Applied Mathematics I

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Chapter 7 Additional Homework

Find and solve the variational equations for $X' = F(X)$

1 $F = (x^2 + xy, x + y^3), X = (-1, 1)$

So, first we compute the Jacobian matrix of this F:

$$DF(X) = \begin{pmatrix} \partial(x^2+xy)_x & \partial(x^2+xy)_y \\ \partial(x+y^3)_x & \partial(x+y^3)_y \end{pmatrix}$$

$$DF(X) = \begin{pmatrix} 2x+y & x \\ 1 & 3y^2 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 1 & 3 \end{pmatrix}$$

Thus, we now find the solution of:

$$\begin{pmatrix} x(t)' \\ y(t)' \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

We row-reduce fairly easily the matrix:

$$\begin{pmatrix} -1 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$$

Thus our solutions are: $x(t) = ae^{-t}$ and $y(t) = be^{2t}$

And our variational solution is:

$$X(t) = \begin{pmatrix} -1 + ae^{-t} \\ 1 + be^{2t} \end{pmatrix}$$

2 $x' = x^{4/3}, x(t) = 27(3-t)^{-3}$

We see that $x(t_0) = 27 * 1/3^3 = 1$

We only have one variable here, thus $DF(X) = \frac{4}{3}x^{1/3} = 4/3$

Thus variational problem: $x' = 4/3x \rightarrow x(t) = ae^{4/3t}$

Thus variational solution is: $X(t) = 27(3-t)^{-3} + ae^{4/3t}$