

#### 4. (40 points)

Insert: 100, 20, 60, 50, 30

Insert: 40, 90, 45, 25, 65

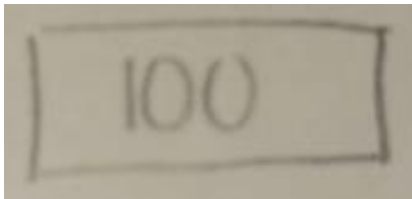
Delete: 90, 20, 60, 45, 50

Delete: 100, 40, 25, 65, 30

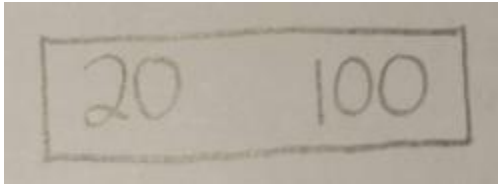
$m = 3$

#### Insert

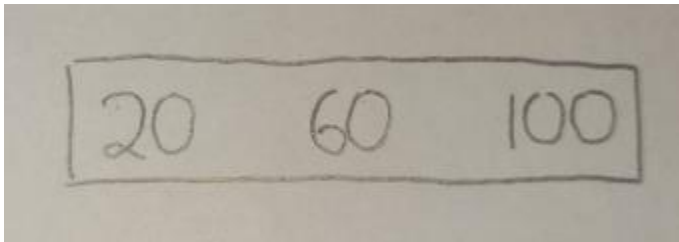
1. 100 becomes the root because there are no other nodes



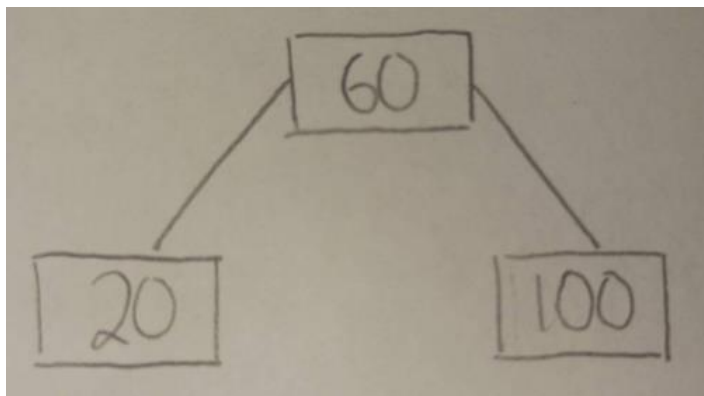
2. 20 goes to the left of 100 ( $20 < 100$ ) and becomes the 2<sup>nd</sup> key



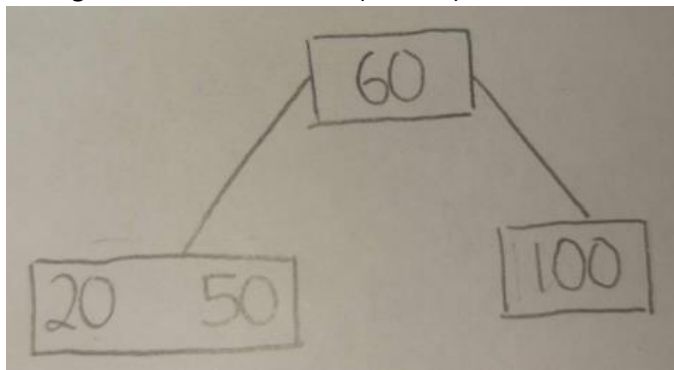
3. 60 goes in between 20 and 100 ( $20 < 60 < 100$ ) and becomes the 3<sup>rd</sup> key, however the maximum number of keys has been surpassed, initiate split



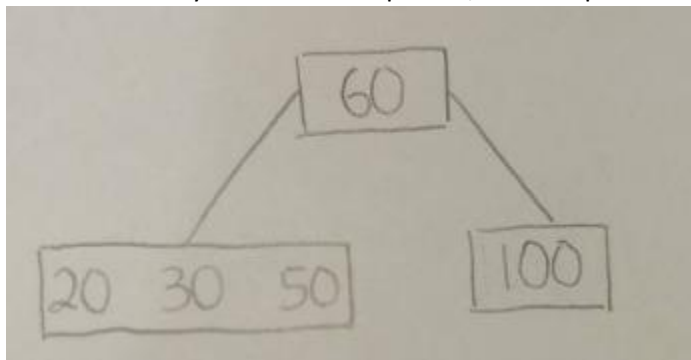
4. To split we take the median of the keys (20, 60, 100) which is 60, to become the parent of the remaining keys



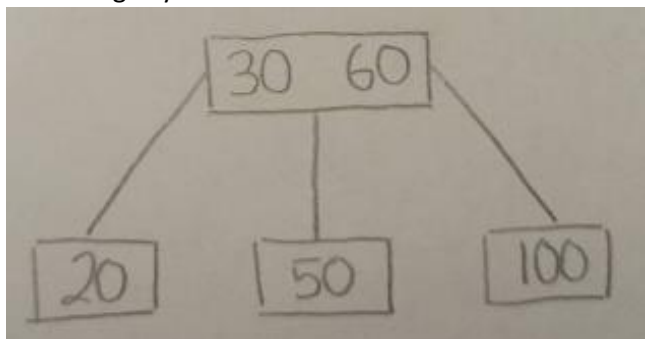
5. 50 goes to the left subtree ( $50 < 60$ ) and becomes the 2<sup>nd</sup> key on 20's right side ( $50 > 20$ )



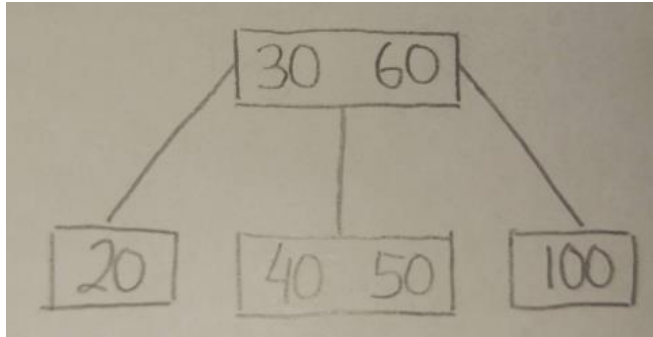
6. 30 goes to the left subtree ( $30 < 60$ ) and becomes the 3<sup>rd</sup> key in between 20 and 50 ( $20 < 30 < 50$ ), however the key limit has been passed, initiate split



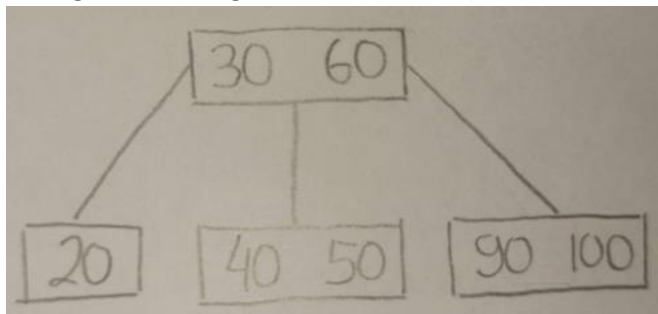
7. To split we take the median of the keys (20, 30, 60) which is 30, to join 60 to become the parent of the remaining keys



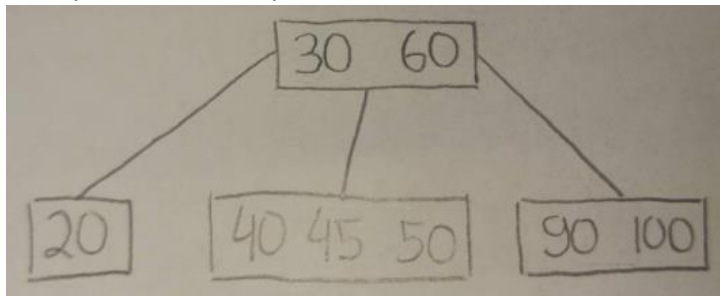
8. 40 goes to the middle subtree ( $30 < 40 < 60$ ) and becomes the 2<sup>nd</sup> key on 50's left ( $40 < 50$ )



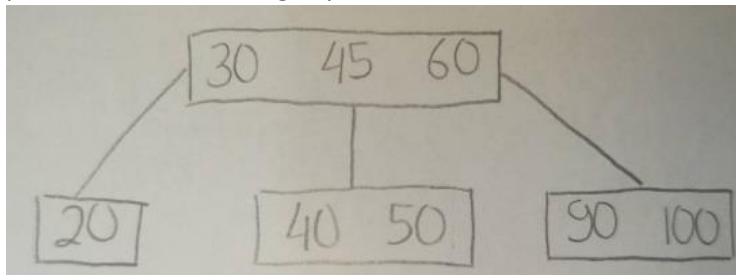
9. 90 goes to the right subtree ( $90 > 60$ ) and becomes the 2<sup>nd</sup> key on 100's left ( $90 < 100$ )



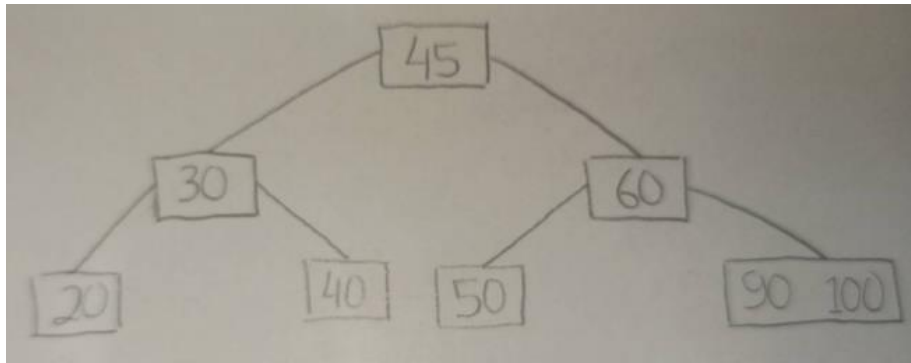
10. 45 goes to the middle subtree ( $30 < 45 < 60$ ) and becomes the 3<sup>rd</sup> key, however the key limit has been passed, initiate split



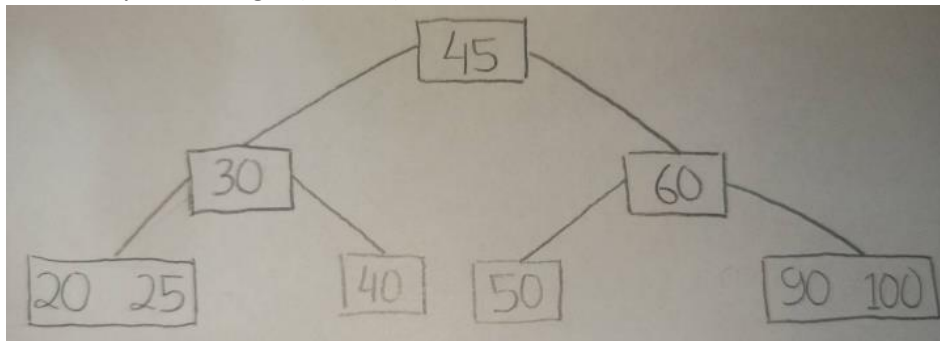
11. To split we take the median of the keys (40, 45, 50) which is 45, to join 30 and 60 to become the parent of the remaining keys



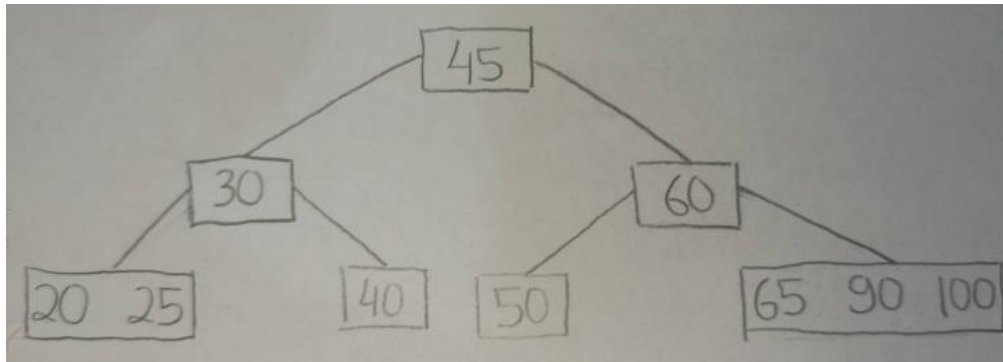
12. Now the limit of keys in the root has been passed, initiate split. To split we take the median of the keys (30, 45, 60) which is 45, become the new root and parent of the remaining keys



13. 25 goes to the left subtree ( $25 < 45$ ) and continues again to the left subtree ( $25 < 30$ ) and becomes the 2<sup>nd</sup> key on 20's right ( $25 > 20$ )



14. 65 goes to the right subtree ( $65 > 45$ ) and continues again down the right subtree ( $65 > 60$ ) and becomes the 3<sup>rd</sup> key to the left of 90 ( $65 < 90$ ), however the key limit has been passed, initiate break



15. To split we take the median of the keys (65, 90, 100) which is 90, to join 60 to become the parent of the remaining keys

