Clasa a XI-a C

$\boxed{Ex\ 1\ 1\text{a}2}$ $f \in \mathbb{Q}[X]$, $\boxed{f = X^3 + X^2 + X + 1}$

$$\boxed{(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3}$$

$$f(1+i) = (1+i)^3 + (1+i)^2 + (1+i) + 1$$

$$\boxed{i^3 = i^2 \cdot i = -i}$$

$$(1+i)^3 = 1^3 + 3\cdot 1^2 \cdot i + 3 \cdot 1 \cdot i^2 + i^3 = 1 + 3i - 3 - i = -2 + 2i$$

$$(1+i)^2 = 1^2 + 2 \cdot 1 \cdot i + i^2 = 1 + 2i - 1 = 2i$$

$$f(1+i) = -2 + 2i + 2i + 1 + i + 1 = 5i$$

Nr. complexe: $z \in \mathbb{C}$: $z = a + bi$ $(a, b \in \mathbb{R})$

Conjugatul lui z: $\boxed{\bar{z} = a - bi}$

$$\boxed{(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3}$$

$$f(1-i\sqrt{3}) = (1-i\sqrt{3})^3 + (1-i\sqrt{3})^2 + (1-i\sqrt{3}) + 1$$

$$(1-i\sqrt{3})^3 = 1^3 - 3\cdot 1^2\cdot(i\sqrt{3}) + 3\cdot 1\cdot(i\sqrt{3})^2 - (i\sqrt{3})^3 =$$

$$= 1 - \cancel{3\sqrt{3}i} - 3\cdot 3 + \cancel{i\cdot 3\sqrt{3}} = 1 - 9 = -8$$

$$(1-i\sqrt{3})^2 = 1 - 2\cdot 1\cdot i\sqrt{3} + (i\sqrt{3})^2 = 1 - 2\sqrt{3}i - 3 =$$

$$= -2 - 2\sqrt{3}i$$

$$f(1-i\sqrt{3}) = -8 \cancel{-2} - 2\sqrt{3}i + \cancel{1} - i\sqrt{3} + \cancel{1} = -8 - 3\sqrt{3}i$$

$$\cancel{f(1-i\sqrt{3}) = -8 + 3\sqrt{3}\cdot i}$$

$\boxed{E5}$ a) $f = bX + a$; $f(i) = 1$; $f(1-i) = 1$

$f(i) = 1 \Rightarrow b\cdot i + a = 1 \Rightarrow bi + a - 1 = 0 \Rightarrow$

$(a-1) + b\cdot i = 0 \Rightarrow \begin{cases} a-1=0 \\ b=0 \end{cases} \Rightarrow \begin{cases} a=1 \\ b=0 \end{cases}$

...

$\boxed{E7/p13}$ $f, g \in \mathbb{Z}_p[x]$; $\underline{f+g = g+f}$

a) $\mathbb{Z}_5$; $f = \hat{4}x^2 + \hat{3}x + \hat{1}$

$g = x^3 + x^2 + \hat{2}x + \hat{3}$

$f+g = x^3 + x^2 + \hat{2}x + \hat{3} + \hat{4}x^2 + \hat{3}x + \hat{1} =$

$= x^3 + x^2(\hat{1} + \hat{4}) + x \cdot (\hat{2} + \hat{3}) + \hat{4}$

$\mathbb{Z}_5 = \{\hat{0} ; \hat{1} ; \hat{2} ; \hat{3} ; \hat{4}\}$

$\hat{0} = \{0 ; 5, 10, 15 \ldots\}$

$\hat{1} = \{1, 6, 11, \ldots\}$

$\hat{3} + \hat{4} = 7 \bmod 5 = \hat{2}$

$0 : 5 = 0$ resto

$5 : 5 = 1$, resto

$1 : 5 = 0$ rest 1

$6 : 5 = 1$ rest 1

$f+g = x^3 + \hat{0}x^2 + \hat{0} \cdot x + \hat{1}$

$f+g = x^3 + \hat{1}$.