



$$\begin{aligned} \text{net}H_1 &= w_1x_1 + w_2x_2 + b_1 \\ &= 0.149 \times 0.05 + 0.1995 \times 0.1 + 0.35 \\ &= 0.37744 \end{aligned}$$

$$\begin{aligned} \text{net}H_2 &= w_3x_1 + w_4x_2 + b \\ &= 0.2497 \times 0.05 + 0.2995 \times 0.1 + 0.35 \\ &= 0.3924 \end{aligned}$$

$$\text{out}H_1 = \frac{1}{1 + e^{-\text{net}H_1}} = 0.5932$$

$$\text{out}H_2 = \frac{1}{1 + e^{-\text{net}H_2}} = 0.59686$$

$$\begin{aligned} \text{net}y_1 &= w_5 \times H_1 + w_6 \times H_2 + b_2 \\ &= 0.3589 \times 0.5932 + 0.40866 \times 0.59686 + 0.6 \\ &= 1.05684 \end{aligned}$$

$$\text{out}y_1 = \frac{1}{1 + e^{-\text{net}y_1}} = 0.742088$$

$$\begin{aligned} \text{net}y_2 &= w_7 \times H_1 + w_8 \times H_2 + b_2 \\ &= 0.5113 \times 0.5932 + 0.5613 \times 0.59686 + 0.6 \\ &= 1.23839 \end{aligned}$$

$$\text{out}y_2 = \frac{1}{1 + e^{-\text{net}y_2}} = 0.77928$$

$$E_{\text{total}} = E_{y1} + E_{y2}$$

$$= \frac{1}{2} (\text{tary}_1 - \text{outy}_1)^2 + \frac{1}{2} (\text{tary}_2 - \text{outy}_2)^2$$

$$= \frac{1}{2} (0.01 - 0.7420)^2 + \frac{1}{2} (0.99 - 0.7752)^2$$

$$E_{\text{tot}} = 0.29102$$

\* Backpropagation.

i)  $\Delta w_5$

$$\frac{\partial E_{\text{tot}}}{\partial w_5} = \frac{\partial E_{\text{tot}}}{\partial \text{outy}_1} \times \frac{\partial \text{outy}_1}{\partial \text{nety}_1} \times \frac{\partial \text{nety}_1}{\partial w_5}$$

$$= (\text{outy}_1 - \text{tary}_1) \times \text{outy}_1 (1 - \text{outy}_1) \times \text{outH}_1$$

$$= (0.74208 - 0.01) \times 0.74208 (1 - 0.74208) \times 0.593256$$

$$= 0.083125$$

ii)  $\Delta w_6$

$$\frac{\partial E_{\text{tot}}}{\partial w_6} = \frac{\partial E_{\text{tot}}}{\partial \text{outy}_1} \times \frac{\partial \text{outy}_1}{\partial \text{nety}_1} \times \frac{\partial \text{nety}_1}{\partial w_6}$$

$$= (0.74208 - 0.01) \times 0.74208 (1 - 0.74208) \times 0.596869$$

$$= 0.08363$$

iii)  $\Delta w_7$

$$\frac{\partial E_{\text{tot}}}{\partial w_7} = \frac{\partial E_{\text{tot}}}{\partial \text{outy}_2} \times \frac{\partial \text{outy}_2}{\partial \text{nety}_2} \times \frac{\partial \text{nety}_2}{\partial w_7}$$

$$= (\text{outy}_2 - \text{tary}_2) \times \text{outy}_2 (1 - \text{outy}_2) \times \text{outH}_1$$

$$= (0.7752 - 0.99) \times (0.7752) (1 - 0.7752) \times 0.593256$$

$$= -0.0221921$$



iv)  $\Delta \omega_8$

$$\frac{\partial \epsilon_{tot}}{\partial \omega_8} = \frac{\partial \epsilon_{tot}}{\partial out_{y2}} \times \frac{\partial out_{y2}}{\partial net_{y2}} \times \frac{\partial net_{y2}}{\partial \omega_8}$$

$$= (0.7752 - 0.99) \times (0.7752)(1 - 0.7752) \times 0.596869$$

$$= -0.022327$$

v)  $\Delta \omega_1$

$$\frac{\partial \epsilon_{tot}}{\partial \omega_1} = \frac{\partial \epsilon_{tot}}{\partial out_{H1}} \times \frac{\partial out_{H1}}{\partial net_{H1}} \times \frac{\partial net_{H1}}{\partial \omega_1}$$

$$\frac{\partial \epsilon_{tot}}{\partial \omega_1} = \left[ \frac{\partial \epsilon_{y1}}{\partial out_{H1}} + \frac{\partial \epsilon_{y2}}{\partial out_{H1}} \right] \times \frac{\partial out_{H1}}{\partial net_{H1}} \times \frac{\partial net_{H1}}{\partial \omega_1}$$

$$\Rightarrow \frac{\partial \epsilon_{y1}}{\partial out_{H1}} = \frac{\partial \epsilon_{y1}}{\partial out_{y1}} \times \frac{\partial out_{y1}}{\partial net_{y1}} \times \frac{\partial net_{y1}}{\partial out_{H1}}$$

$$= (0.74208 - 0.01) \times 0.74208 (1 - 0.74208) \times 0.3589$$

$$= 0.05029$$

$$\Rightarrow \frac{\partial \epsilon_{y2}}{\partial out_{H1}} = \frac{\partial \epsilon_{y2}}{\partial out_{y2}} \times \frac{\partial out_{y2}}{\partial net_{y2}} \times \frac{\partial net_{y2}}{\partial out_{H1}} = [0.7752 - 0.99] \times 0.7752 (1 - 0.7752) \times 0.5113$$

$$= (0.74208 - 0.99) \times 0.74208 (1 - 0.74208) \times 0.40566$$

$$= -0.01912$$

$$\frac{\partial \epsilon_{tot}}{\partial \omega_1} = [0.05029 - 0.01912] \times 0.5932 (1 - 0.5932) \times 0.05$$

$$= 0.031163 \times 0.5932 (1 - 0.5932) \times 0.05$$

$$\frac{\partial \epsilon_{tot}}{\partial \omega_1} = 0.00037$$

vi)  $\Delta \omega_2$

$$\frac{\partial \epsilon_{tot}}{\partial \omega_2} = \frac{\partial \epsilon_{tot}}{\partial out_{H1}} \times \frac{\partial out_{H1}}{\partial net_{H1}} \times \frac{\partial net_{H1}}{\partial \omega_2} = [0.05029 - 0.01912] \times 0.5932 (1 - 0.5932) \times 0.1$$

$$= 0.000761$$



vi)  $\Delta w_3$

$$\frac{\partial \epsilon_{tot}}{\partial w_3} = \frac{\partial \epsilon_{tot}}{\partial out_{h2}} \times \frac{\partial out_{h2}}{\partial net_{h2}} \times \frac{\partial net_{h2}}{\partial w_3}$$

$$\begin{aligned} \frac{\partial \epsilon_{y1}}{\partial out_{h2}} &= \frac{\partial \epsilon_{tot}}{\partial out_{y1}} \times \frac{\partial out_{y1}}{\partial net_{y1}} \times \frac{\partial net_{y1}}{\partial out_{h2}} \\ &= (0.7752 - 0.99) \times (0.7752)(1 - 0.7752) \times 0. \\ &= (0.742088 - 0.01) \times (0.742088)(1 - 0.742088) \times 0.40866 \\ &= 0.057260 \end{aligned}$$

$$\begin{aligned} \frac{\partial \epsilon_{y2}}{\partial out_{h2}} &= \frac{\partial \epsilon_{tot}}{\partial out_{y2}} \times \frac{\partial out_{y2}}{\partial net_{y2}} \times \frac{\partial net_{y2}}{\partial out_{h2}} \\ &= (0.7752 - 0.99) \times 0.7752(1 - 0.7752) \times 0.5613 \\ &= -0.020999 \end{aligned}$$

$$\begin{aligned} \frac{\partial \epsilon_{tot}}{\partial w_3} &= [0.05726 - 0.02099] \times 0.59688(1 - 0.59688) \times 0.05 \\ &= 0.000436 \end{aligned}$$

vii)  $\Delta w_4$

$$\begin{aligned} \frac{\partial \epsilon_{tot}}{\partial w_4} &= \frac{\partial \epsilon_{tot}}{\partial out_{h2}} \times \frac{\partial out_{h2}}{\partial net_{h2}} \times \frac{\partial net_{h2}}{\partial w_4} \\ &= (0.7752 - 0.99) \times 0.7752(1 - 0.7752) \times \\ &= [0.05726 - 0.02099] \times 0.59688(1 - 0.59688) \times 0.01 = 0.0008830 \end{aligned}$$

New  $w_i$

$$\Delta w_1 = w_1 - \eta \times \frac{\partial \epsilon}{\partial w_1} = 0.1497 - 0.5 \times 0.0003759 = 0.149592$$

$$\Delta w_2 = w_2 - \eta \times \frac{\partial \epsilon}{\partial w_2} = 0.19955 - 0.5 \times 0.000761 = 0.199175$$

$$\Delta w_3 = w_3 - \eta \times \frac{\partial \epsilon}{\partial w_3} = 0.2495 - 0.5 \times 0.00043 = 0.24953$$

$$\Delta w_4 = w_4 - \eta \times \frac{\partial \epsilon}{\partial w_4} = 0.2995 - 0.5 \times 0.00088 = 0.299054$$

$$\Delta w_5 = w_5 - \eta \times \frac{\partial \epsilon}{\partial w_5} = 0.3589 - 0.5 \times 0.083121 = 0.317353$$

$$\Delta w_6 = w_6 - \eta \times \frac{\partial \epsilon}{\partial w_6} = 0.40866 - 0.5 \times 0.083631 = 0.3668$$

$$\Delta w_7 = w_7 - \eta \times \frac{\partial \epsilon}{\partial w_7} = 0.5113 - 0.5 \times (-0.02219) = 0.5223$$

$$\Delta w_8 = w_8 - \eta \times \frac{\partial \epsilon}{\partial w_8} = 0.5613 - 0.5 \times (-0.02232) = 0.5725$$

THE END