in Fig. 2. At the points a, b, c, d, f, g, h, and i, there exists a pentagonal or hexagonal mask which is completely included in that region, while at e all these eight masks contain points of both the region and the surrounding one. If we add a 3×3 mask as a neighborhood of the point (x, y), then we can smooth even a 3×3 region without destroying the shape. The variances of these nine masks are compared with each other, and the average gray level of the least-variance mask is given to the point (x, y).

Generally, in order to reduce strongly the amplitude of the noise fluctuation, the neighborhood for averaging should be large. Smoothing by averaging over a

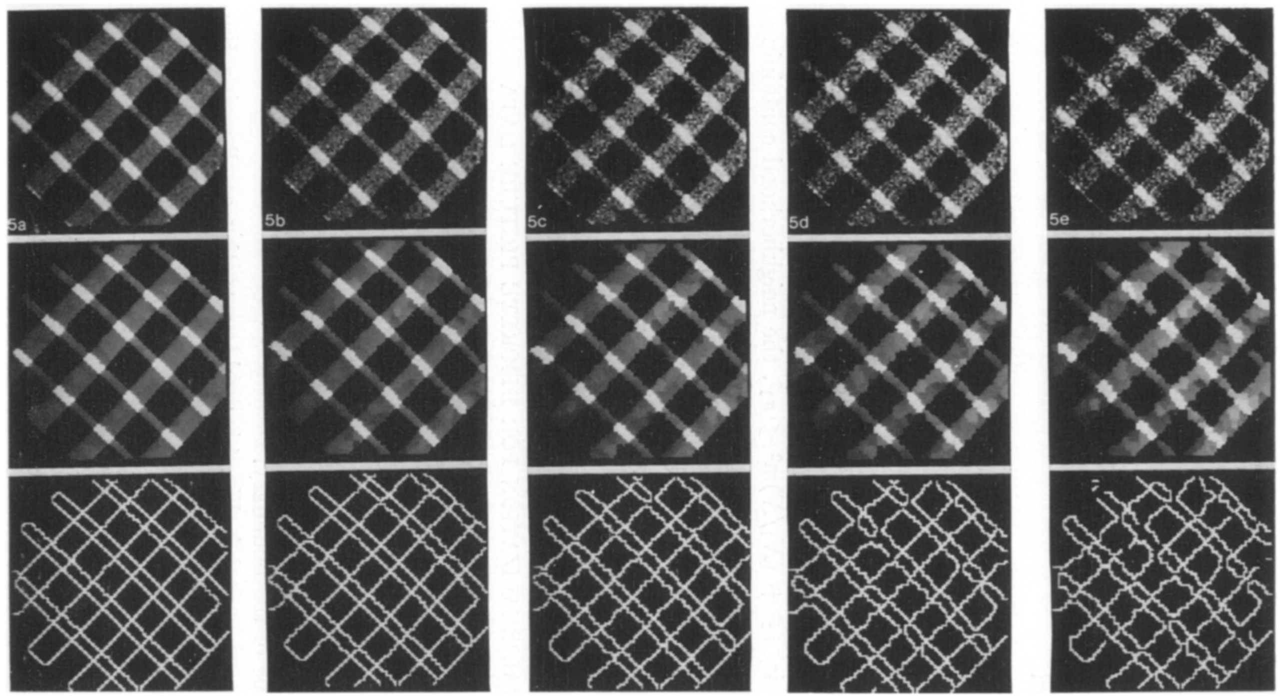


FIG. 5. Result of the edge preserving smoothing of a simple artificial pattern which has three different regions whose average gray levels are 48, 108, and 168, respectively. The pictures on the top row are corrupted by Gaussian noise with standard deviations 10, 20, 30, 40, and 50, respectively ((a) to (e)). The pictures on the middle row are the results of the edge preserving smoothing after ten iterations. The pictures on the bottom row are the results of ordinary differentiation and thresholding of the smoothed patterns. Isolated noise is clearly removed and the boundaries of regions are almost completely preserved.

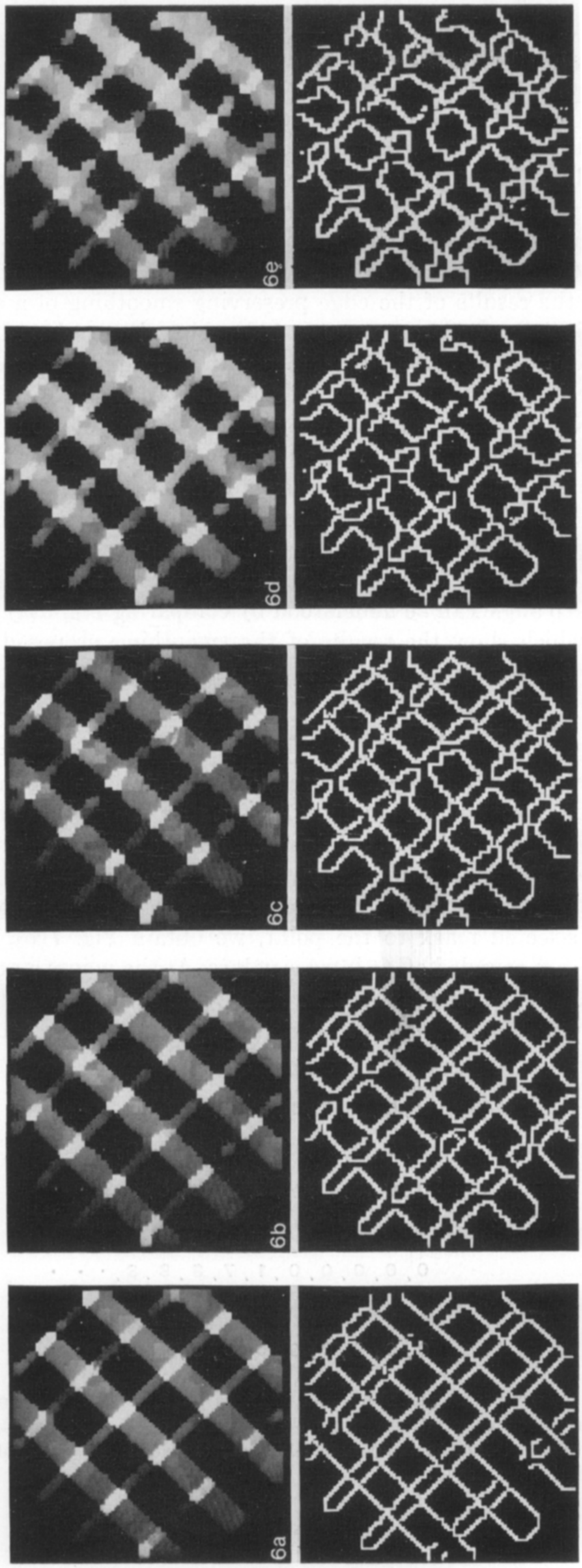


FIG. 6. Result of smoothing the noisy artificial patterns of Fig. 5(a) to (e) by the method proposed in [3] (upper row). Result of the same differentiation and thresholding as in Fig. 5 (bottom row). By this smoothing, sharp corners of regions are destroyed and several regions are smoothed out.

large neighborhood, however, smooths out small regions and the details of the boundaries. When we use small neighborhoods of area 5×5 around each point in a picture, as discussed above, in order to preserve the details of boundaries, we cannot strongly reduce the noise. However the iteration of this smoothing operation can give the same effect as averaging over a large neighborhood at once. We apply this process repeatedly until the gray levels of almost all points in a picture do not change.

Figure 5 shows the results of the edge preserving smoothing of a simple pattern, which is artificially made on a 100×100 grid and quantized to 256 gray levels. The picture has three different regions with average gray levels 48, 108, and 168, respectively. Gaussian noise is added to this pattern with the standard deviations 10, 20, 30, 40, and 50, respectively. The noisy pictures on the top row of Fig. 5 are made by this operation. The pictures on the middle row show the results of the edge preserving smoothing after ten iterations. The pictures on the bottom row show the edges of the smoothed pictures as the result of ordinary differentiation and thresholding. The edges and the angles of each region are preserved almost completely even if the picture is very noisy. This ability of smoothing at a sharp angle can be understood by comparing Fig. 5(a) to (e) with Fig. 6(a) to (e), which show the results of the smoothing proposed in [3] by using the five 3×3 square neighborhoods shown in Fig. 1.

5. SHARPENING OF A BLURRED EDGE

This smoothing program not only removes the noise but also sharpens a blurred edge. This effect can be understood by considering a simple one-dimensional example. Figure 7(a) shows a one-dimensional digital blurred edge. The numbers under each gray level denote the mean and the variance of the 3×1 mask centered at each point. If we select the minimum variance mask and give the average gray level of the selected mask to the point, we obtain Fig. 7(b), where the average gray levels are rounded off to integer values. At the points a and b on the blurred edge we have the new gray levels 1 and 7, respectively, which approach the gray levels of the left and right flat regions. Figure 7(c) shows the result of

							a	b				
(a)	GRAY LEVEL	0,	0,	0,	0,	1,	3,	5,	8,	8,	8,	· · ·
	MEAN	0,	0,	0,	1/3,	4/3,	3,	16/3,	7,	8,	8,	· · ·
	VARIANCE (×3)	0,	0,	0,	4/9,	14/3,	8,	38/3,	6,	0,	0,	· · ·
(b)		0,	0,	0,	0,	0,	1,	7,	8,	8,	8,	· · ·
(c)		0,	0,	0,	0,	0,	0,	8,	8,	8,	8,	· · ·

FIG. 7. Sharpening of a blurred one-dimensional edge. (a) A blurred one-dimensional edge; numbers under each gray level denote the mean and the variance of the 3×1 mask centered at each point. (b) Result of smoothing the one-dimensional blurred edge of (a). (c) Result of smoothing (b); the blurred edge is sharpened.

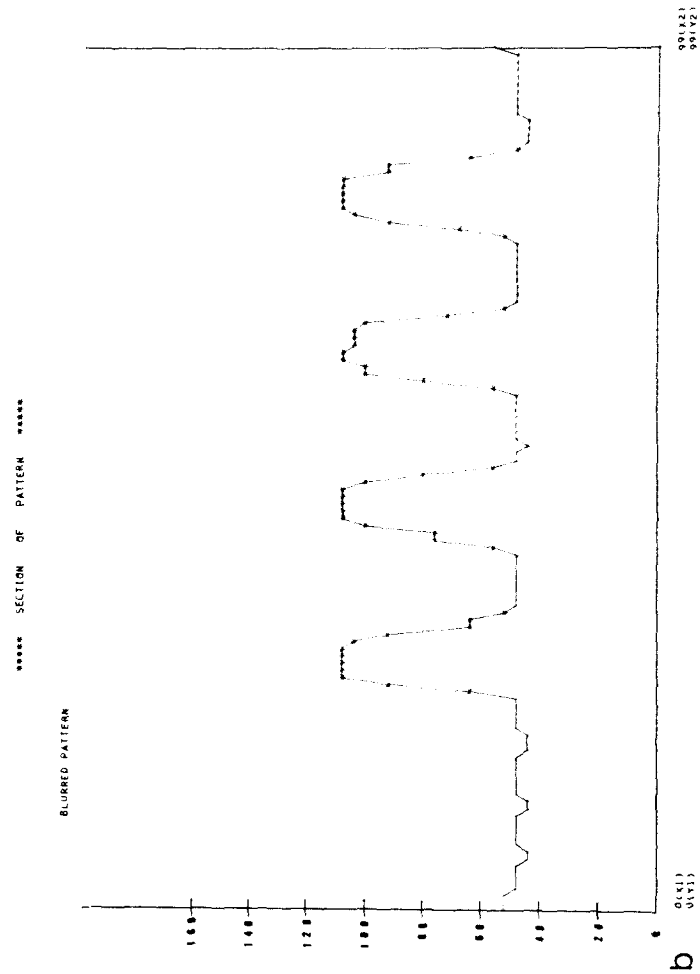
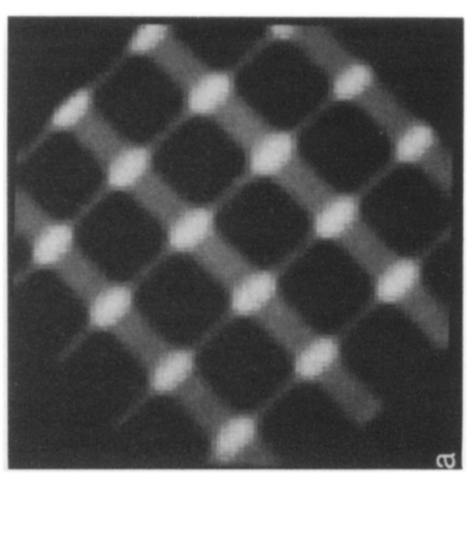

***** SECTION OF PATTERN *****
BLURRED PATTERN
160
140
120
100
80
60
40
20
0
0(X1)
0(Y1)
99(X2)
99(Y2)
b

FIGURE 8


a

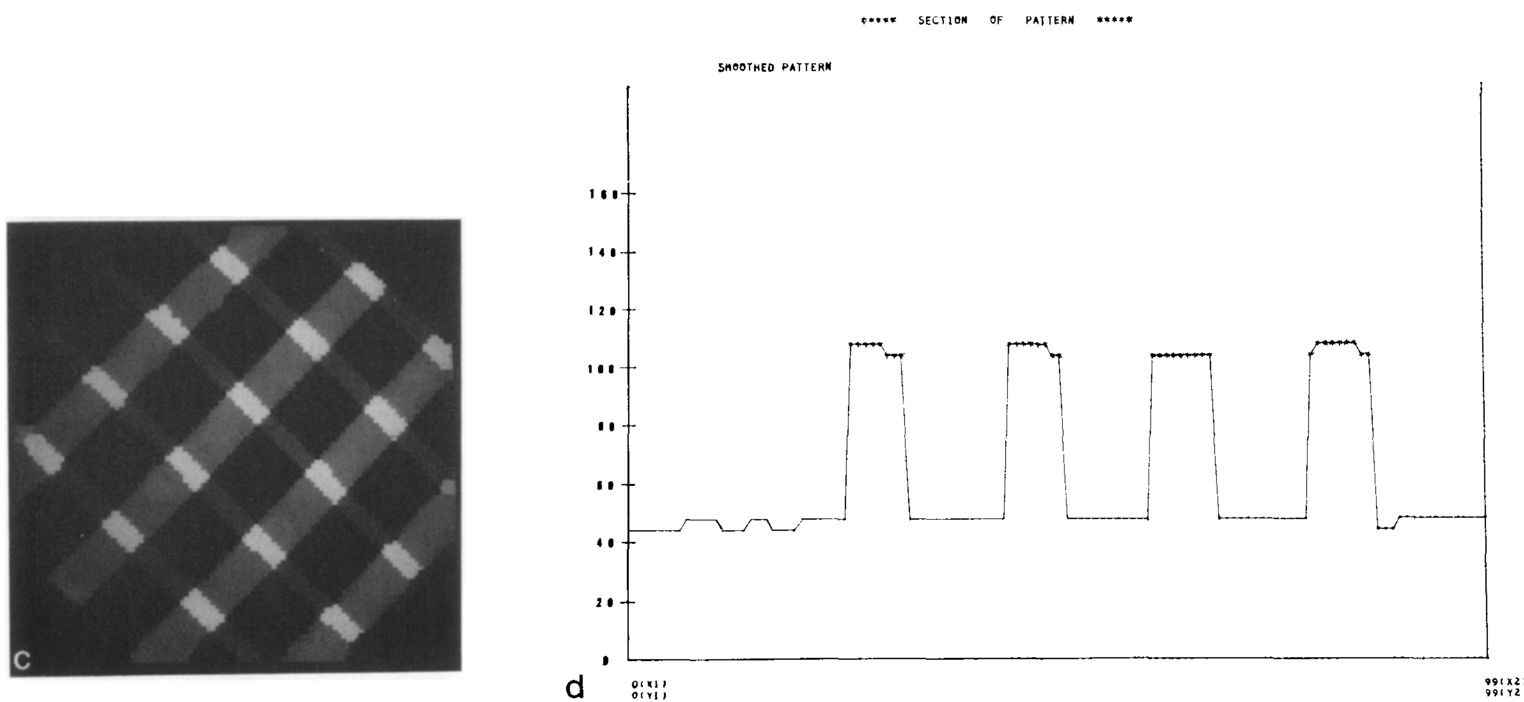


FIG. 8. (a) A blurred artificial pattern; the noisy artificial pattern of Fig. 5(a) is blurred by averaging over a 5×5 neighborhood at each point in the picture. (b) The cross section of (a) along the diagonal line from the upper left to the lower right corner. (c) Result of edge preserving smoothing of (a) after ten iterations. (d) The cross section of (c) along the diagonal line. Blurred edges are sharpened and the shapes of all regions are completely preserved.

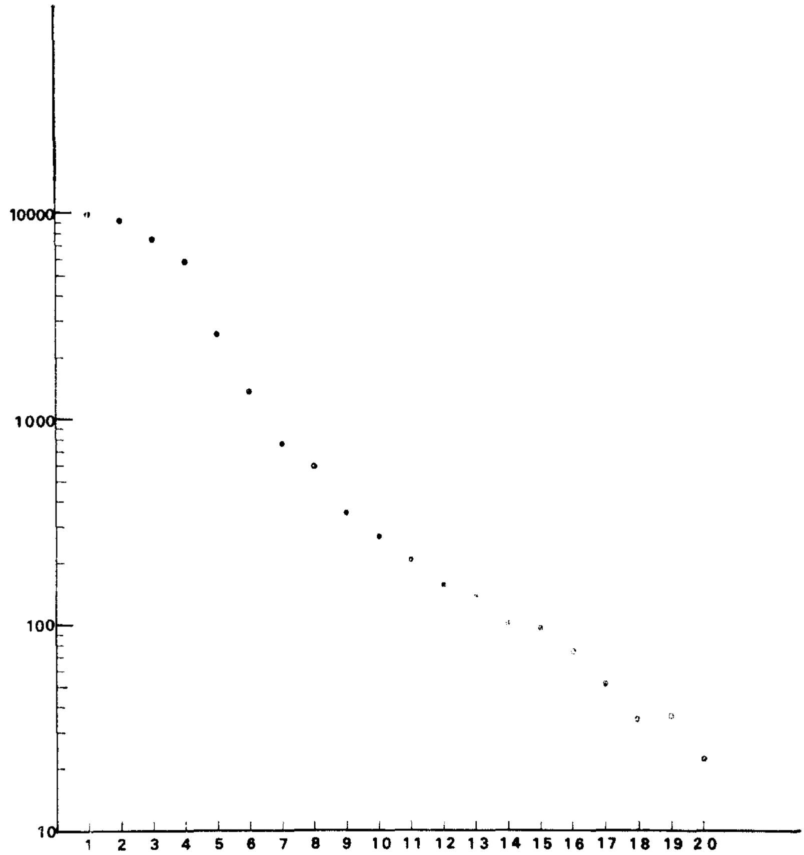


FIG. 9. Convergence of edge preserving smoothing of the noisy artificial pattern of Fig. 5(c); the horizontal axis shows the number of iterations and the vertical axis shows the number of points whose gray levels are changed by smoothing (on a log scale).

smoothing Fig. 7(b), where the original blurred edge is completely sharpened. This function works in essentially the same way on two-dimensional digital pictures. Figure 8(a) and (b) show blurred artificial patterns and their cross sections along the diagonal line. Figure 8(c) and (d) show the results of the smoothing and the corresponding cross sections. Blurred edges are sharpened.

6. CONVERGENCE

The fluctuation of the gray level is gradually reduced by several iterations of smoothing. Once a point has a neighborhood of constant gray level, its gray level is never changed by smoothing. Therefore the number of points whose gray levels are changed by smoothing will gradually decrease to zero. Figure 9 shows a typical example of this phenomenon for the artificial pattern of Fig. 5, where the horizontal axis shows the number of iterations and the vertical axis the number of points whose gray levels are changed. Though this curve does not decrease to zero exactly, the gray level changes after ten iterations are very small, i.e., 1 or 2. Therefore the process can be regarded as converged. Figure 10 shows the results of 1, 2, 3, 4, and 5 iterations of smoothing. In this simple artificial pattern

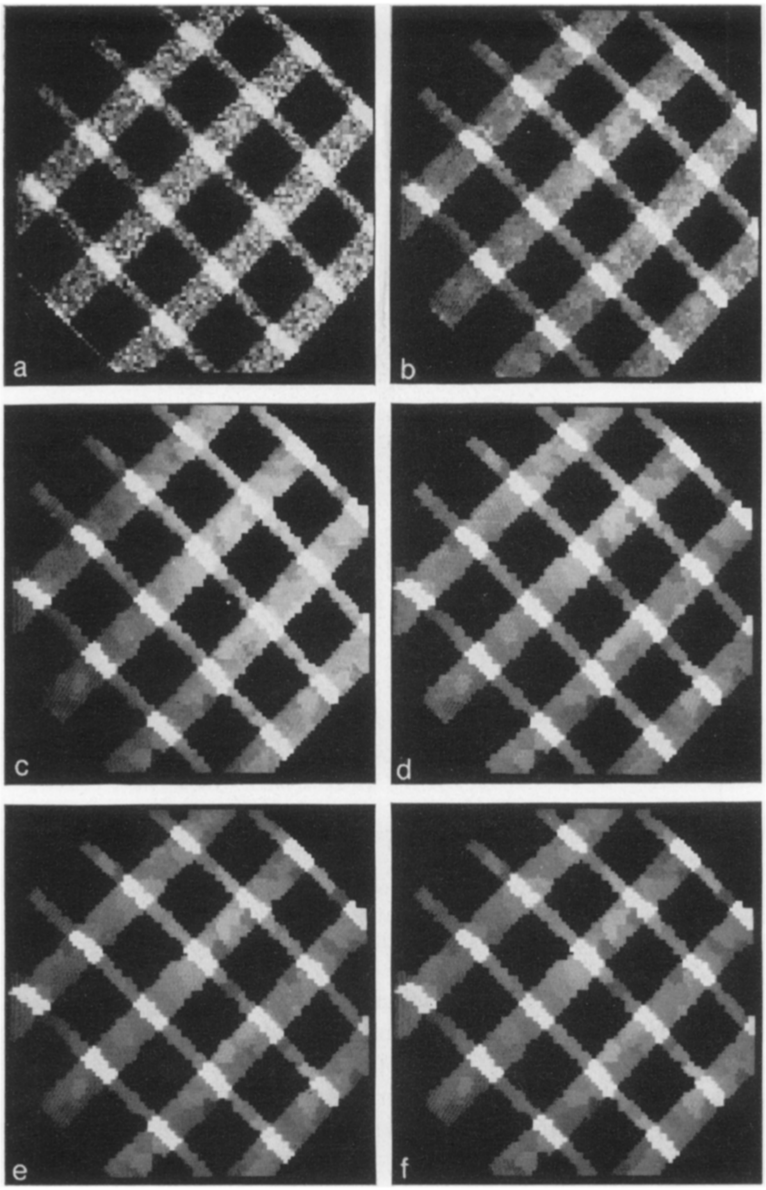


FIG. 10. (a) A noisy artificial pattern (the same as Fig. 5(c)). (b)–(f) Results of edge preserving smoothing after 1, 2, 3, 4, and 5 iterations, respectively.

several iterations are sufficient for practical use. The number of iterations needed for convergence depends on the amplitude of noise fluctuation and on the shapes of regions in a picture.

7. APPLICATION TO A NATURAL SCENE

We have applied this smoothing to a picture of an aerial photograph in order to see its effect on a complex natural scene. Figure 11 shows the original picture

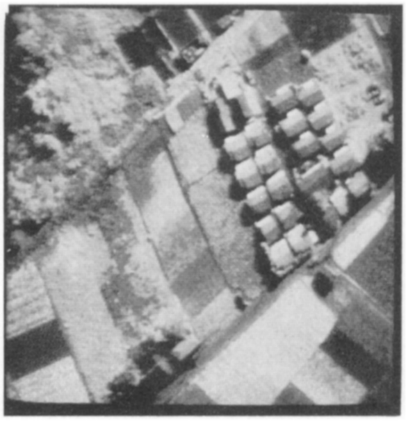


FIG. 11. A digitized aerial photograph.

FIG. 12. Result of ordinary differentiation and thresholding of Fig. 11.

of an aerial photograph which is sampled on a 256 × 256 grid and quantized to 256 gray levels. This picture contains many small houses and textured areas such as woods (upper left side of the scene). Figure 12 shows the result of ordinary differentiation and thresholding of Fig. 11, in which we cannot identify the boundaries of regions because there is much isolated noise. Figure 13 shows the result of smoothing, and Fig. 14 is the result of the same differentiation and thresholding as in Fig. 12. Figure 15(a), (b) show the cross sections of Fig. 11, 13, respectively, along the same horizontal line, (120, 200) to (239, 200). Almost all isolated noise is clearly removed, and all edges are sharpened. As the complex boundaries of houses in the residential area keep their shapes completely, we can identify the roof of each house.

This experiment shows that we can use this smoothing method as preprocessing for the segmentation of complex natural scenes.

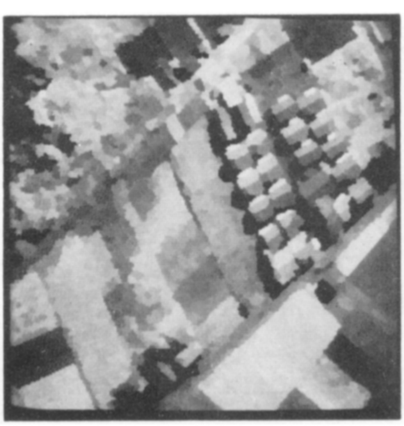


FIG. 13. Result of edge preserving smoothing of Fig. 11 after 20 iterations.

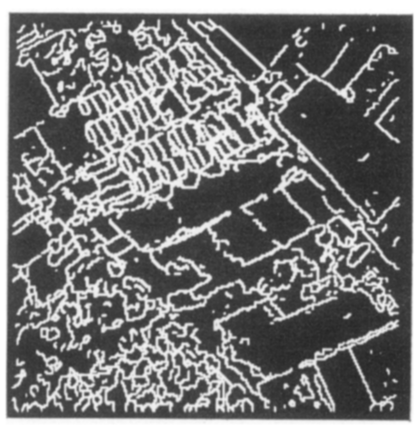


FIG. 14. Result of the same differentiation and thresholding of Fig. 13 as in Fig. 12.

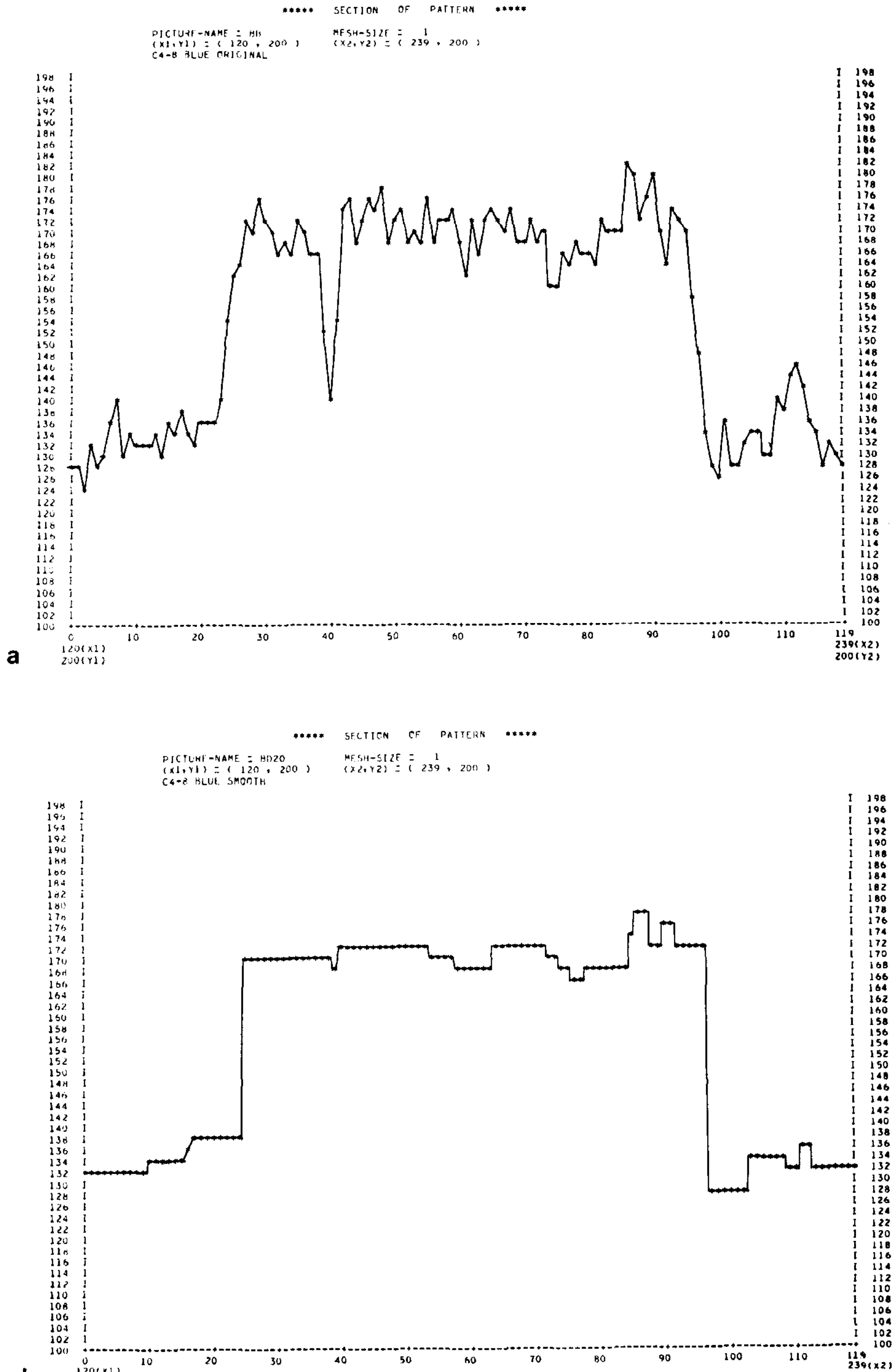


FIG. 15 (a), (b). Cross sections of Figs. 11, 13 along the horizontal line (120, 200) to (239, 200).

8. CONCLUSION

A new smoothing method is presented which removes noise without blurring a sharp edge, nor destroying the details of a boundary of a region. This smoothing also has the ability to sharpen blurred edges. By using this smoothing method we can easily extract a homogeneous region or a boundary between regions even in a complex natural scene. Therefore this can be used as a powerful preprocessing method for the segmentation of a picture.

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