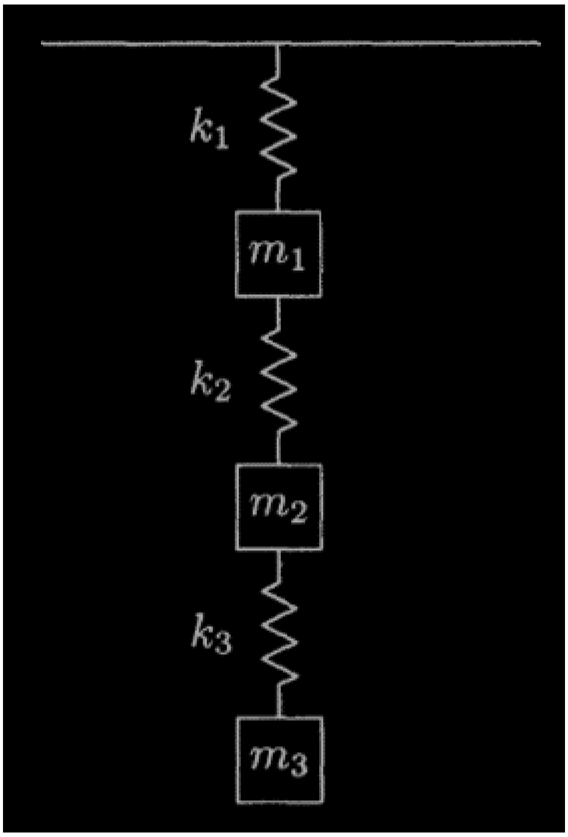

The equation of motion of the system can be derived using Newton's second law of motion, which states that the force acting on a body is equal to its mass times acceleration.

Let's denote the acceleration of each mass as y1", y2", and y3" the displacement of each mass as y1, y2, and y3.

Using F = ma (or ma = F as written in the equations below)

For mass 1

$$m_1 y_1'' = -k_1 y_1 + k_2(y_2 - y_1)$$

For mass 2

$$m_2 y_2'' = -k_2(y_2 - y_1) + k_3(y_3 - y_2)$$

For mass 3

$$m_3 y_3'' = -k_3(y_3 - y_2)$$

In matrix form it would be MY'' = -KY

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$Y'' = \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \end{bmatrix}$$

Bonus m1 = 2, m2 = 2, m3 = 4. k1 = k2 = k3 = 1

MY'' = -KY

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$Y'' = \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \end{bmatrix}$$

This is a standard set of differential equations of an oscillator for which we try the solution.

$$Y = \begin{bmatrix} a_1 e^{-i\delta_1} \\ a_2 e^{-i\delta_2} \\ a_3 e^{-i\delta_3} \end{bmatrix} e^{i\omega t} = a e^{i\omega t}$$

Here a_i is the amplitude, and δ_i is the initial phase.

$$Y'' = -\omega^2 \begin{bmatrix} a_1 e^{-i\delta_1} \\ a_2 e^{-i\delta_2} \\ a_3 e^{-i\delta_3} \end{bmatrix} e^{i\omega t} = -\omega^2 a e^{i\omega t}$$

Plugging it into the equation

$$M\left(-\omega^2 a e^{i\omega t}\right) = -K a e^{i\omega t}$$

$$(K - \omega^2 M) a e^{i\omega t} = 0$$

We ignore the trivial solution a = 0 (and the exponent can't be 0 anyways for all $t \geq 0$)

Hence, to have multiple such solutions, the matrix $(K - \omega^2 M)$ must be singular.

Hence $\det(K - \omega^2 M) = 0$

$$K - \omega^2 M = \begin{bmatrix} 2 - 2\omega^2 & -1 & 0 \\ -1 & 2 - 2\omega^2 & -1 \\ 0 & -1 & 1 - 4\omega^2 \end{bmatrix}$$

$$(2-2\omega^2)\big((2-2\omega^2)(1-4\omega^2)-1\big)-1+4\omega^2=0$$

Solving this we get 6 roots. Approximately those values are.

$$w = \pm 0.25171$$

Or

$$w = \pm 0.79767$$

Or

$$w = \pm 1.2451$$

Natural frequencies are the positive ones.