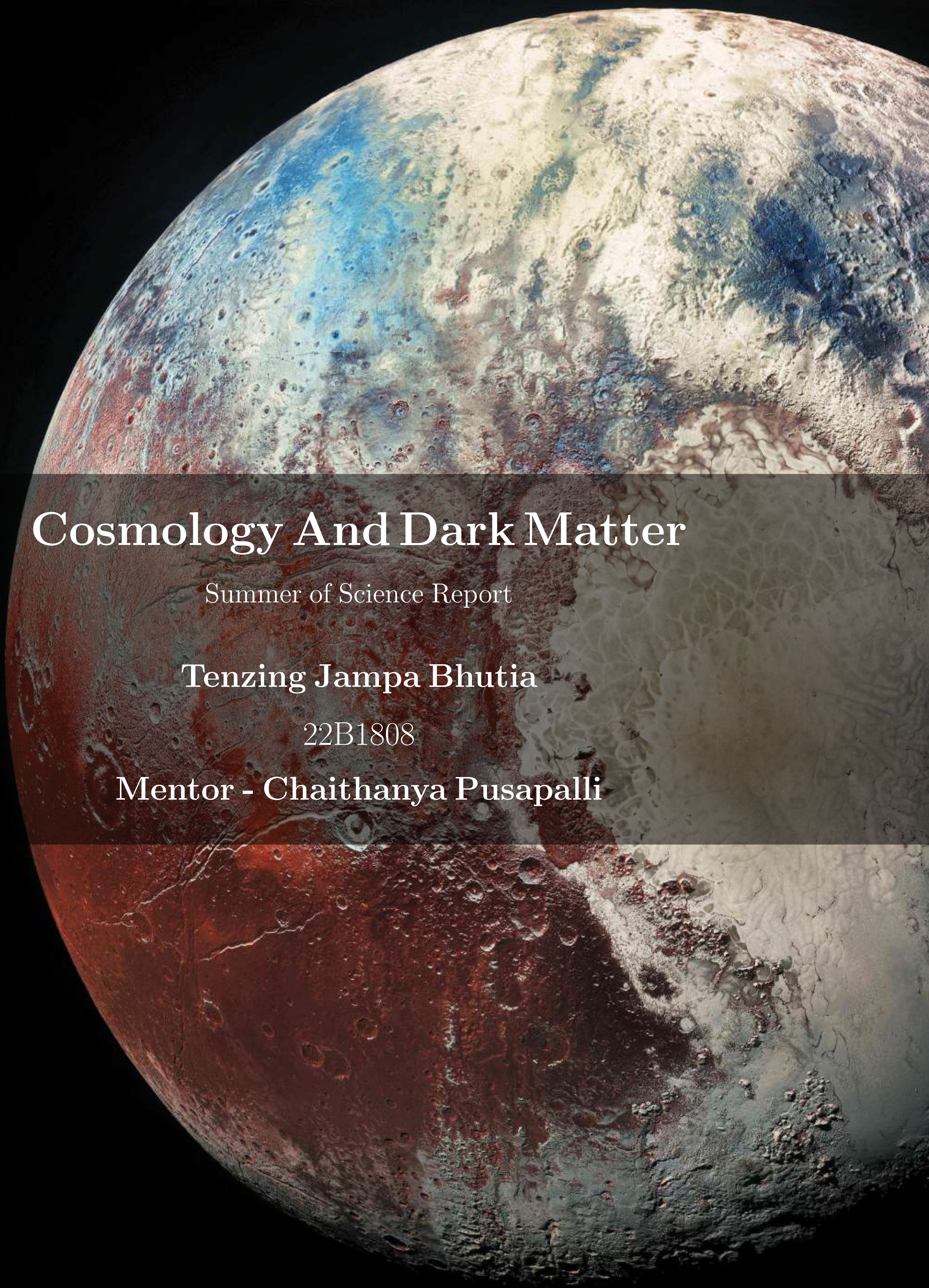

Cosmology And Dark Matter

Summer of Science Report

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1 Introduction to Cosmology

1.1 The Expanding Universe

The fact that the Universe is expanding has been discovered in the last century and is backed by Einstein's ground breaking discovery of General Relativity. This revelation made the Scientific world open to a whole new possibility that the Universe in fact started from a singular point which was a very hot and dense place after which it is now ever expanding.
We then move on to the success of Big Bang theory and the research which backs it up those being :

- The Hubble Diagram
- The Big Bang Nucleosynthesis
- The Cosmic Microwave Background

With this we begin with measuring the distance between us and the objects in the galaxy. Since in the beginning of time the distance between us and the other galaxies were very much near so to measure distances keeping that in mind we use the mechanism of scale factor a whose value in present time is set to 1. We also take notes of the *comoving distances* which measures the distance between the coordinates. In Fig1.1 (*taken from Modern Cosmology by Scott Dodelson*) the comoving distance remains constant but the physical distance changes with time.

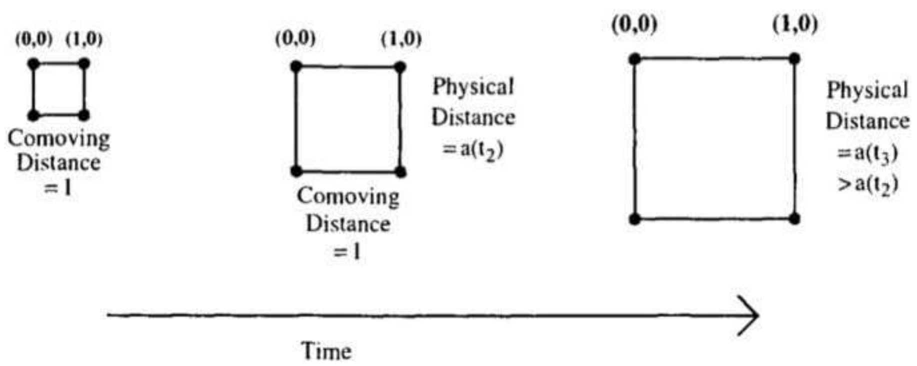


Fig1.1

The geometry of the universe, whether flat, open, or closed, defines its smoothness. Imagine two particles starting off parallel. In a flat (Euclidean) universe, they stay parallel. General relativity connects this flatness to a critical energy density, roughly $10^{-29}\ g/cm^3$. When the density is higher, the universe is closed, causing the particles to converge over time, akin to lines converging at the poles of a sphere. This positive curvature is in three spatial dimensions for the universe, unlike the two-dimensional sphere. Conversely, if the density is lower, the universe is open, and particles diverge like marbles rolling off a saddle.

To trace the universe's history, we monitor the scale factor a over time t. General relativity describes how a evolves with the universe's energy. Initially, a grows quickly with time ($a \propto t^{1/2}$), but later, its growth rate slows ($a \propto t^{2/3}$). This change results from different dominant energy forms at different times: radiation in the early universe and nonrelativistic matter later. Measuring the scale factor's changes allows us to deduce the universe's energy content. Recent observations indicate that the growth rate of the scale factor has decelerated, suggesting the influence of a new form of energy.

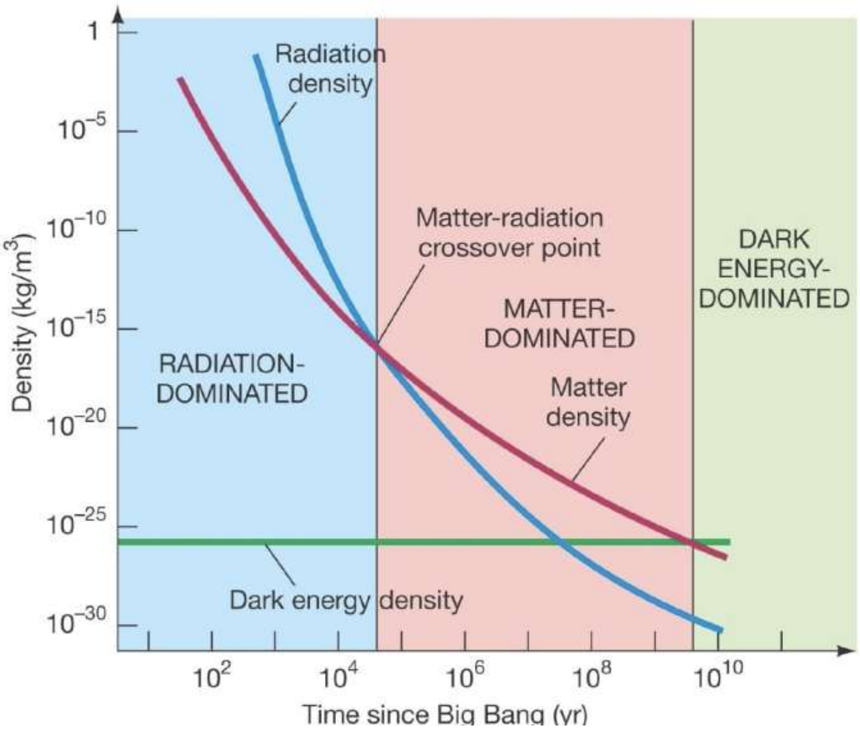


Fig1.2 this figure shows the change in density of different types of matter and energy since the beginning of time

To understand how the scale factor changes and how it relates to energy, we begin by defining the Hubble rate, which measures how fast the scale factor is changing:

$$H(t) \equiv \frac{\dot{a}}{a}$$

In a flat, matter-dominated universe where $a \propto t^{2/3}$, the Hubble rate H is given by:

$$H = \frac{2}{3}t^{-1}$$

An important test for this cosmological model is to determine the current Hubble rate, H_0, and the present age of the universe, t_0. Here, the subscript 0 denotes the current value of a quantity. In a flat, matter-dominated universe, the product $H_0 t_0$ should be equal to $\frac{2}{3}$.

Generally, the scale factor's evolution is described by the Friedmann equation:

$$H^2(t) = \frac{8\pi G}{3}\left[\rho(t) + \frac{\rho_{cr} - \rho_0}{a^2(t)}\right]$$

where $\rho(t)$ is the universe's energy density at time t, and ρ_0 is its current value. The critical density is defined as:

$$\rho_{cr} \equiv \frac{3H_0^2}{8\pi G}$$

determined using Newton's constant G.

Now to apply Einstein's equation, we need to understand how the energy density changes over time. This is hard because ρ includes multiple components, where each evolves very differently. Considering the nonrelativistic matter first, the energy of such a particle is its rest mass energy, which remains constant over time. Therefore, the energy density of many such particles is the rest mass energy times the number density. When the scale factor a was smaller, the densities were higher. Since number density is inversely proportional to volume, it scales as a^{-3}. Thus, the energy density of matter is proportional to a^{-3}.

Now consider the photons in the cosmic microwave background (CMB), which today have a well-measured temperature $T_0 = 2.725 \pm 0.002K$(Mather et al., 1999). A photon with energy $k_B T_0$ today has a wavelength of $\lambda = \frac{hc}{k_B T_0}$ In the early universe, when the scale factor was smaller, this wavelength was also smaller. Since photon energy is inversely proportional to its wavelength, the energy of a photon was higher by a factor of $\frac{1}{a}$. Now if we apply this to the thermal bath of photons says that the temperature of the plasma changes with time

$$T(t) = T_0/a(t)$$

Thus meaning that during early times the Temperature was way hotter than today.

1.2 The Hubble Diagram

As we now know that the Universe is expanding we should therefore constantly see the other galaxies getting away from us as time passes by. The wavelength of light rays and sound gets stretched as it travels more to deifine this factor we introduce redshift z

$$1 + z \equiv \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{1}{a}$$

This explains how the light we see from stars currently is much more longer than when it was first emitted. For lower reshifts we use the standard Doppler formula which has $z \simeq \frac{v}{c}$

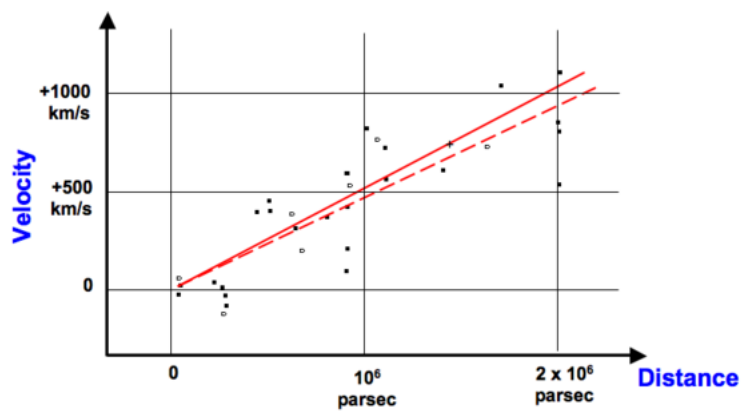


Fig1.3 The figure shows the velocities of different galaxies vs the distance in Mpc

Hubble concluded that the velocity of the galaxies increase with the distance. So we take the distance between two galaxies as $d = ax$ where x is the comoving distance, but when

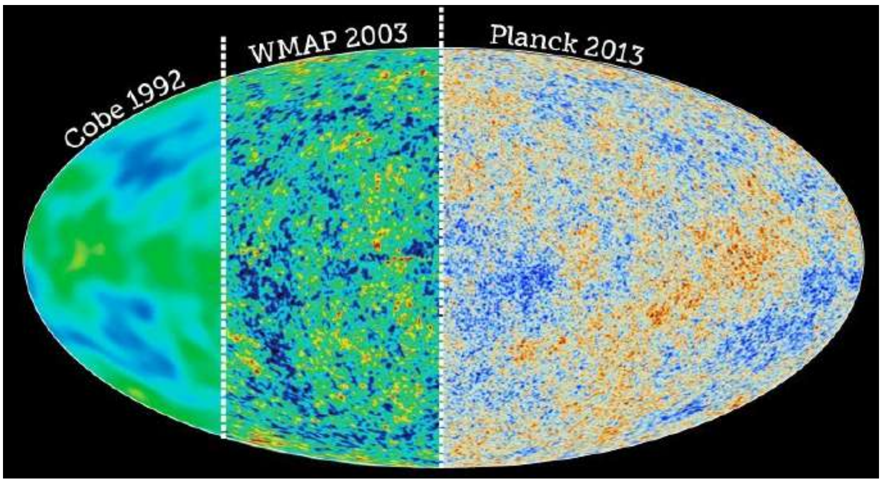
there's no comoving motion i.e $\dot{x} = 0$, the relative velocity is then $v = \dot{d}$ and therefore gives $\dot{a}x = Hd$.
Where H is the slope also known as Hubble Constant which is $H = 500 kmsec^{-1} Mpc^{-1}$

1.3 Big Bang Nucleosynthesis

In the early stages of the universe, it was incredibly hot and dense, with temperatures reaching approximately one MeV per cubic meter. At such high temperatures, neutral atoms and bound nuclei could not exist because any that formed would be instantly destroyed by the abundant high-energy photons. As the universe expanded and its temperature dropped below the binding energies of common nuclei, light elements began to form. By studying the early universe's conditions and nuclear reaction rates, we can predict the initial abundances of all the elements. The combined density of both the protons and neutron is known as baryon density, most of the matter in the universe is made of three quarks known as baryons.
BBN (Big Bang Nucleosynthesis) allows us to measure the baryon density of the universe. As these densities decrease with the universe's expansion (scaling as a^{-3}), measurements of light elements can indicate today's baryon density. Primordial deuterium is particularly accurate in determining baryon density, pinpointing it to a few percent of the critical density. Baryons, or ordinary matter, make up about 5% of the critical density, while total matter density is estimated to be around 20-30%. This discrepancy suggests the existence of nonbaryonic dark matter.

1.4 The Cosmic Microwave Background

The Cosmic Microwave Background (CMB) offers us a glimpse into the universe as it existed 300,000 years ago. Photons, from the CMB last interacted with an electron during a redshift of 1100. Have been travelling through space since, by observing these photons today we can guess what went into the stages of the universe making them a valuable tool for exploring its origins. Interactions with electrons help stabilize photons before they eventually break down resulting in what's known as a body. The COBE spacecrafts findings confirmed the existence of a hole within the CMB. Penzias and Wilsons discovery of background radiation at 3K, in the mid 1960s provided evidence supporting the Big Bang theory over explanations even though their observations were limited to just one wavelength.



2 General Relativity

2.1 Introduction

To study Cosmology it requires us to be vary with Einstein's General Relativity. We have to be aware of the concepts such as the metric and geodesics. General Relativity was introduced by Albert Einstein in 1915, which redefines gravity as the warping of spacetime by mass and energy, rather than a traditional Newtonian force. Massive bodies like stars and planets bend the spacetime around them, influencing the trajectories of nearby objects, which follow these curved paths. This theory has been validated through various observations and experiments, like the deflection of light by gravity (gravitational lensing) and the accurate predictions of different planetary orbits. General Relativity is essential for understanding complex astronomical phenomenons like black holes and the universe's expansion. In this chapter we are going to get a small taste of the algebra related to General Relativity.

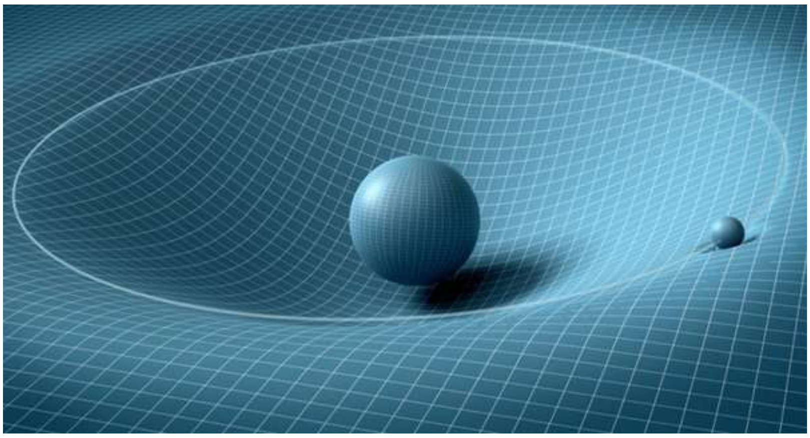


Fig2.1 This is a visualisation of the space time fabric where two objects warp the fabric around them with their mass

2.2 The FRW

As we know about the Cartesian coordinate system that gives us the physical distance between two points dx and dy as $(dx)^2 + (dy)^2$, when we change the system to polar i.e, dr and $d\theta$ we cannot directly write the distance now as $(dr)^2 + (d\theta)^2$. For this we need to notice that with a very small difference in θ the distance in the polar coordinates varies very much, thus we then find the distance to be $(dr)^2 + r^2(d\theta)^2$. This explains how the distance is invariant but the perspective changes when we change the metric.
The metric's significant advantage is that it integrates gravity with it. Instead of treating gravity as an external force affecting particles in a gravitational field, the metric allows us to describe particles moving freely in a curved spacetime, where the metric isn't universally Euclidean.
In four-dimensional spacetime, the invariant interval includes both spatial and temporal components, with indices ranging from 0 to 3 (time being $dx_0 = dt$ and the other three for spatial coordinates i.e x,y and z).

$$ds^2 = \sum_{\mu.\nu=0}^{3} g_{\mu\nu} dx_\mu dx_\nu$$

We'll use the convention that repeated indices imply summation even though I wrote the summation sign before. The metric tensor $g_{\mu\nu}$ is symmetric, featuring four diagonal and six off-diagonal components, linking coordinate values to the physical measure of the interval ds^2 (often called proper time). Special relativity operates in Minkowski spacetime with the metric $g_{\mu\nu} = \eta_{\mu\nu}$

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To explain the expanding universe, we can look at the grid in Fig1.1 , where two grid points move apart, their separation always proportional to the scale factor. If the comoving distance now is $x_0$the physical distance between two points at an earlier time t was $a(t)x_0$. In a flat universe, the metric closely resembles the Minkowski metric, with distances scaled by the factor $a(t)$. This results in the Friedmann-Robertson-Walker (FRW) metric for an expanding, flat universe.

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{bmatrix}$$

2.3 Geodesic Equations

In the context of Minkowski space, particles follow straight trajectories unless influenced by a force. In more complex spacetimes, these trajectories, known as geodesics and are way more elaborate. To describe this, we generalize the form of Newton's law without forces, for an expanding universe :

$$\frac{d^2x}{dt^2} = 0$$

Considering a simple case: If a particle moves on a flat 2D plane. In Cartesian coordinates $x^i = (x, y)$, the equations of motion are straightforward. However, in polar coordinates $x'^i = (r, \theta)$, the equations for free particles appear differently because the basis vectors for r and θ change throughout the plane. Hence, $\frac{d^2x'^i}{dt^2} = 0$ does not imply that each coordinate r and θ individually satisfies $\frac{d^2x'^i}{dt^2} = 0$.

To derive the motion equations in polar coordinates, we now start with the Cartesian equations and transform them using the transformation matrix $\frac{\partial x^i}{\partial x'^j}$. For converting Cartesian to polar coordinates in 2D, where $x = r\cos\theta$ and $y = r\sin\theta$, this matrix is derived.

$$\frac{\partial x^i}{\partial x'^j} = \begin{bmatrix} cosx'^2 & -x'^1 sinx'2 \\ sinx'^2 & x'^1 cosx'2 \end{bmatrix}$$

This makes us conclude the geodesic equation as

$$\frac{d[\frac{dx^i}{dt}]}{dt} = \frac{d[\frac{\partial x^i}{\partial x'^j}\frac{dx'^j}{dt}]}{dt}$$

Now if the transformation from the Cartesian coordinates to the new coordinates was linear then the derivative relating to the Transformation matrix clears out, making our new geodesic equation in our new basis will still be $\frac{d^2 x'^i}{dt^2} = 0$.
As for the polar coordinates our transformation isn't linear making the geodesic equation

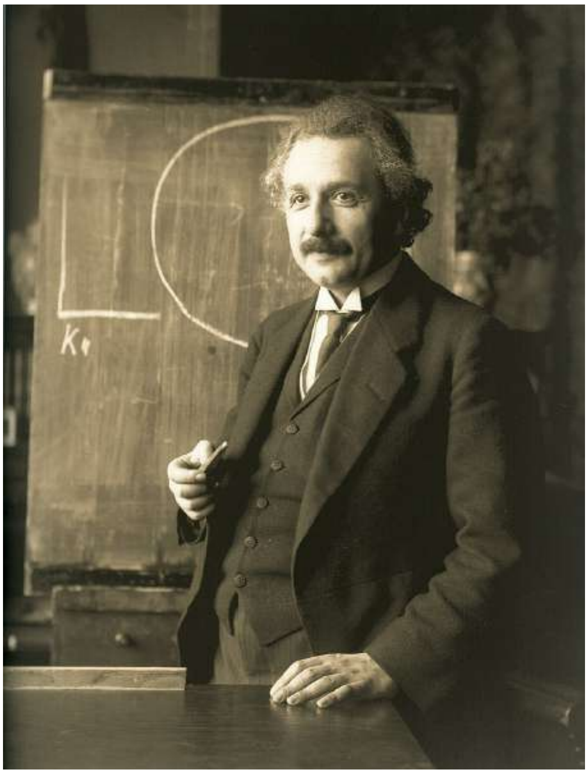
$$\frac{d[\frac{\partial x^i}{\partial x'^j}]}{dt} = \frac{\partial[\frac{dx^i}{dt}]}{\partial x'^j} = \frac{\partial^2 x^i}{\partial x'^j \partial x'^k}\frac{dx'^k}{dt}$$

After doing all the maths (changing the indices j to k) we then reach to the final equation as

$$\frac{d^2 x^i}{dt^2} + \left[\left(\left\{\frac{\partial x}{\partial x'}\right\}^{-1}\right)'_i \frac{\partial^2 x'}{\partial x' \partial x'^k}\right]\frac{dx^k}{dt}\frac{dx^j}{dt} = 0. \tag{1}$$

We can now substitute the coefficient of $(\frac{dx'^k}{dt})(\frac{dx'^j}{dt})$ with the *Christoffel symbol* Γ^l_{jk}. This symbol then in the case of Cartesian coordinates vanishes, but normally doesn't as it then leads how the geodesics work.

2.4 Einstein Equations



In this excerpt we come to see that in General Relativity we can add gravity in the metric as well. We observe that General Relativity associates the metric with the energy (mass) in the Universe. We then notice how the Einstein equations co-relate to the energy-momentum tensors

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 8\pi G T_{\mu\nu}$$

Where $G_{\mu\nu}$ is the *Einstein* tensor; $R_{\mu\nu}$ the *Ricci* tensor; $\mathcal{R}$ the Ricci scalar($\mathcal{R} \equiv g^{\mu\nu}R_{\mu\nu}$); G the Newtonian constant; and lastly $T_{\mu\nu}$ is the energy-momentum tensor.

We could rewrite the Ricci tensor in the form of Christoffel symbol,

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu.\alpha} - \Gamma^{\alpha}_{\mu\alpha.\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}$$

We use the convention of having commas taken as derivative in x here $[A_{a,b} \equiv \frac{\partial A_{a,b}}{\partial x^a}]$
The above equation thus has only two terms left where either $\mu = \nu = 0$ or $\mu = \nu = i$, so considering both to be zero we minimise the equation to

$$R_{00} = -\Gamma^{i}_{0i.0} - \Gamma^{i}_{j0}\Gamma^{j}_{oi}.$$

Now from $[\Gamma^{i}_{0j} = \Gamma^{i}_{j0} = \delta_{ij}\frac{\dot{a}}{a}]$ we can rewrite the equation as

$$R_{00} = -\delta_{ii}\frac{\partial\frac{\dot{a}}{a}}{\partial t} - (\frac{\dot{a}}{a})^2\delta_{ij}\delta ij \ \ = -3\frac{\ddot{a}}{a}$$

The factor 3 arises since δ_{ii} gives the sum over all the three spatial coordinates
If we simplify the Ricci scalar now

$$\begin{aligned}\mathcal{R} &\equiv g^{\mu\nu}R_{\mu\nu}\\ &= -R_{00} + (\frac{1}{a^2})R_{ii}\\ &= 6[\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2] \qquad\qquad \text{[since the three factor from spatial indices]}\end{aligned}$$

All this now leads us to the equation for the homogeneous Universe which is given by:

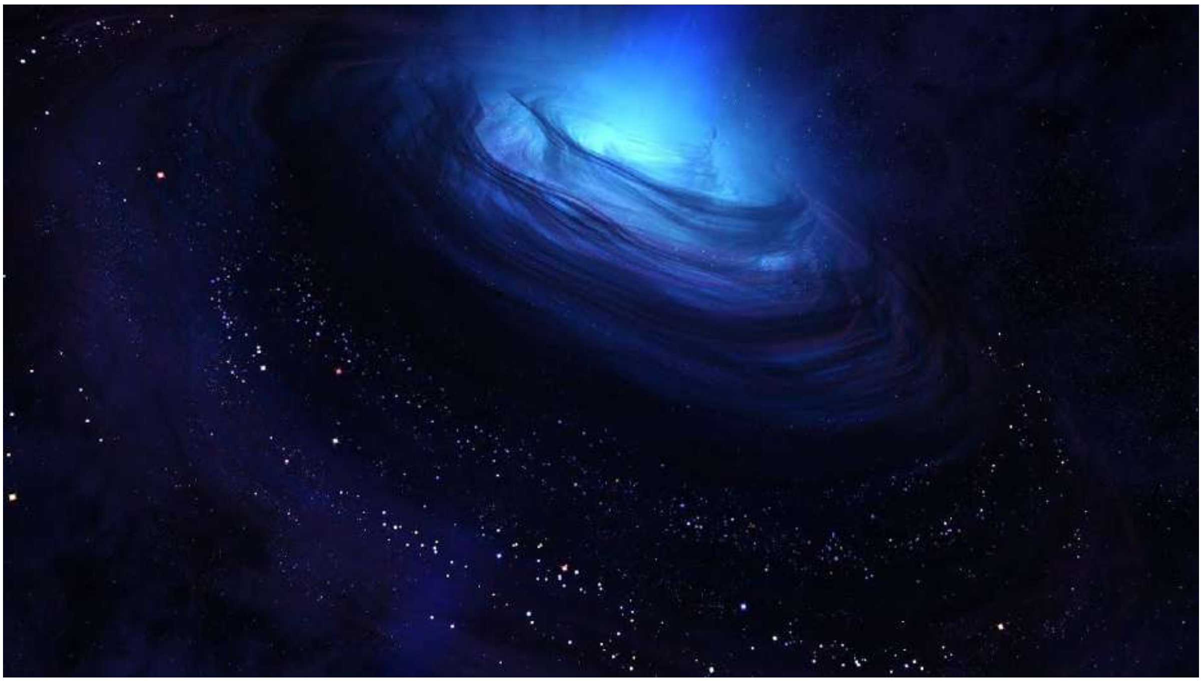
$$R_{00} - \frac{1}{2}g_{00}\mathcal{R} = 8\pi G T_{00}$$

3 Dark Matter

3.1 Introduction

In the recent century the world of science has come to know about the fact that out of all the matter in the universe nearly 98% is dark and not visible to the different spectrum of lights (only some percentage barely). This mysterious dark matter is still not fully understood by physicists and cosmologists in the world and thus its name.

In 1933 Zwicky first discovered the existence of dark matter when he was taking the velocities of the galaxies in the Coma cluster and noticed how the luminous matter were transcending their attributed velocity. When Smith worked on it in 1936, there was the same conclusion with the Virgo cluster galaxies. The mass to light ratio was awfully higher then expected.



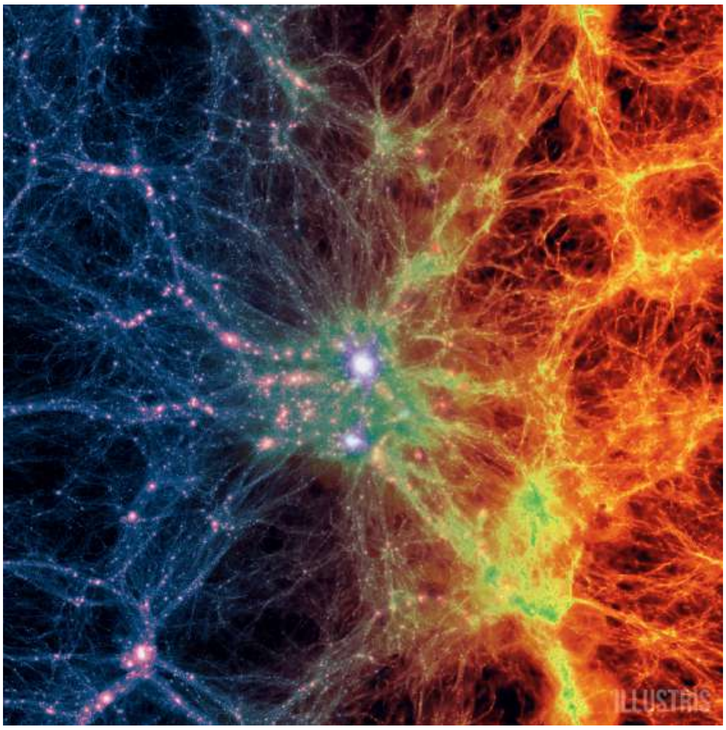
In 1959, Kahn and Woltjer studied the speeds at which the Milky Way and the Andromeda Galaxy (M31) are moving towards each other. They were using this information to estimate the mass of the Local Group, the galaxy cluster that includes both galaxies. They then found that the mass was much greater than what could be accounted for by the visible stars and gas, leading them to conclude that most of the Local Group's mass must be dark matter.

Later in the January of 1975 the first conference for Dark Matter was held. It was suggested that dark matter might be baryonic, then some type of very low massed ionized gas. Also coming to the suggestion that dark matter may be non-baryonic matter (neutrino)

Now since by this time the world came to know about the abundance of dark matter in this Universe it was supposed to explain/influence all the formations of large structures like the galaxy clusters and the speed for the expansion of the Universe. Although till this day only some properties of the dark matter are known to us.

3.2 Dark Matter Candidates

The main candidates for the dark matter are baryonic matter, non-baryonic matter, WIMPs and MACHOs. Since the approximation of the contribution of luminous matter in the Universe is $\Omega_{lum} \approx 0.0051$ making just around 2% of the total matter in the Universe meaning that the remaining 98% are dark matter. The fact that the total distribution of the total matter in the Universe is $\Omega_M = 0.3$ and only a small part of dark matter is similar to familiar matter. The fact that Ω_{lum} (luminous matter) is less than Ω_{bar} shows that there are two kinds of dark matter: baryonic and non-baryonic.



Age of the Universe is right now estimated to be around 13.8 billion years by observing the Cosmic Microwave Background Radiation, the Universe was firstly dominated by radiation after which the age of matter came by. Now during this shift if the matter were all expected to be baryonic, the overly dense regions would not have been able to collapse which is contrary to what the an-isotrophies in the CMBR suggest. But this property can be shown by non-baryonic matter.

Now if the dark matter particles move at near the speed of light making them *Hot Dark Matter* the formation of smaller structures wouldn't be possible as they would spread out of the dense regions in the spacetime. Thus the only correct explanation for forming of the galaxies would be from bottom to top,i.e the formation of smaller regions first and then they merge with other stuffs to create clusters. This can be done by baryonic matter which move slower than the speed of light and thus the prime candidate is called *Cold Dark Matter*.

3.2.1 WIMPs and MACHOs

WIMPs or Weakly Interacting Massive Particles are the tiny non baryonic particles. They interact with the help of gravity and weak nuclear forces and their masses range from Gev to Tev, making them slow enough at the time of matter-radiation equality to behave as *cold dark matter* (CDM). They do not interact with normal matter or electromagnetism so the only moment we see them is when they decay or annihilate into photons.

MACHOs or Massive Astrophysical Compact Halo Objects are large objects which are believed to be failed stars since they emit very small light emission. However, many non-baryonic objects could also act as MACHOs. Unlike common belief, MACHOs detectable through microlensing aren't necessarily limited by the Universe's baryonic mass fraction and could, in theory, make up all dark matter, though this is unlikely based on current constraints. They interact very minimally with baryonic matter similar to the CDM. So these two elements are big candidates for the Dark Matter.

3.3 Distribution of Dark Matter

Till the current observations and simulations we have encountered,we have seen that CDM(*Cold Dark Matter*) shows how the structure of large structures have many voids, filaments and walls which somewhat look like a spider's web. Its size is around $100 - 120Mpcs$.
When there are regions with very dense matter they collapse and the spherical dark matter halos form, where the baryons collapse to create luminous galaxies. The Universe now contains dark halos from the size of dwarf galaxies($\approx 10^6 solarmasses$) to large galaxy clusters($\approx 10^15solarmasses$). Larger halos contain smaller sub-halos, with galaxy clusters containing many galaxy-mass halos, and galaxies containing dwarf-galaxy halos.

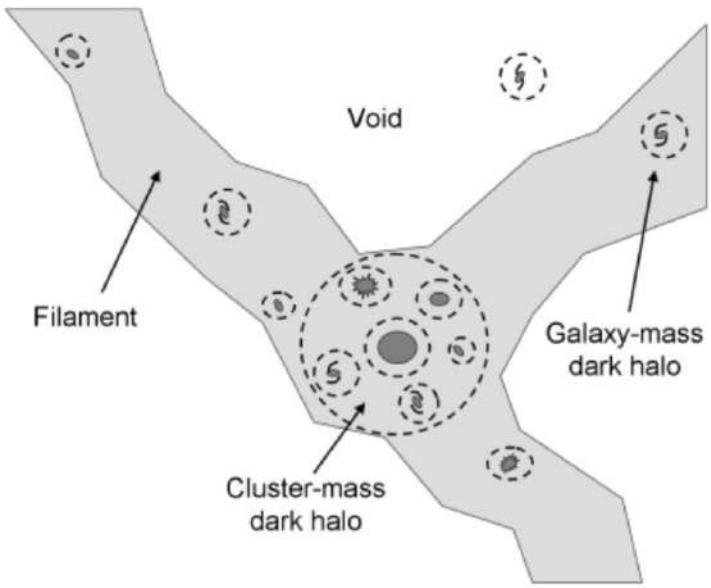


Fig3.1 This figure shows the formation of the voids in the Universe

The dark halos help the large galaxies to maintain their spherical shape or else they would progress to form into bar like galaxies. This was shown by *Ostriker and Peebles (1973).*

While a baryonic bulge might suffice, not all disk galaxies have them. Weak gravitational lensing, where distant objects are slightly distorted by matter in foreground objects, also supports the existence of CDM halos.

However, there are other possible explanations for some phenomena thought to be caused by dark halos. For example, HI flaring, where neutral hydrogen spreads out more at greater distances from the galaxy center, suggests a rounder mass distribution and has been used to support the idea of dark halos. But a thick dark matter disk could also cause this effect.

UPDATED POA

1. *Until* 10^{th} *July :* Thermal WIMP and Warm Dark Matter
2. *Until* 20^{th} *July :* Intro and dealing with Astrophysics
3. *Until* 29^{th} *July :* Axions

References

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