CMPT300 Operating Systems I - Assignment 4 Virtual Memory Questions

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Question 1

Given the following:
Virtual Memory space = 2^{32} bytes = 32 bits to represent
Physical memory space = 2^{18} bytes = 18 bits to represent
Page size is 4096 bytes = 2^{12} bytes = 12 bits to represent

The binary representation of the virtual address '11123456' (hexadecimal) is

Virtual Address

0001 0001 0001 0010 0011	0100 0101 0110
Virtual Page Number	Page Offset

The lower 12 bits are the page offset, since the page size is 2^{12} bytes and require 12 bits to represent all bytes within the page.

The remaining upper 20 bits are the Virtual Page Number (VPN), used to look for the corresponding Physical Page Number (PPN) in the Page Table.

If the target page is not currently in the cache, we encounter a page fault. The operating system picks a page to evict, for the new page to load into. Combining the PPN and the Offset bits will create the physical address needed to establish the corresponding location. Since the physical memory space requires 18 bits to represent, the PPN will simply be $18 - 12 = 6$ bits in the physical address. The physical address will have the form:

Physical Address

_ _ _ _ _ _	0100 0101 0110
Physical Page Number	Page Offset

Software: The software operations involve the managing the page tables, including updating the page table when the pages are loaded or evicted, as well as handling page faults. The software searches through the page table with the given VPN, and returns the corresponding PPN for the hardware.

Hardware: The harware operations involve the actual translation of the virtual address to a physical address. The hardware uses the PPN obtained from the page table from the software operations. The hardware performs an operation to combine the PPN and the original offset from the virtual address. This forms the physical address.

Question 2

The effective access time (EAT) is calculated as follows:

EAT = (1 - Page Fault Rate) * Memory Access Time + Page Fault Rate * (Page Fault Overhead)

Where Page Fault Overhead is the weighted average of the service times:
Page Fault Overhead = $0.7 * 20ms + 0.3 * 8ms$
Page Fault Overhead = $16.4ms = 16,400,000ns$

Plugging in the values of 200 for the EAT, and the values of Memory Access Time and Page Fault Overhead:

$$200 = (1 - \text{ Page Fault Rate}) * 100 + \text{ Page Fault Rate } * 16,400,000$$

$$200 = 100 - 100(\text{Page Fault Rate}) + 16,400,000(\text{Page Fault Rate})$$

$$100 = (\text{Page Fault Rate})(16,400,000 - 100)$$

$$100/16,399,900 = (\text{Page Fault Rate})$$

$$\text{Page Fault Rate} = 6.1 * 10^{-6}ns = 0.0000060976ms$$

This means that the maximum acceptable page-fault rate for an effective access time of no more than 200 nanoseconds is $0.0000060976ms$ per Page Fault.