CS 815 Assignment 3

Camera Calibration and Fundamental Matrix Estimation with RANSAC

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Objectives

Design algorithms to estimate

1. Camera Projection Matrix
2. Fundamental Matrix
3. Fundamental Matrix with RANSAC

Camera Projection Matrix:

A camera projection matrix is a set of matrices that is used to map a 2D image to a 3D coordinate.

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \cong \begin{pmatrix} u*s \\ v*s \\ s \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

(u, v) - 2D image coordinates
M - Projection matrix
(X, Y, Z) - location of those points in 3D world

The following is in the form of AM = B

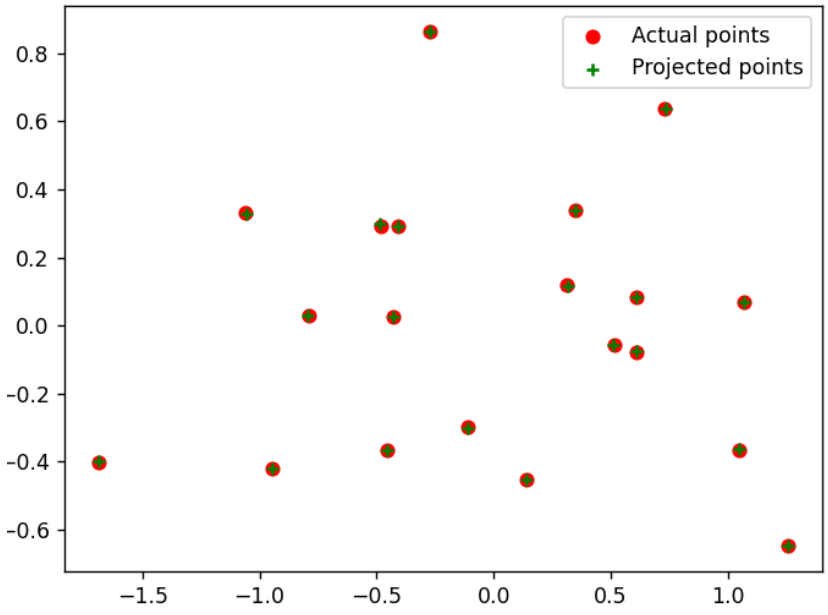
$$AM = B \Rightarrow A.A^T.M = A^T.B \Rightarrow M = (A.A^T)^{-1}A^T.B$$

This can be solved by inverting the A matrix, in order to get the value of M.
The value of m34 is set to 1.

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & & & \vdots & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_n \\ v_n \end{bmatrix}$$

Projection Matrix:

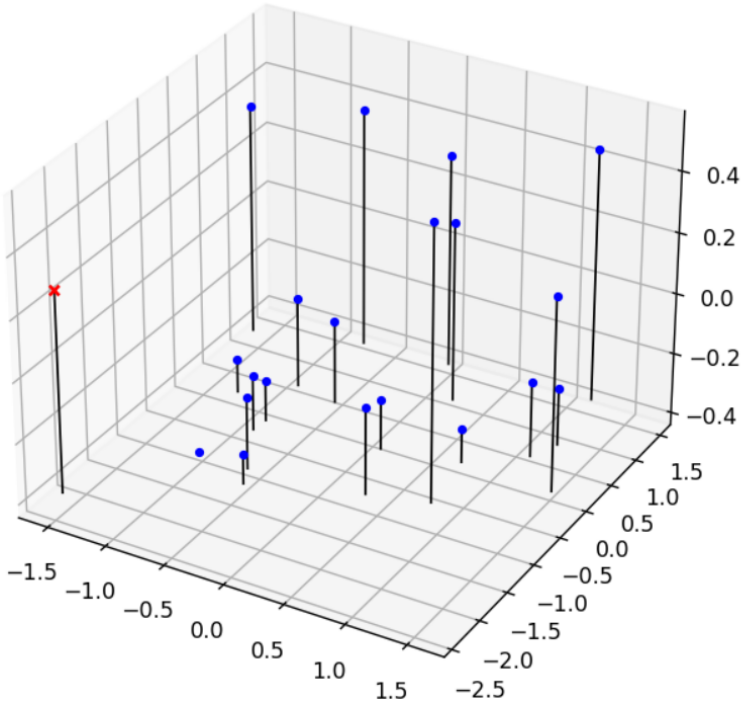
The projection matrix is
[[0.76785834 -0.49384797 -0.02339781 0.00674445]
 [-0.0852134 -0.09146818 -0.90652332 -0.08775678]
 [0.18265016 0.29882917 -0.07419242 1.   ]]



Calculation of Camera center:

$$CC = -Q^{-1}M_4$$

$$M_4 \rightarrow M(:, 0{:}3)$$



<-1.5126, -2.3517, 0.2827> is found out to be the camera centre

Calculation of Fundamental Matrix:

Two cameras are used to provide images. To say, the same scene is captured using two different angles, or positions.

The aim of calculating the fundamental matrix is to find identical points in those two images that have been captured from two different directions.

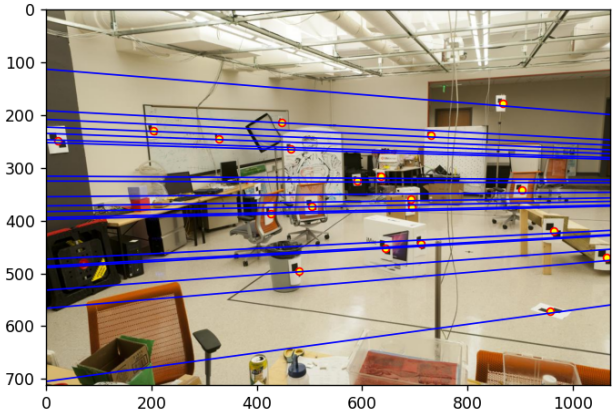
For this, points from both images have to be taken.

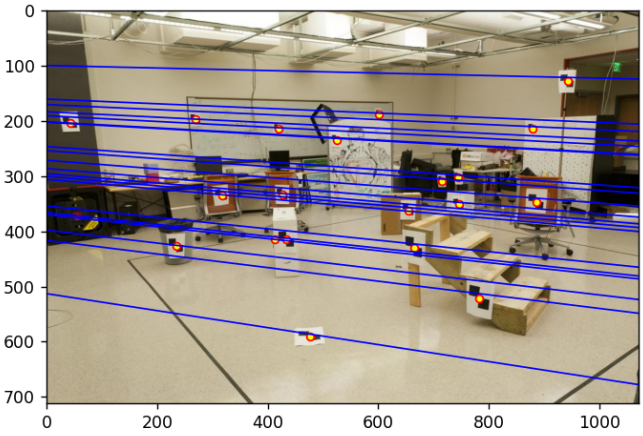
$$\begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = 0$$

A simplified version of the equation is

$$\begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_v' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = 0$$

Using the algorithm that was used for the calculation of Camera Centre, SVD is used to estimate a second ranked matrix by setting the lowest singular value as 0 and using SVD for the second time.
Once the fundamental matrix is calculated, epipolar lines can be plotted



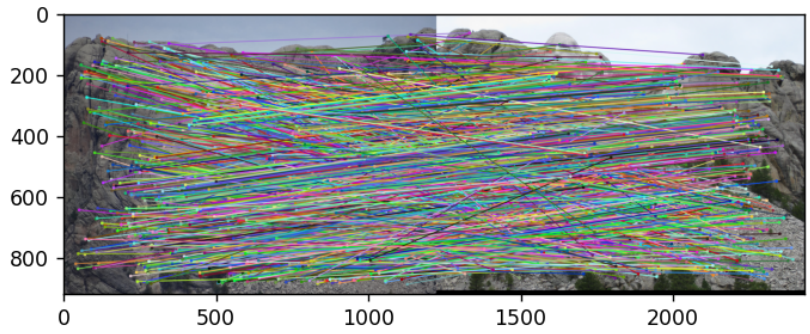


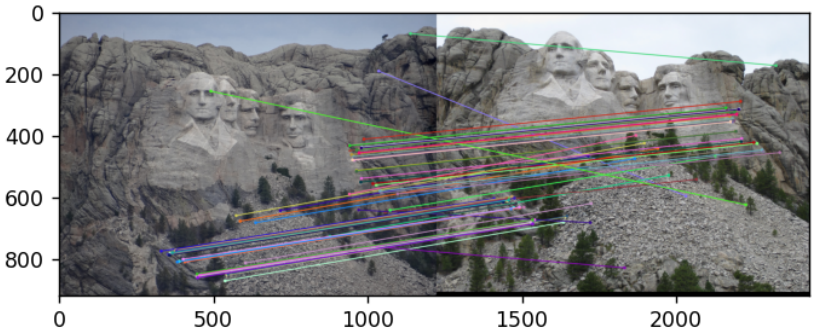
Calculation of Fundamental Matrix using RANSAC:

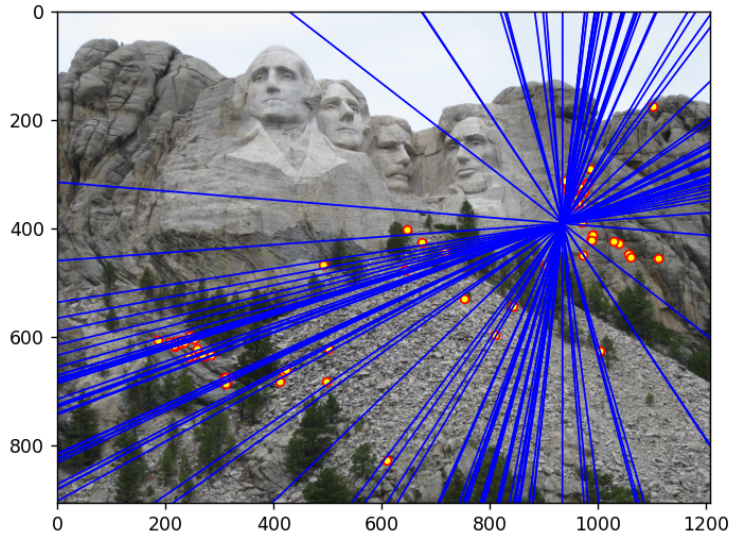
For RANSAC algorithm, 8 points on the image, that have been identified by the SIFT detector, are used.
Using those 8 points, a fundamental matrix is created.
From those, the inliers are selected.
This cycle is run several times, and each time the number of inliers is compared to the cycle which had the most inliers.
The fundamental matrix is kept updated each time the number of inliers increases.

Results:

Mount Rushmore:



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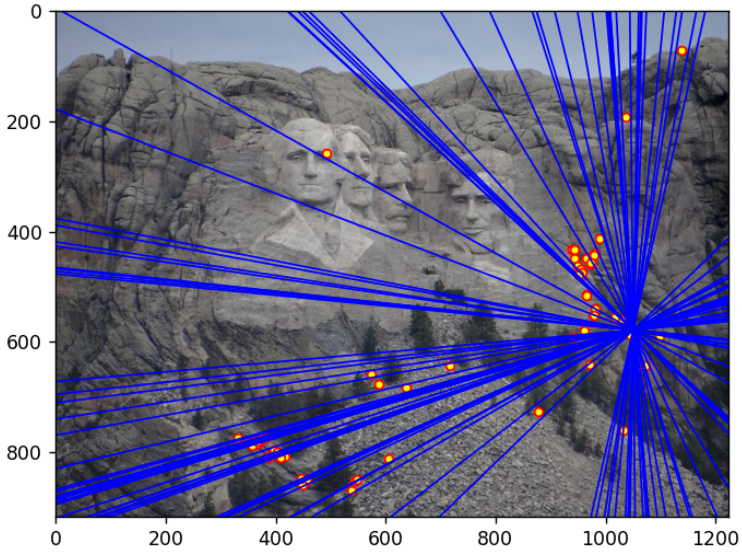
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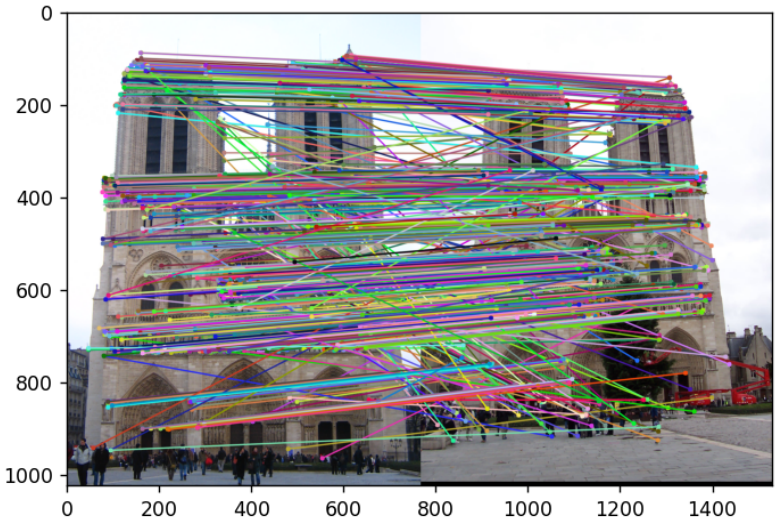
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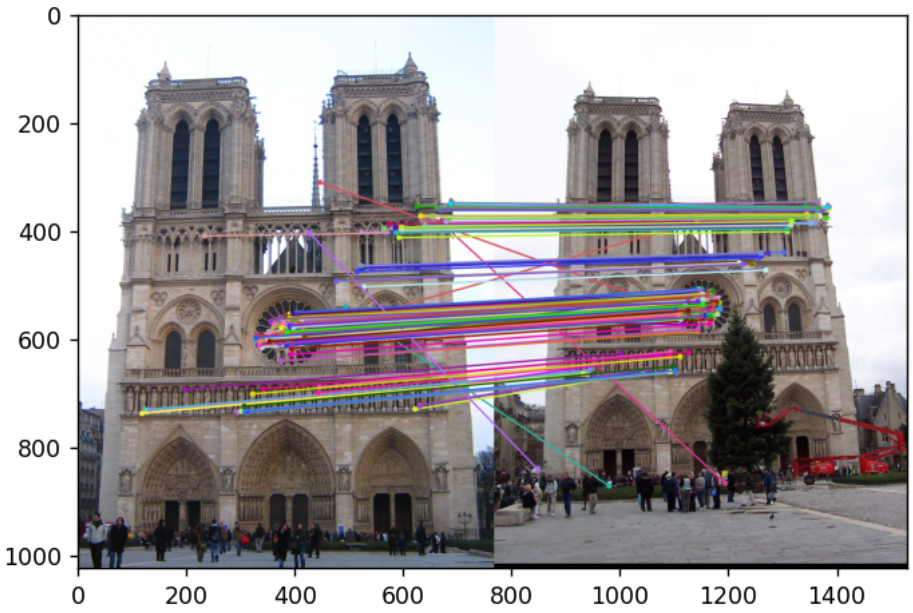
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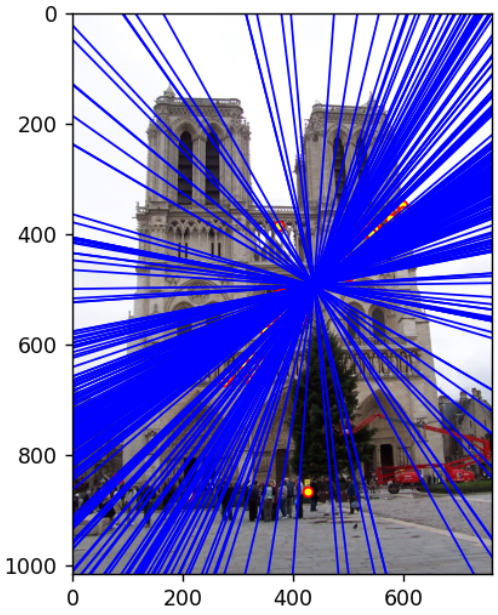
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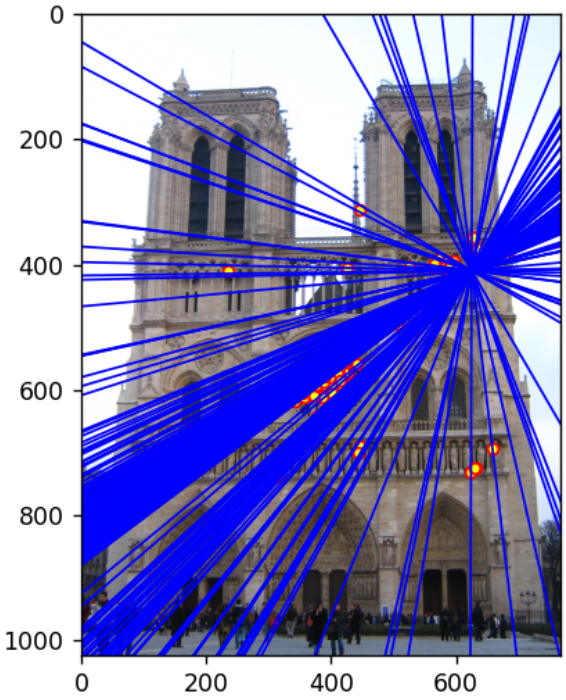
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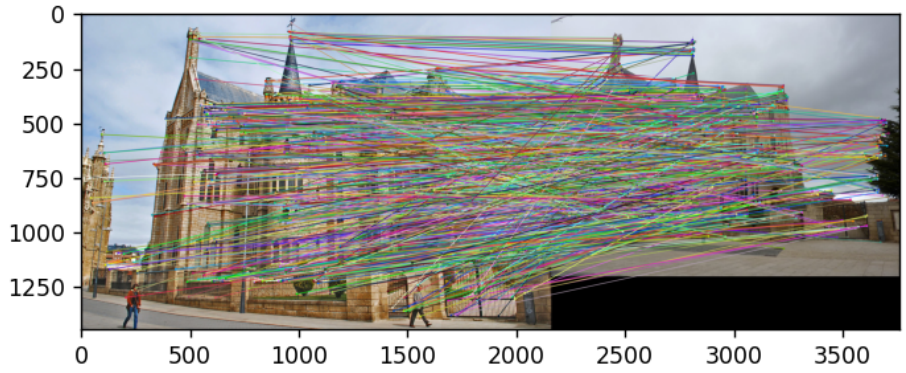

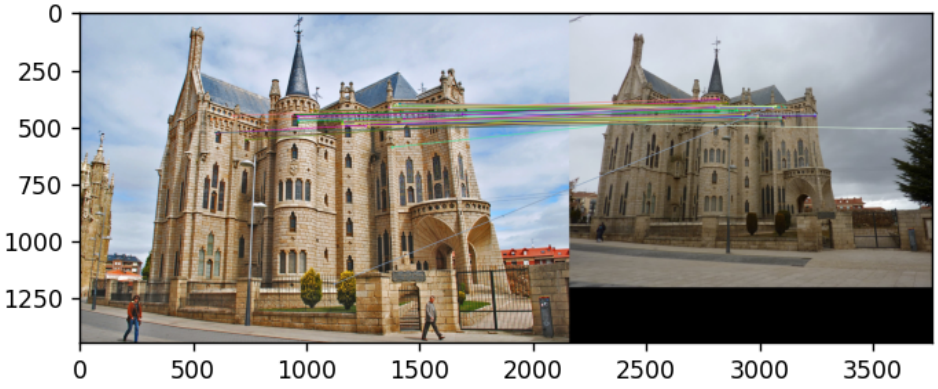

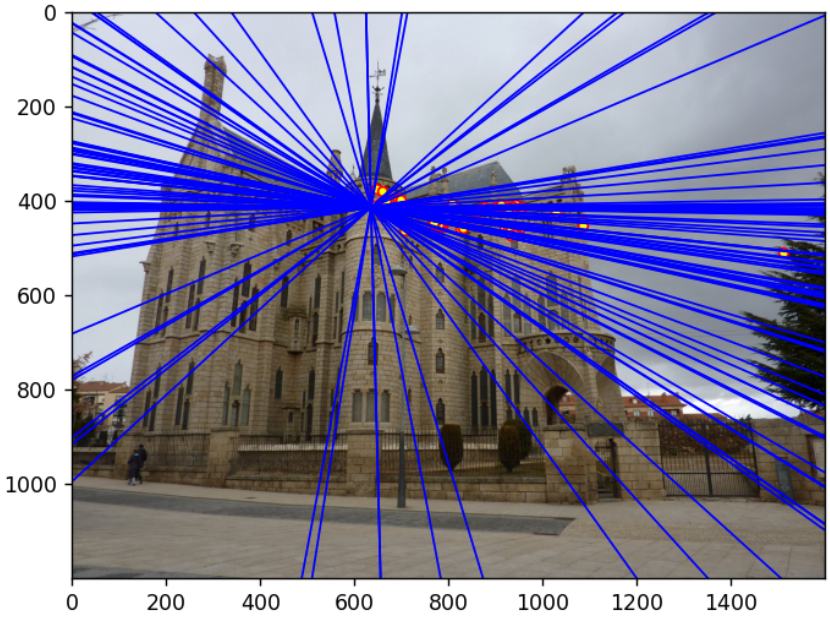
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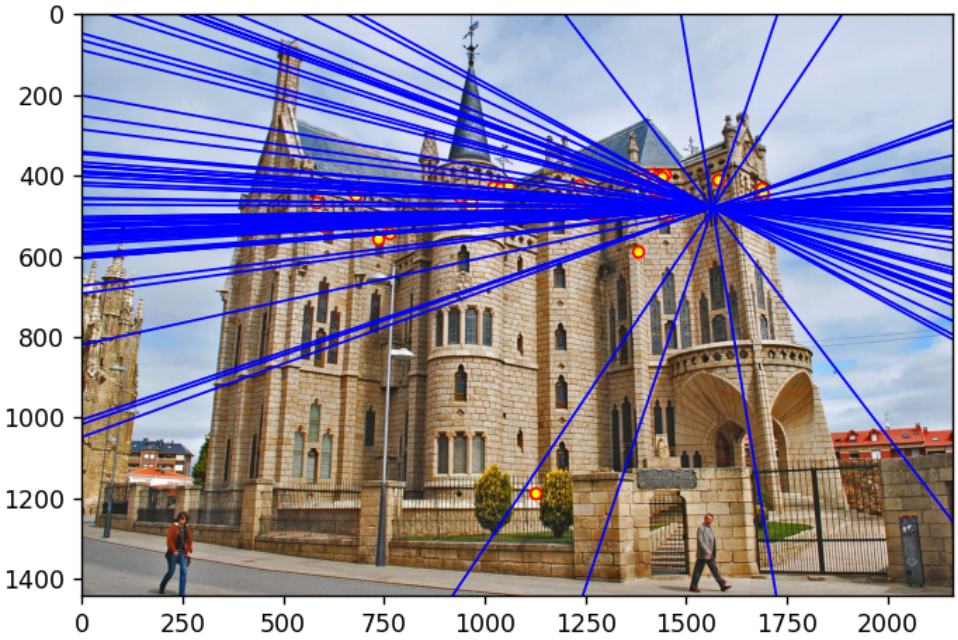
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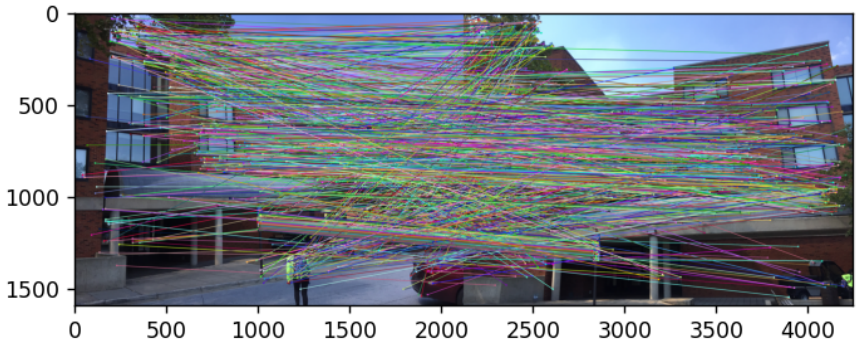
Woodruff: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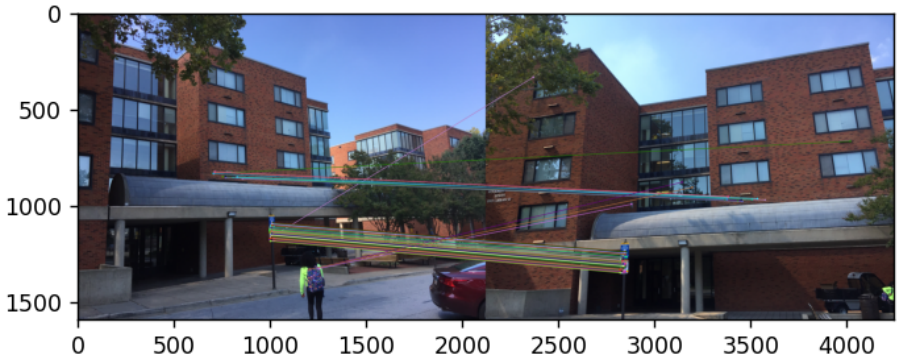
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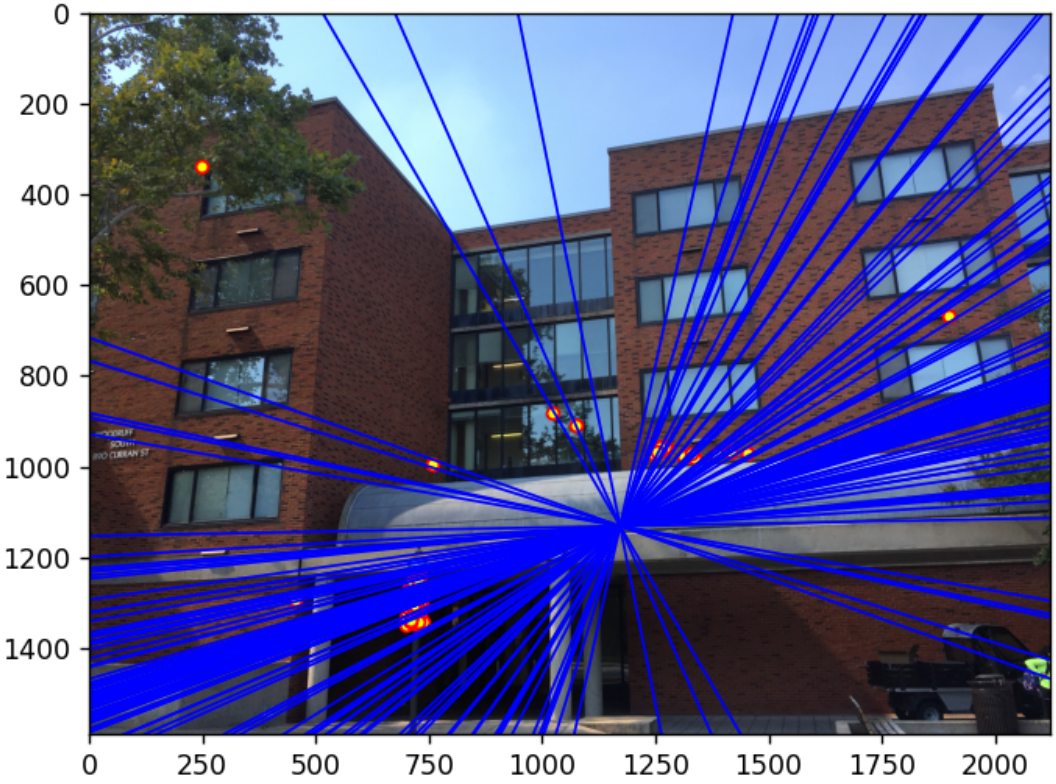
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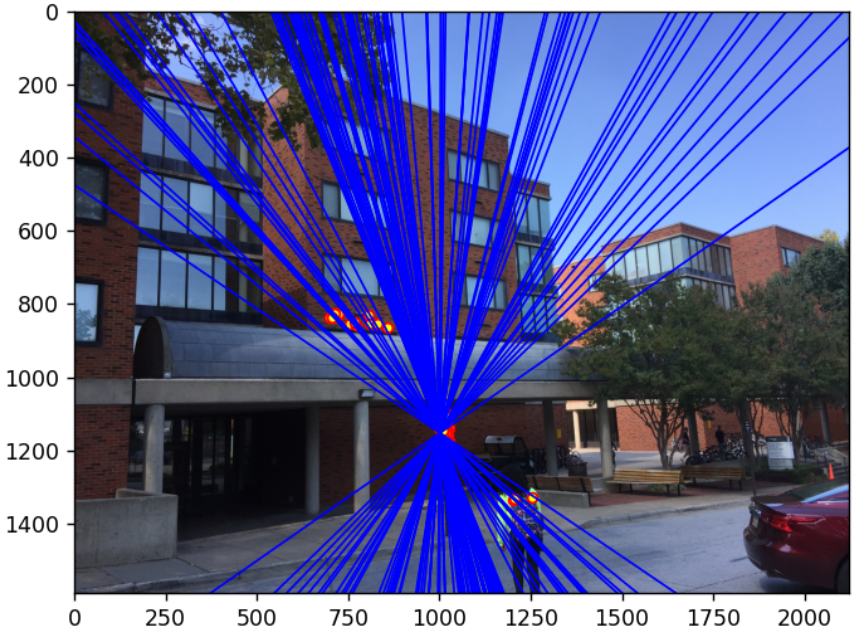
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Conclusion:
The projection matrix was calculated to map 2D points into the corresponding 3D points. For this, a fundamental matrix was calculated. The calculated fundamental matrix was also used to improve the feature matching algorithms. The RANSAC algorithm was used for the noisy data.