Clasa a XII-a C

- OPTIONAL -

Culegere BAC, pag 71

7) $b = \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} \in \mathbb{N}.$

$b = \frac{\sqrt{1}-\sqrt{2}+\sqrt{2}-\sqrt{3}+\dots+\sqrt{7}-\sqrt{8}+\sqrt{8}-\sqrt{9}}{-1} = \frac{\sqrt{1}-\sqrt{9}}{-1} =$

$b = \frac{1-3}{-1} = \frac{-2}{-1} = 2 \in \mathbb{N}$

9) $a, b \in \mathbb{Q}$; $(\sqrt{2}+\sqrt{6})^2 = a - b\sqrt{3} \Rightarrow$

$2 + 2\sqrt{2}\cdot\sqrt{6} + 6 = a - b\sqrt{3} \Rightarrow 8 + 2\sqrt{12} = a - b\sqrt{3}$

$\Rightarrow 8 + 4\sqrt{3} = a - b\sqrt{3} \Rightarrow \begin{cases} a = 8 \\ b = -4 \end{cases}$

6 $a = \left(\sqrt{168} + 4\sqrt{\frac{21}{2}} - 6\sqrt{\frac{14}{3}} \right) \cdot \left(\sqrt{4\frac{2}{3}} \right)^{-1}$

$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

$a = \left(2\frac{\sqrt{42}}{1} + \frac{4\cdot\sqrt{21}}{\sqrt{2}} - \frac{6\sqrt{14}}{\sqrt{3}} \right) \cdot \sqrt{\frac{3}{14}} \Rightarrow$

$a \frac{b}{c} = \frac{a \cdot c + b}{c}$

$a = \frac{2\sqrt{6\cdot 42} + 4\sqrt{3\cdot 21} - 6\sqrt{2\cdot 14}}{\sqrt{14}} \cdot \frac{\sqrt{3}}{\sqrt{14}} \Rightarrow$

$a = \frac{12\sqrt{7} + 12\sqrt{7} - 12\sqrt{7}}{\sqrt{2}\cdot\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{2}\cdot\sqrt{7}} \Rightarrow$

$a = \frac{12\sqrt{7}}{\sqrt{2}\cdot\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{2}\cdot\sqrt{7}} \Rightarrow a = \frac{12}{2} \Rightarrow a = 6$

$6 \in \mathbb{N}.$

(10|8) Dc. $x \in [0,51]$, a.t. $a = \sqrt{x+49} + \sqrt{x+625}$

$a \in [32, 36]$

$x \in [0,51] \Leftrightarrow 0 \leq x \leq 51 \Rightarrow \begin{cases} 49 \leq x+49 \leq 100 \\ 625 \leq x+625 \leq 676 \end{cases}$

$\Rightarrow \begin{cases} \sqrt{49} \leq \sqrt{x+49} \leq \sqrt{100} \\ \sqrt{625} \leq \sqrt{x+625} \leq \sqrt{676} \end{cases} \Rightarrow \begin{cases} 7 \leq \sqrt{x+49} \leq 10 \\ 25 \leq \sqrt{x+625} \leq 26 \end{cases} \Rightarrow$

$\Rightarrow 7+25 \leq a \leq 10+26 \Rightarrow 32 \leq a \leq 36$

$\Rightarrow$

11/8 $E(x,y) = \sqrt{x^2-2x+5} + \sqrt{y^2+6y+10}$; $x, y \in \mathbb{R}$;

$E(x,y) \geq 3$

$x^2-2x+5 = x^2-2x+1+4 = (x-1)^2+4$

$(x-1)^2 \geq 0$ $(\forall) x \in \mathbb{R}$ $\Rightarrow (x-1)^2+4 \geq 4$, $(\forall) x \in \mathbb{R} \Rightarrow$

$\Rightarrow \sqrt{(x-1)^2+4} \geq \sqrt{4}$, $(\forall) x \in \mathbb{R}$ (1°)

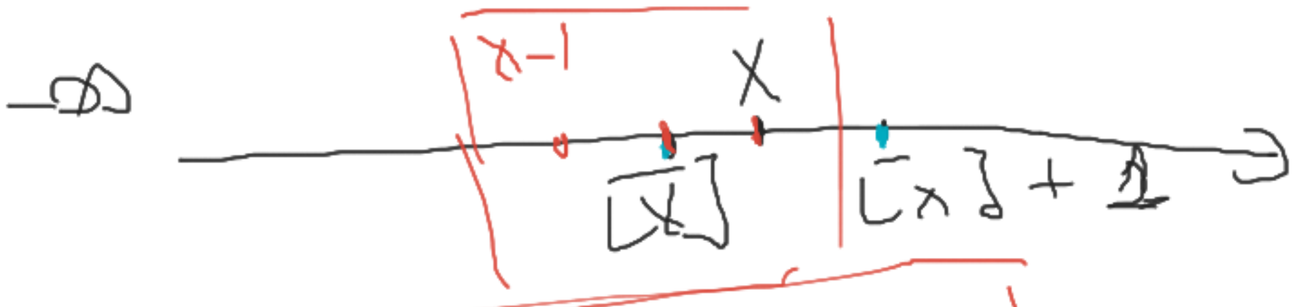
$y^2+6y+10 = y^2+6y+9+1 = (y+3)^2+1$

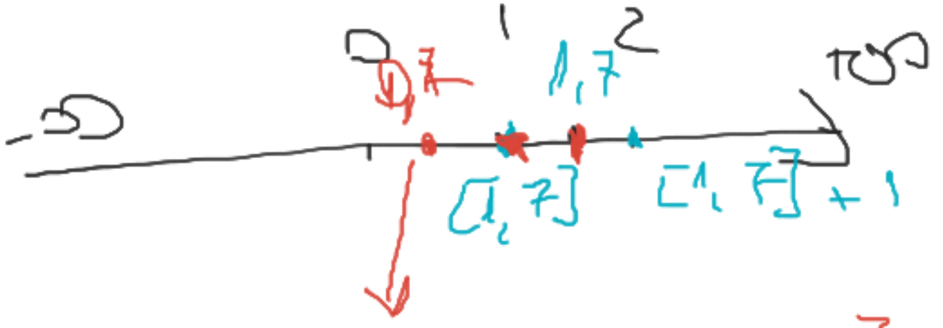
$(y+3)^2 \geq 0$, $(\forall) y \in \mathbb{R} \Rightarrow (y+3)^2+1 \geq 1$, $(\forall) y \in \mathbb{R} \Rightarrow$

$\Rightarrow \sqrt{(y+3)^2+1} \geq \sqrt{1}$, $(\forall) y \in \mathbb{R}$ (2°)

Din relatiile (1°) si (2°) $\Rightarrow E(x,y) \geq \sqrt{4} + \sqrt{1}$, $(\forall) x, y \in \mathbb{R}$

$\Rightarrow E(x,y) \geq 3$, $(\forall) x, y \in \mathbb{R}$.





$$[x] \le x < [x] + 1$$

$$x - 1 < [x] \le x$$

$$1{,}7 - 1 \le [1{,}7] < 1{,}7$$

$$[x] + [x+1] + [x+2] = 24$$

(10) $x - 1 < [x] \le x$

$x + 1 - 1 < [x+1] \le x + 1$

$x + 2 - 1 < [x+2] \le x + 2$

$\Rightarrow x - 1 + x + x + 1 < [x] + [x+1] + [x+2] \le x + x + 1 + x + 2 \Rightarrow$

$\Rightarrow 3x \le 24 \le 3x + 3 \Rightarrow \begin{cases} 3x \le 24 \\ 3x + 3 \ge 24 \end{cases} \Rightarrow \dots$