

Phys 426: Reading 1

Q1: What does density (tend) to depend on?

Consider the density of fresh water or dry air. What two thermodynamic variables does the density of the fluid depend on, and in what sense?

Density, the mass per unit volume, depends on temperature and pressure. An increase in temperature is akin to an increase in kinetic energy on a microscopic scale, generally causing expansion of the fluid decreasing the density. The relation between density and temperature is not a linear relationship. Some fluids, water amongst them, can increase in density at certain temperatures. Water around freezing will increase in density up until a temperature of around 4°C due to the lattice structure of ice and water at these temperatures.

An increase in pressure involves increased intermolecular forces. The pressure depends on the number and intensity of collisions of fluid particles colliding with its container or boundary. Compressing a fluid is often an easy way to increase the pressure, naturally this involves increasing the density of the fluid. In practice this may not always be so straightforward, but an increase in pressure should always result in an increased density (all else remaining constant).

Q2: What is pressure?

Briefly explain what pressure is in terms of molecular motion. If we put a surface in the fluid what effect does the pressure have on the surface? Why doesn't pressure have a direction?

Pressure is the measure of force applied to a unit of surface area. It is effectively measuring the force and frequency of particles of a fluid colliding with a surface in contact. It's a scalar relation between the normal of the surface area and the force acting upon it. We can represent this in general by the equation:

$$P = F/A.$$

The pressure on a surface submerged in a fluid will feel a pressure of $P = \rho gh$ with density ρ , height h , and the acceleration due to gravity g . The pressure will apply to all sides submerged within the fluid. It will apply equally, assuming a static fluid, to all sides varying only slightly with elevation (change in gravitational acceleration).

If we assess the pressure on a surface at an infinitesimally small point, we will find a single value for pressure (approximately) for any position. Therefore, pressure is a magnitude without a vector (can be represented by a field) is scalar.

Q3: How does pressure support water columns?

Suppose we have a U-shaped pipe with a constant cross-sectional area of 0.1m^2 . Water filled up to 1 m above the bottom of the U. A weight of 10 kg is placed on a plunger on one side of the pipe. What will the water levels be in each side of the pipe?

We can assume that the water will have a consistent density throughout. Therefore, we can determine the volume of water displaced by the weight using the density:

$$V = \frac{m}{\rho} = \frac{10\text{kg}}{1000\left(\frac{\text{kg}}{\text{m}^3}\right)} = 0.01\text{m}^3$$

We then use $V = A \cdot h$ where A is the cross-sectional area to solve for the height of the displaced water:

$$0.01\text{m}^3 = h(0.1\text{m}^2) \rightarrow h = 0.1\text{m}$$

Therefore, the displacement of water is roughly 10 centimeters. That will be 5 centimetres on each side of the tube from the initial position.

Q4: Kinematics: How does “advection” change the Eulerian perception of a quantity?

Suppose the concentration of Caffeine in a river is measured to be 100 g/m³ at one station, and 150 g/m³ at a station 100 km upstream. if the river is flowing downstream at 1 m/s, what do you think the concentration will be at the downstream station three hours later?

Given that the speed of the water is constant, we assume that the rate at which the caffeine is lost is constant. We can find the rate at which the caffeine is lost by finding the time it takes for the water to travel from one station to the next:

$$\frac{100 \cdot 10^3 \text{ m}}{1 \left(\frac{\text{m}}{\text{s}} \right)} \cdot \left(\frac{1 \text{ hour}}{3600 \text{ seconds}} \right) = 27.77 \text{ hours}$$

$$\frac{50 \left(\frac{\text{g}}{\text{m}^3} \right)}{27.77 \text{ hours}} = 1.8 \left(\frac{\text{g}}{\text{m}^3 \cdot \text{hour}} \right)$$

So, for 3 hours travelling down the stream, the concentration of caffeine 3 hours later should be:

$$100 \left(\frac{\text{g}}{\text{m}^3} \right) - 3 \text{ hours} \cdot 1.8 \left(\frac{\text{g}}{\text{m}^3 \cdot \text{hour}} \right) = 94.69 \frac{\text{g}}{\text{m}^3}$$