Empirical Testing of Kelvin-Helmholtz Instability Wave-Vortex Behavior: Introduction

Why it matters

Kelvin-Helmholtz instability (KHI) is the instability described at the interface between two horizontal parallel fluid streams with unequal densities moving with respect to each other. Any occurrence of fluid shear of varying fluid densities will have potential to be unstable. Once the destabilizing affect of the shear overcomes any stability from stratification at a surface, the interface is no longer stable and may display the nonlinear instability properties we will seek. This phenomenon is critical in any study involving dynamic interaction of multiple fluids. We often see the effects of KHI in the wave-like billows of clouds and in the vortices amongst turbulence created when milk is poured into coffee. In modern day studies, most KHI research revolves around numerical simulations of astronomical effects, such as neutron star mergers[1] or plasma interactions on a magnetopause[2]. These are amongst a multitude of topics and studies of other interstellar fluid body interaction. It is especially prevalent in ocean and atmospheric studies as fluids of varying densities are constantly interacting. Naturally, KHI tends to quickly contribute to and create the turbulence in systems of flowing fluids.

The Plan

In this experiment, we intent is to replicate KHI inspired in nature by the experiments of Professor Stephen A. Thorpe[3]. Using a similar setup, nonlinear KHI will be induced between stratified fluids and recorded via camera to be compared post-procedure. The plan is to fill a large rectangular glass or plastic tube of large length compared to a small cross section with water of a density ρ_1 up to a halfway point. We then carefully and slowly fill the space underneath this fluid with a second fluid, saltwater, of density ρ_2. Once the apparatus is a full as reasonably able, the apparatus is slowly tilted until it is approximately horizontal taking care to not mix the fluids. Once set in an equilibrium position the apparatus is tilted a small angle suddenly to allow the fluids to flow. The resulting movement will result in a KHI with visible waves-like vortices given that the instability due to shear between the fluids overcomes the stabilizing stratification due to the densities of the fluids. The purpose of the experiment is to assess the wave behavior of the KHI and how it may change as the density gradient is varied. The primary objective of the experiment specifically, is to test the wavelength of the instability vortices for varying fractional density difference between the fluids. We will also assess the time between the tilt and the establishment of KHI, and the amplitude of the resulting instability for the variation in interfacial density. Comparison's will be made to the theoretical results from equations derived from the method of normal modes for small-amplitude disturbance in accelerating stratified miscible fluids.

Theory

Firstly, since there is significant consideration of the inertial and viscous forces at play, we need to consider how viscosity could impact the results. Thankfully for a low ranging Reynolds Number (ratio of inertial to viscous forces), the early growth stages for shear flow in viscous flow do not vary significantly from that of an inviscid flow[4]. We will note the Reynolds number in the discussion of the results and their validity:

$$R_e = \frac{\mu L}{\nu} = \frac{\rho u L}{\mu}$$

In which ν is the kinematic viscosity of the fluid We will consider this with thought for the implication of viscosity of the fluids on the acceleration of the flow. In the case of small angles however, the ratio of the velocity gradient at z = 0 in a viscous flow to that at z = 0 in an inviscid flow is[5]:

$$Q = \frac{2}{1 + \left(1 + \frac{4}{\pi}(\left(\frac{\nu t}{\kappa \tau}\right)^{\frac{1}{2}}\right)}$$

In which κ is the molecular diffusivity of salt and τ is the diffusion time across the interface. This value will be approximately one for low viscosity. Thankfully, this will not be a large issue in the case of water. In terms of the amplitude of the instability vortices, we must first refer to the Richardson number:

$$R_i = \frac{g}{\rho}\left(\frac{\nabla\rho}{\nabla u^2}\right) = \frac{N^2}{\left(\frac{du}{dz}\right)^2}$$

Where N is the buoyancy frequency. Clearly, we should desire to minimize the Richardson number to have it at a value less than 0.25, above which there is no instability[5]. In the case of instability, we can then compare the wavelengths of the instability for varying density difference. It has been confirmed that the wavelength is directly related to the thickness of the density interface[6], although the precision of this relation is far from ideal. We will collect data for wavelength controlling for density and make direct comparison with known data and theory as described in Professor Thorpe's studies.

Citations

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